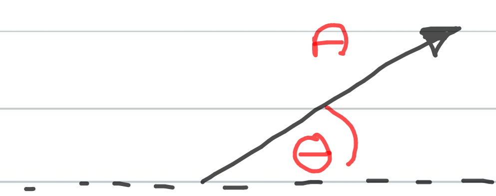
جامعة قطر
QATAR UNIVERSITY

2 Chapter

Force VECTORS

Vectors → Magnitude (A) / direction (θ)



Ex. Force & Velocity

Scalars } only Quantity (+ / −), No-direction

Ex: Mass & volume

$\vec{A} = \vec{B}$ } → same Magnitude / same direction

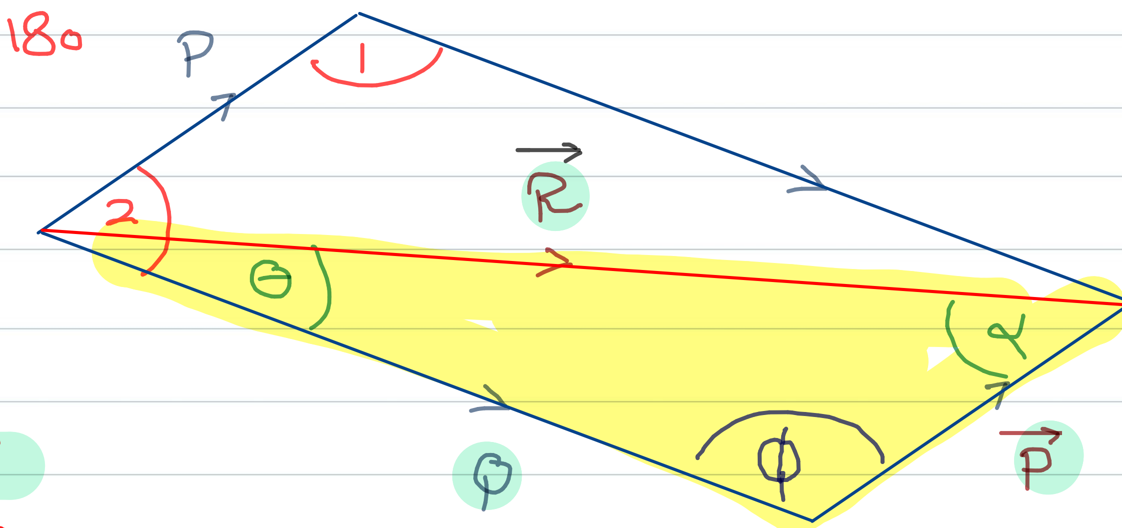
$A = 1$ $\quad B = 4$

$\vec{A} + \vec{B} = 5$

Parallelogram law } only two Forces

Trignometric Method $\vec{R} = \vec{P} + \vec{Q}$

$\hat{1} + \hat{2} = 180$



Case ①

Given $\vec{P}$, $\vec{Q}$ Required $\vec{R}$ → Mag / dir

① Conclude internal angle between $\vec{P}$ & $\vec{Q}$ (φ)

② Cos-law ⇒ Mag

$$R = \sqrt{P^2 + Q^2 - 2PQ\cos\phi}$$

③ Direction ⇒ Sin law

$$\frac{P}{\sin\theta} = \frac{Q}{\sin\alpha} = \frac{R}{\sin\phi}$$

Case ②

Given $\vec{R}$ Required $\vec{P}$, $\vec{Q}$

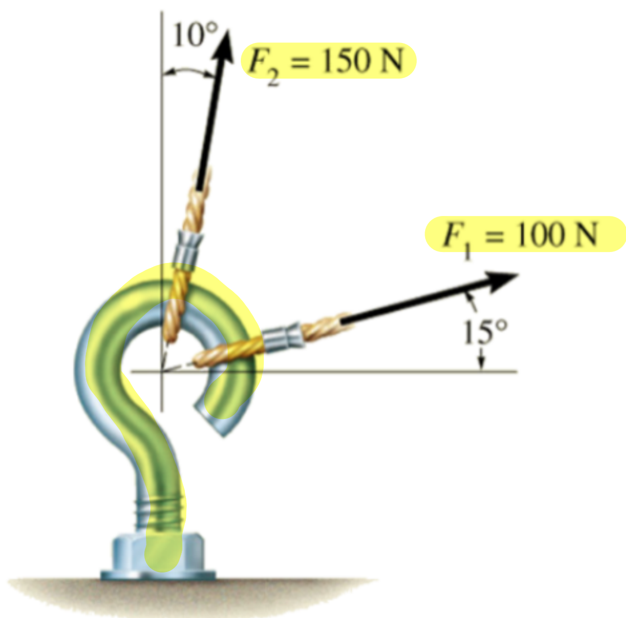
① Conclude all internal angles (θ, α & φ)

② Sin-law

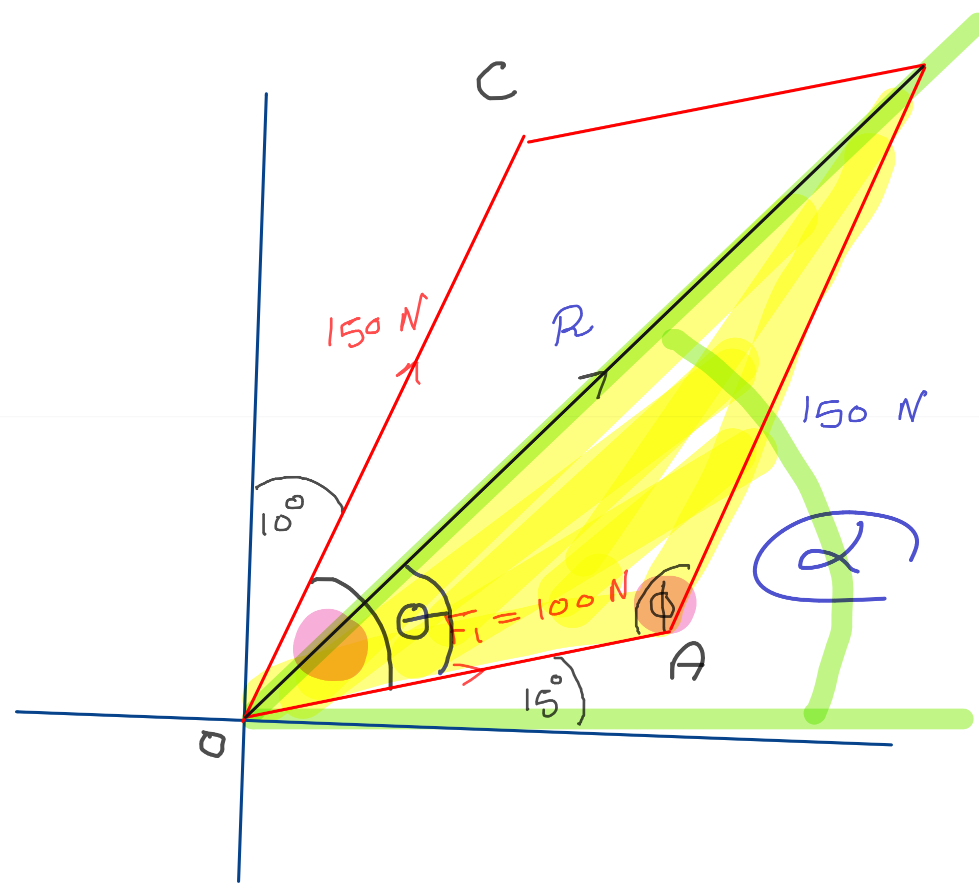
$$\frac{P}{\sin\theta} = \frac{Q}{\sin\alpha} = \frac{R}{\sin\phi}$$

EXAMPLE 2.1

The screw eye in Fig. 2–10*a* is subjected to two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$. Determine the magnitude and direction of the resultant force.



$\vec{R} = ???$



angle CoA = 90 − 15 − 10 = 65

$\phi = 180 - 65 = 115$

$$R = \sqrt{100^2 + 150^2 - 2(100)(150)\cos 115}$$

$R = 213$ (N)

Direction → sin law

$\alpha = 15 + \theta$

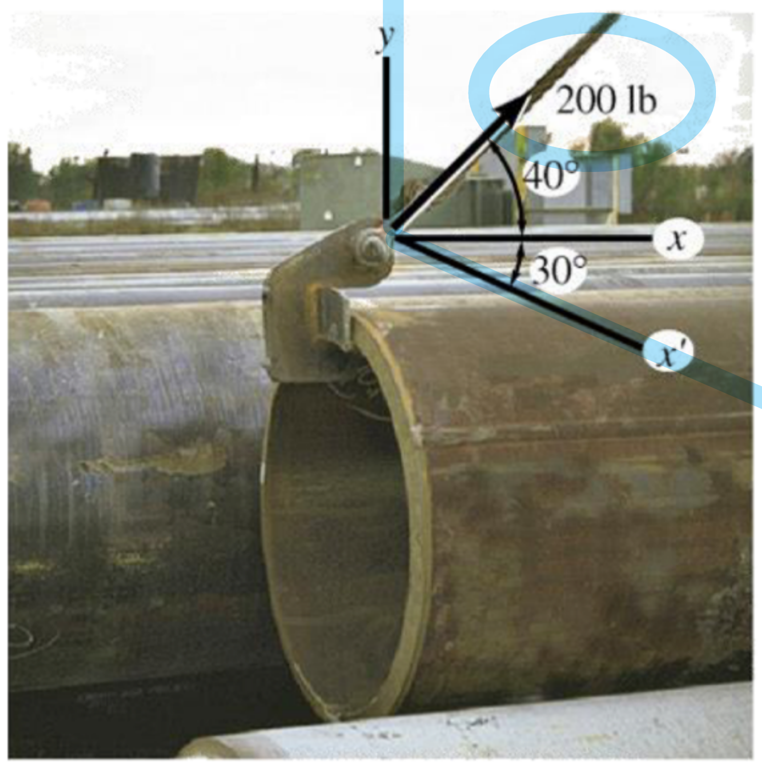
$$\frac{150}{\sin\theta} = \frac{213}{\sin 115}$$

$$\theta = \sin^{-1}\left(\frac{150 \sin 115}{213}\right) = 39.7^\circ$$

$\alpha = 15 + 39.7^\circ = 54.7^\circ$

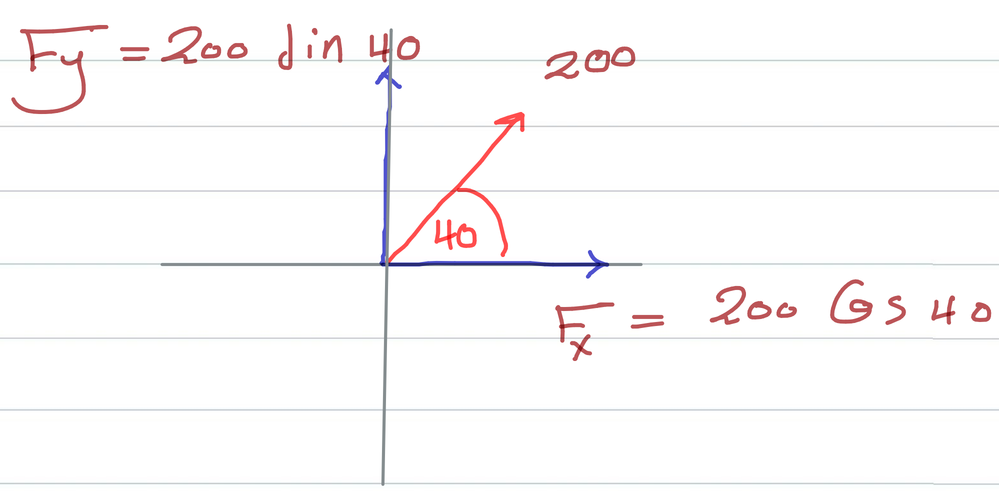
EXAMPLE 2.2

Resolve the 200-lb force acting on the pipe, Fig. 2–11*a*, into components in the (a) *x* and *y* directions, and (b) *x′* and *y* directions.



(a)

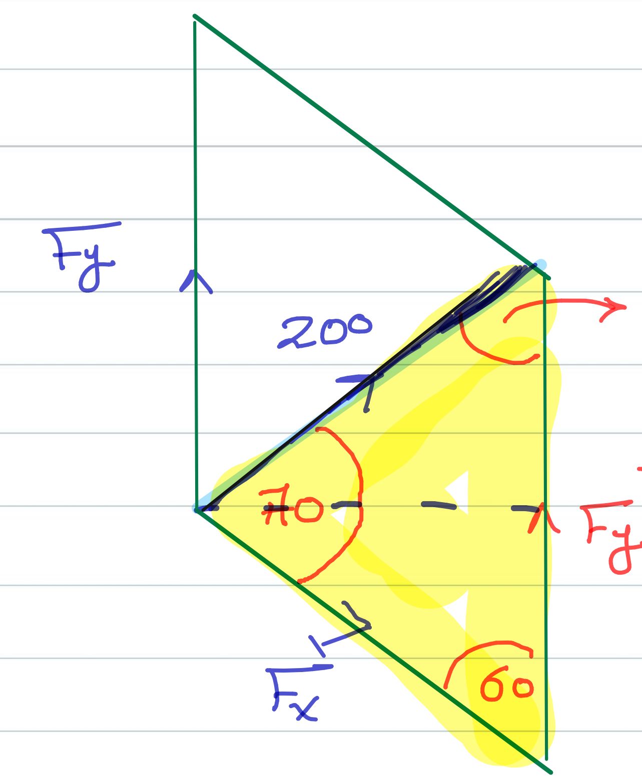
(a)



$F_x = 200 \cos 40 = 153 \text{ lb}$

$F_y = 200 \sin 40 = 129 \text{ lb}$

(b)



Sin-law :-

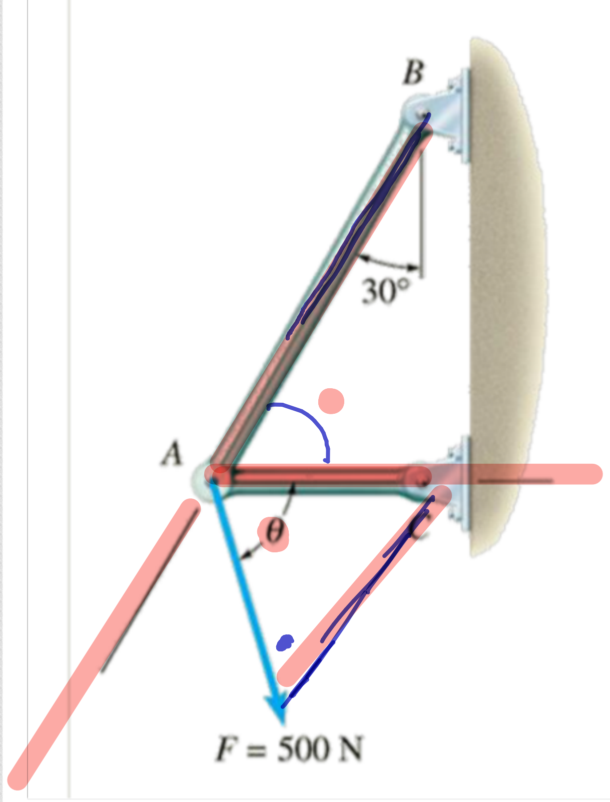
$$\frac{F_x'}{\sin 50} = \frac{F_y}{\sin 70} = \frac{200}{\sin 60}$$

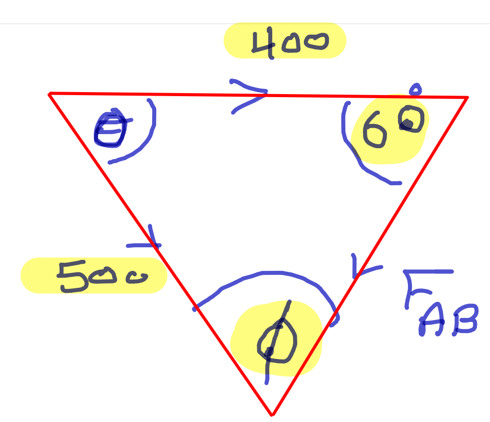
$$F_x' = \frac{200 \sin 50}{\sin 60} = 177 \text{ lb}$$

$$F_y = \frac{200 \sin 70}{\sin 60} = 217 \text{ lb}$$

EXAMPLE 2.3

The force **F** acting on the frame shown in Fig. 2–12*a* has a magnitude of 500 N and is to be resolved into two components acting along members *AB* and *AC*. Determine the angle θ, measured *below* the horizontal, so that the component $\mathbf{F}_{AC}$ is directed from *A* toward *C* and has a magnitude of 400 N.





$$\frac{400}{\sin \phi} = \frac{500}{\sin 60}$$

$$\sin \phi = \frac{400 \sin 60}{500}$$

$$\phi = 43.9^\circ$$

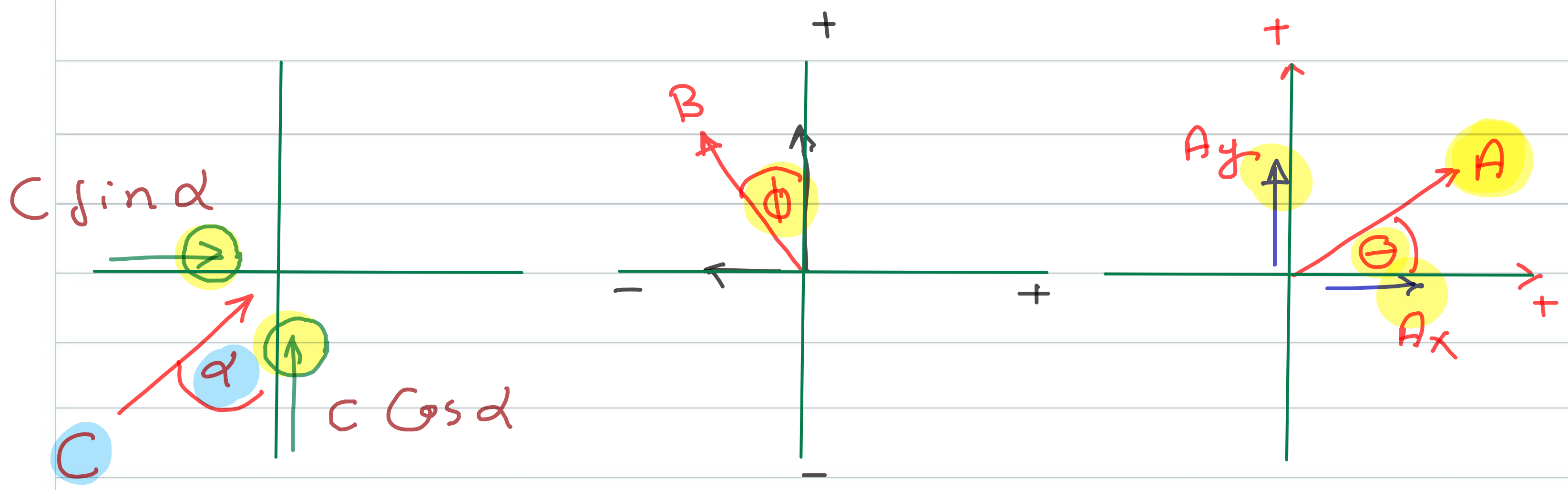
$$\theta = 180 - 60 - 43.9$$

$$= 76.1^\circ$$

$$F_{AB} = \sqrt{400^2 + 500^2 - 2(400)(500)\cos 76.1}$$

$$= 561 \text{ N}$$

Components



$$C_x = C \sin\alpha$$

$$C_y = C \cos\alpha$$

$$B_x = -B \sin\phi$$

$$B_y = +B \cos\phi$$

$$B = B_x \hat{i} + B_y \hat{j}$$

$$A_x = A \cos\theta$$

$$A_y = A \sin\theta$$

$$A = \sqrt{A_x^2 + A_y^2}$$

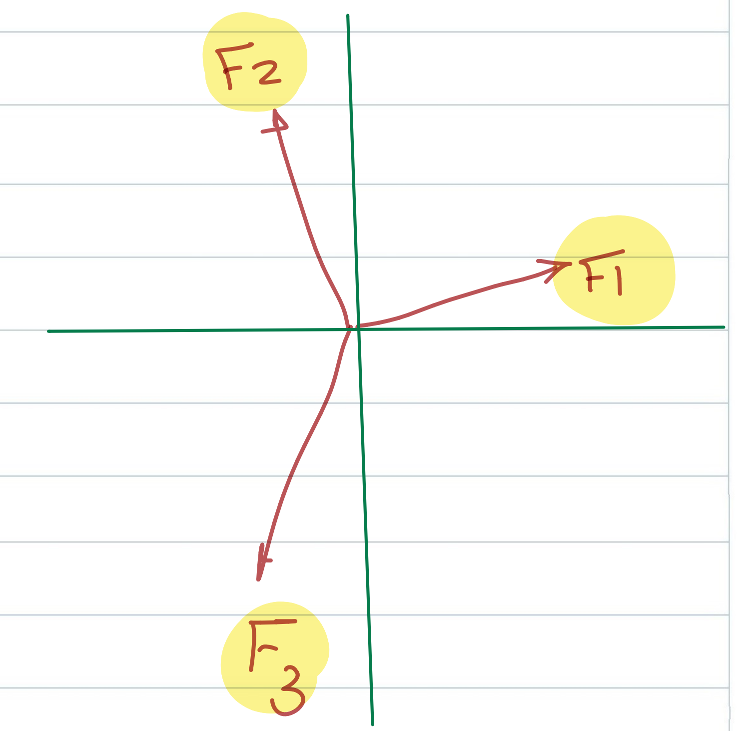
$$\theta = \tan^{-1} \frac{A_y}{A_x}$$

Coplanar Force Resultants

(1) Resolve

F_{1x} F_{2x} F_{3x}

F_{1y} F_{2y} F_{3y}



(2) $R_x = \Sigma F_x$ $\xrightarrow{+}$

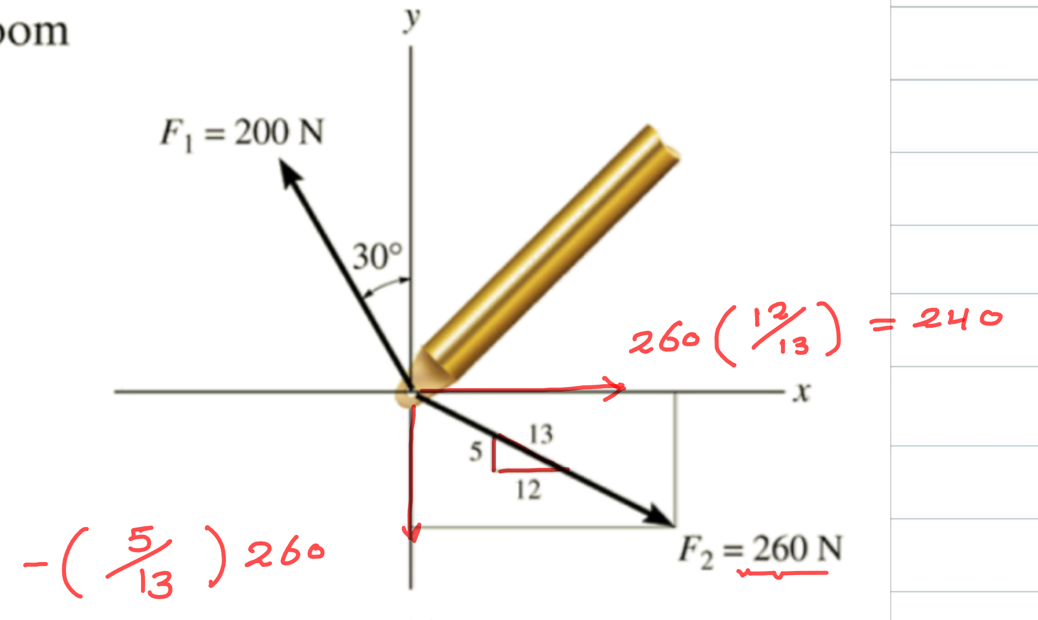
$R_y = \Sigma F_y$ $\uparrow +$

(3) $R = \sqrt{R_x^2 + R_y^2}$

(4) $\theta = \tan^{-1} \frac{R_y}{R_x}$

EXAMPLE 2.5

Determine the x and y components of $\mathbf{F}_1$ and $\mathbf{F}_2$ acting on the boom shown in Fig. 2–17a. Express each force as a Cartesian vector.



(a)

$F_{1x} = -200 \sin 30 = -100$

$F_{1y} = 200 \cos 30 = 173$

$\vec{F_1} = (-100)\hat{i} + (173)\hat{j}$

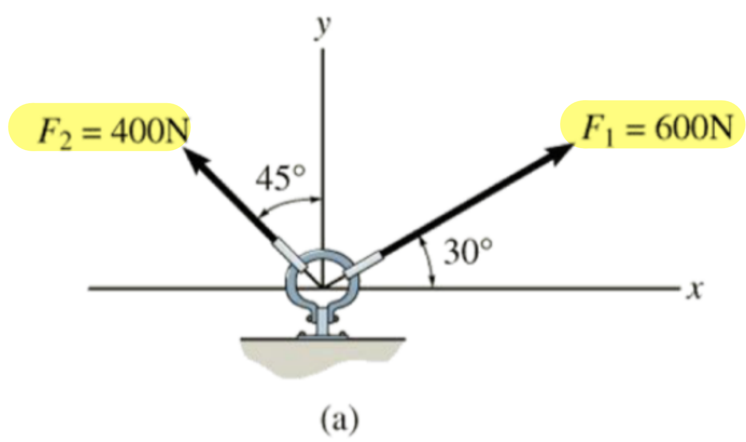
$F_{2x} = 240$

$F_{2y} = -100$

$F_2 = (240)i + (-100)\hat{j}$

EXAMPLE 2.6

The link in Fig. 2–18*a* is subjected to two forces $\mathbf{F}_1$ and $\mathbf{F}_2$. Determine the magnitude and orientation of the resultant force.



(a)

* $F_{1x} = 600 \cos 30 = 519.6$

$F_{1y} = 600 \sin 30 = 300$

$F_{2x} = -400 \sin 45 = -282.8$

$F_{2y} = 400 \cos 45 = 282.8$

* $R_x = \Sigma F_x \quad \xrightarrow{+}$

$= 519.6 - 282.8 = 236.8$

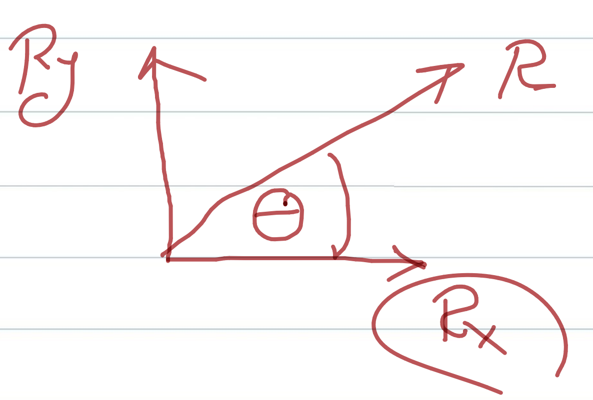
$R_y = 300 + 282.8 = 582.8 \text{ N}$

* $R = \sqrt{R_x^2 + R_y^2}$

$= \sqrt{(236.8)^2 + (582.8)^2}$

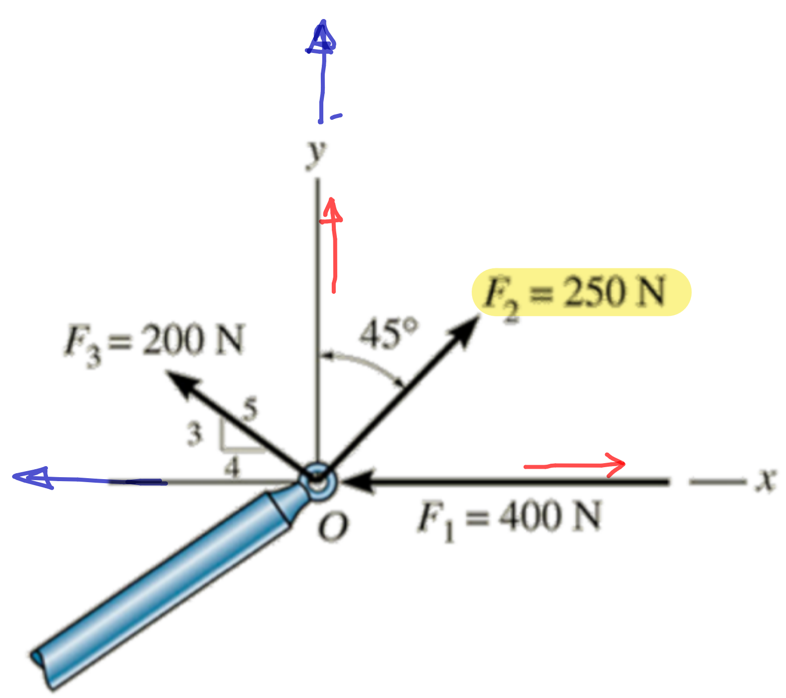
$= 629.1 \text{ N}$

* $\theta = \tan^{-1} \dfrac{582.8}{236.8} = 67.9°$



EXAMPLE 2.7

The end of the boom O in Fig. 2–19a is subjected to three concurrent and coplanar forces. Determine the magnitude and orientation of the resultant force.



(a)

* $F_{1x} = -400$

$F_{1y} = 0$

$F_{2x} = 250 \sin 45 = 177$ N

$F_{2y} = 250 \cos 45 = 177$ N

$F_{3x} = 200\ (4/5) = -160$

$F_{3y} = 200\ (3/5) = 120$

* $F_{Rx} = \Sigma F_x \quad \xrightarrow{+}$

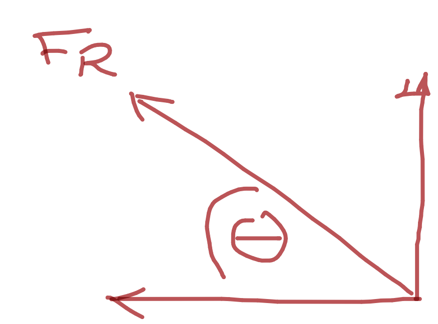
$= -400 + 177 - 160 = -383$

$F_{Ry} = \Sigma F_y \quad \uparrow +$

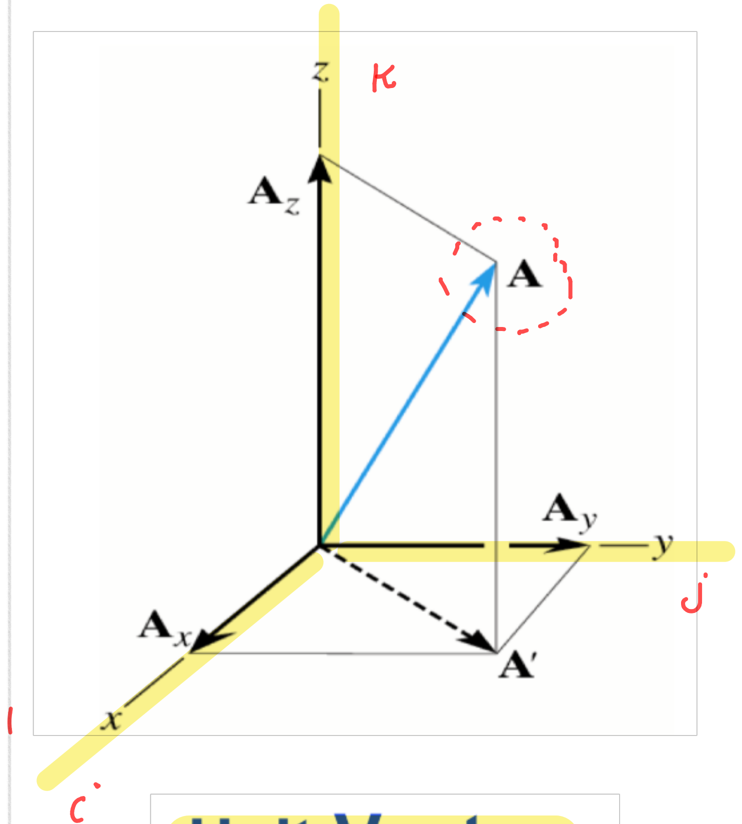
$= 177 + 120 = 297$ N

* $F_R = \sqrt{(-383)^2 + (297)^2} = 485$

* $\theta = \tan^{-1} \dfrac{297}{-383} = -37.8^\circ$



2.7. Cartesian Vectors



Unit Vectors in Coordinate Directions:

$\vec{i}$, $\hat{i}$: Unit vector in the x-direction

$\vec{j}$, $\hat{j}$: Unit vector in the y-direction

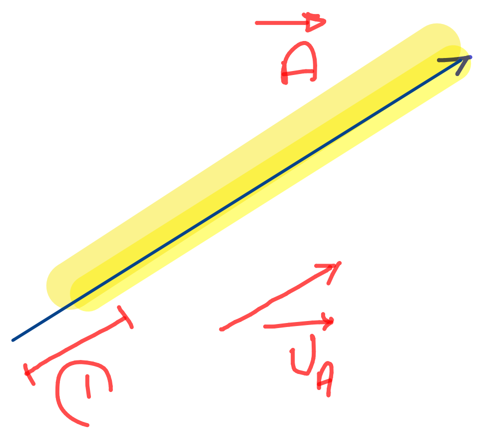
$\vec{k}$, $\hat{k}$: Unit vector in the z-direction

$$A = A_x\,\hat{i} + A_y\,\hat{j} + A_z\,\hat{k}$$

Unit Vectors

$$\vec{U}_A = \frac{\vec{A}}{|A|}$$

$$\vec{A} = |\vec{A}|\,\vec{U}_A$$



Magnitude

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

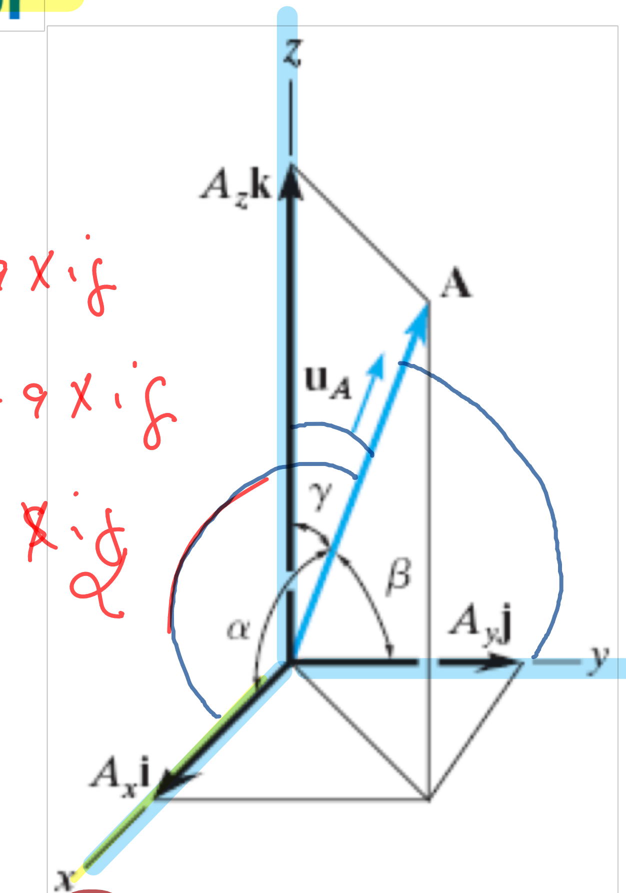
Direction of a Cartesian Vector

Direction angles :-

α angle with + x-axis

β angle with + y-axis

γ angle with + z axis



Direction Cosines of A is

$$\boxed{\cos\alpha} = \frac{A_x}{A}$$

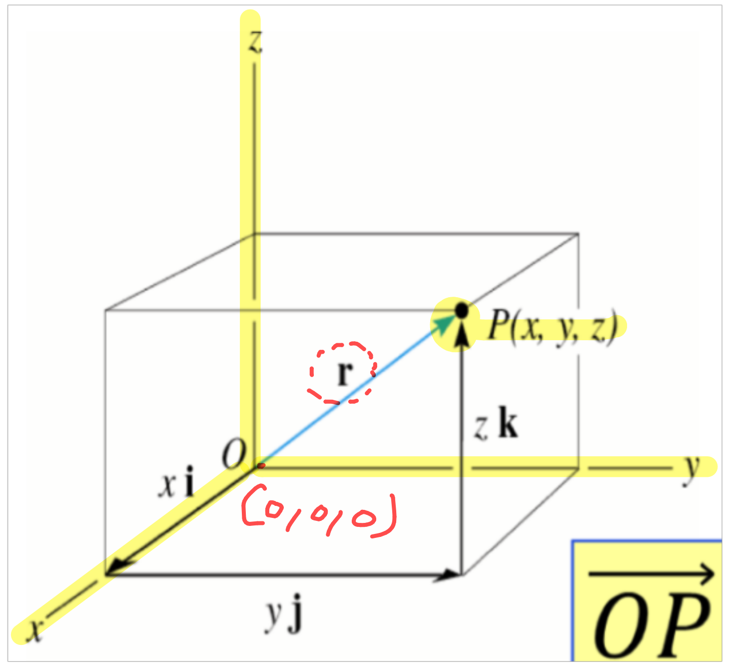
$$\boxed{\cos\beta} = \frac{A_y}{A}$$

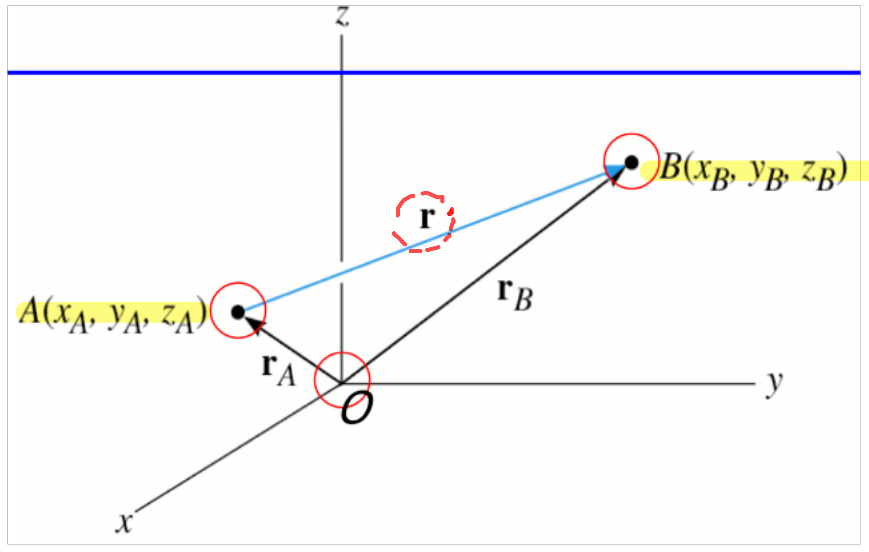
$$\boxed{\cos\gamma} = \frac{A_z}{A}$$

2.9. Coordinates of Relative Position Vectors

$$\vec{r} = \vec{OP}$$

$$= x\hat{i} + y\hat{j} + z\hat{k}$$





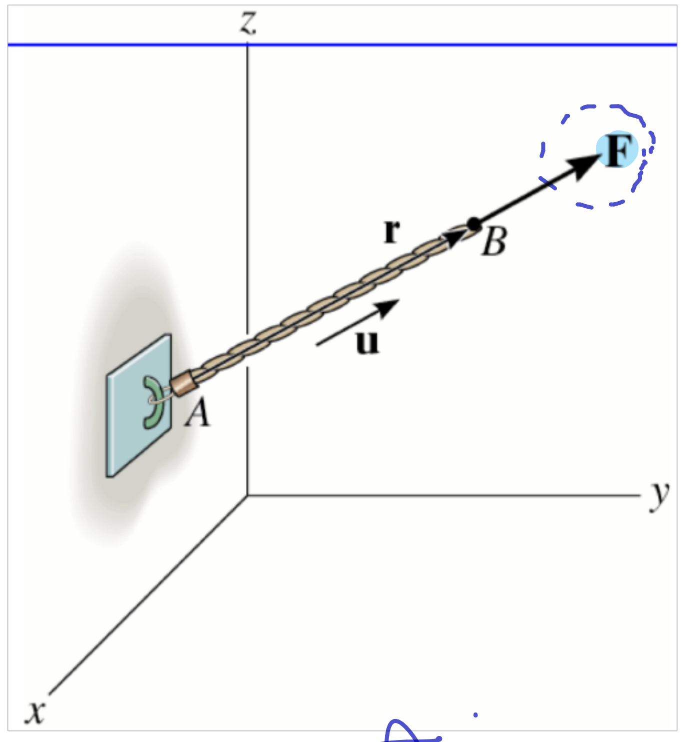
$$\vec{r}_{AB} = \vec{r}_B - \vec{r}_A$$

$$= (x_B - x_A)\hat{i} + (y_B - y_A)\hat{j} + (z_B - z_A)\hat{k}$$

2. 10. Force Along a Line

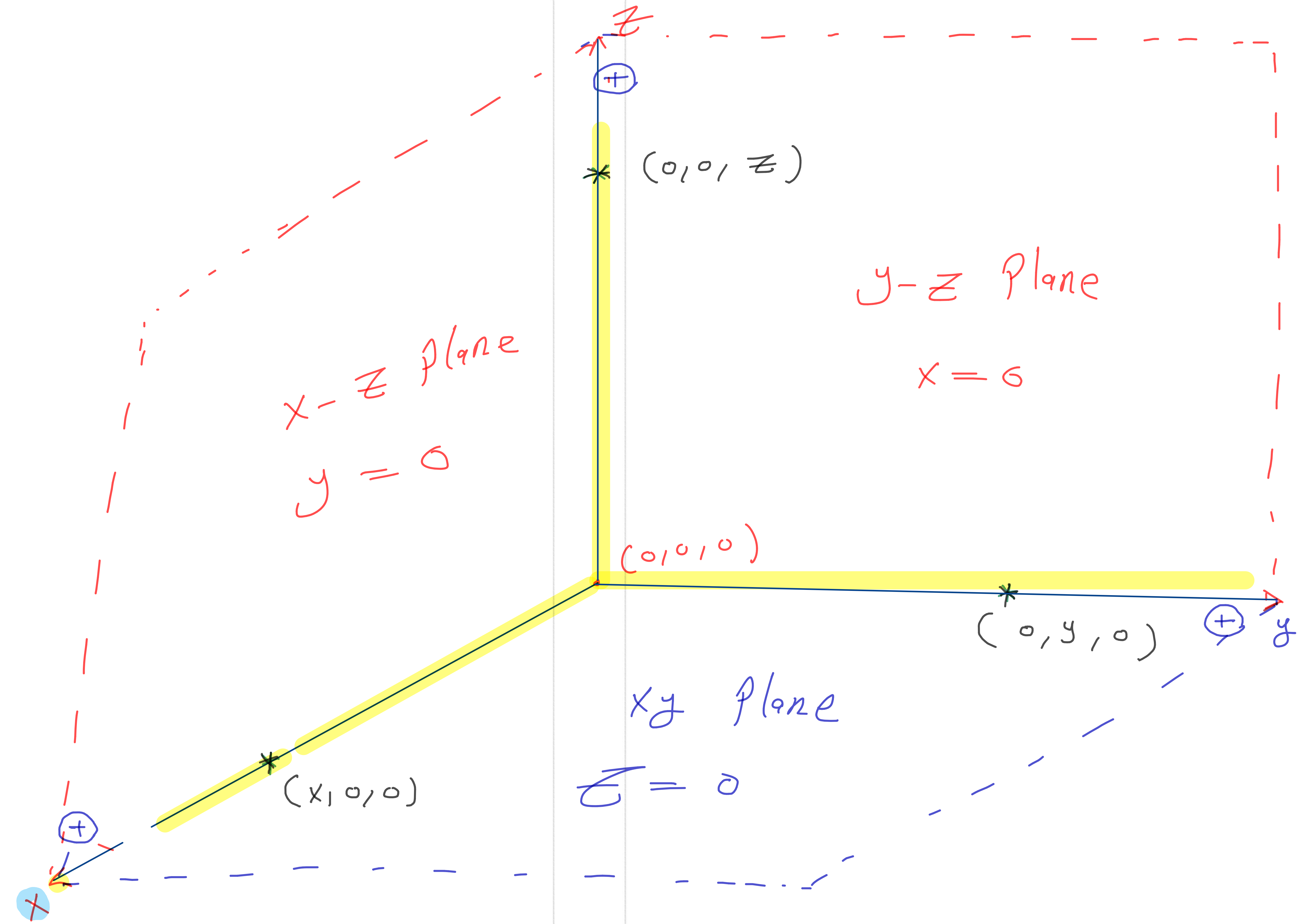
$$\vec{F} = F\,\vec{U}_{AB}$$

$$= F\,\frac{\vec{AB}}{|\vec{AB}|}$$

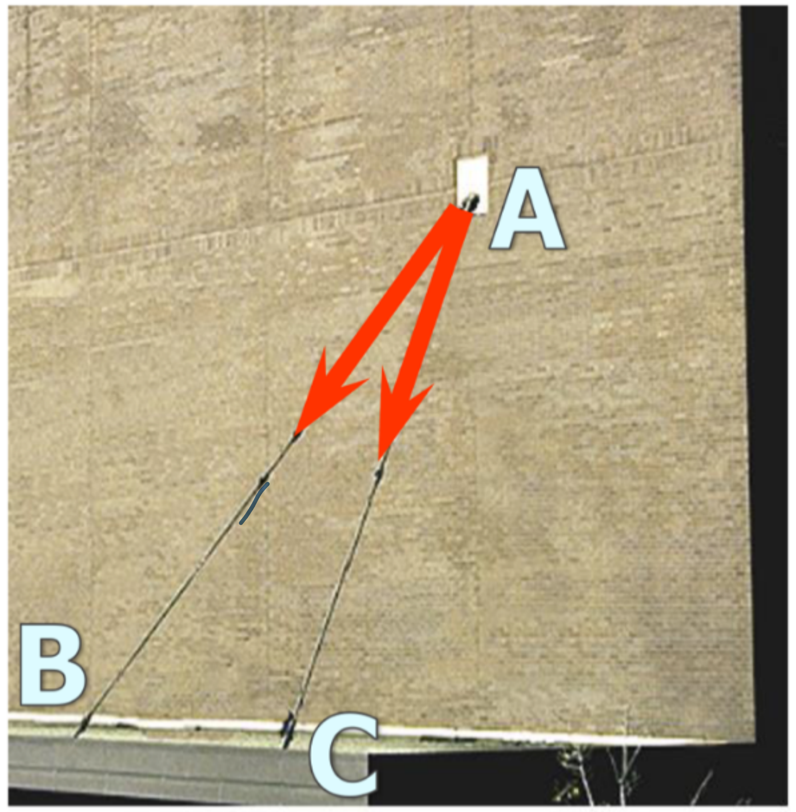


Direction of Force = Direction of Cable

Unit vector of Force = Unit vector of Cable


z
+
(0, 0, z)
y-z Plane
x = 0
x-z plane
y = 0
(0, 0, 0)
(0, y, 0)
+
y
xy Plane
z = 0
(x, 0, 0)
+
x

Example



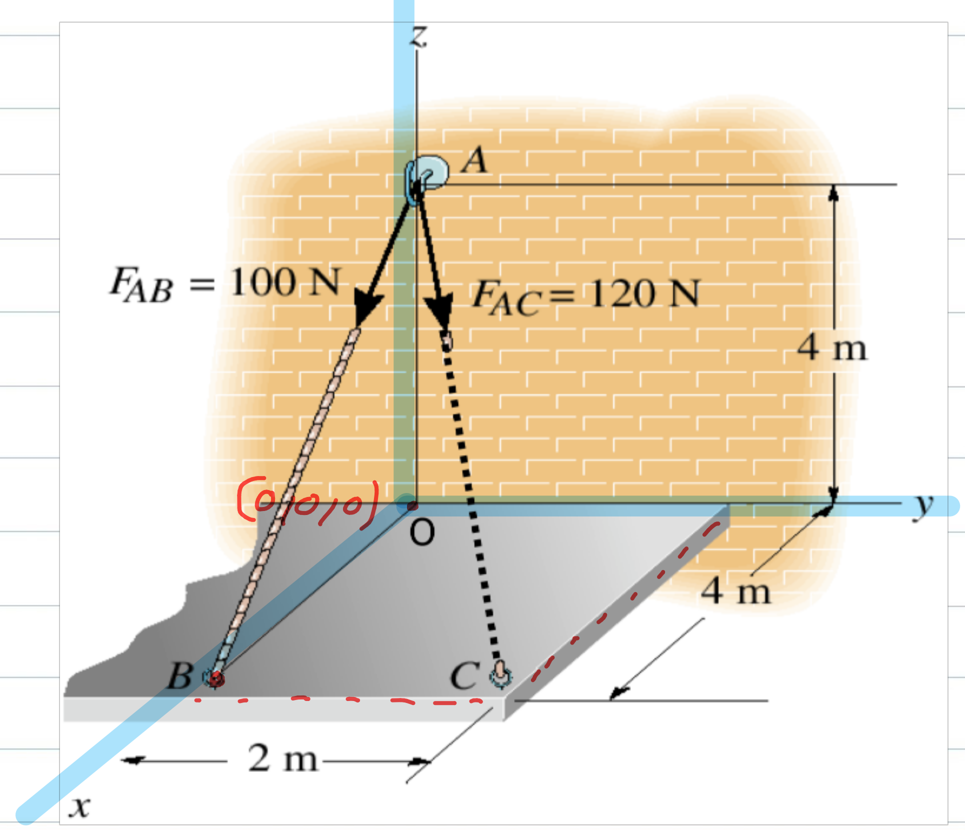
The roof is supported as shown. If the cables exert forces of **F_{AB} = 100 N** and **F_{AC} =120 N** on the wall hook at A, determine the magnitude of the resultant force acting at A.

Step (A) Identify the absolute coordinates of all points (x, y, z)

$A = (0, 0, 4)$

$B = (4, 0, 0)$

$C = (4, 2, 0)$



Step (B) Identify the absolute position vectors

$$\vec{r}_A = 0\,\hat{i} + 0\,\hat{j} + 4\,k$$

$$\vec{r}_B = 4\,i + 0\,j + 0\,K$$

$$r_C = 4\,i + 2\,j + 0\,K$$

Step (C) Identify the position vectors of the mechanical elements

$$\vec{AB} = \vec{r}_B - \vec{r}_A = (4)\,i + (0)\,j + (-4)\,k$$

$$|\vec{AB}| = \sqrt{4^2 + (-4)^2} = 5.66$$

$$\vec{U}_{AB} = \frac{\vec{AB}}{|AB|} = \frac{4i + 0j - 4k}{5.66}$$

$$= \frac{4}{5.66}\,\hat{i} - \frac{4}{5.66}\,k$$

$$\vec{AC} = \vec{r}_C - \vec{r}_A = 4i + 2\hat{j} - 4\hat{k}$$

$$|\vec{AC}| = \sqrt{(4)^2 + (2)^2 + (-4)^2} = 6 \text{ (N)}$$

$$\vec{U}_{AC} = \frac{\vec{AC}}{|\vec{AC}|} = \frac{4i + 2j - 4k}{6}$$

$\vec{U}_{AC} = \frac{4}{6}\, i + \frac{2}{6}\, j - \frac{4}{6}\, k$

Step (E) Identify the force vectors

$\vec{F} = F\,\vec{U}$

$\vec{F}_{AB} = 100\,\vec{U}_{AB} = 100\left(\frac{4}{5.66}\,\hat{i} - \frac{4}{5.66}\,\hat{k}\right)$

$= 70.7\,\hat{i} - 70.7\,\hat{k}$

$\vec{F}_{AC} = 120\,\vec{U}_{AC} = 120\left(\frac{4}{6}\, i + \frac{2}{6}\, j - \frac{4}{6}\, k\right)$

$= 80\,\hat{i} + 40\, j - 80\, k$

Step (F) Find the resultant force

$\vec{F}_R = \vec{F}_{AB} + \vec{F}_{AC}$

$= (70.7 + 80)\,i + (40)\,j + (-70.7 - 80)\,k$

$= 150.7\,\hat{i} + 40\,j - 150.7\,k$

Step (G) Identify the magnitude and direction of the resultant

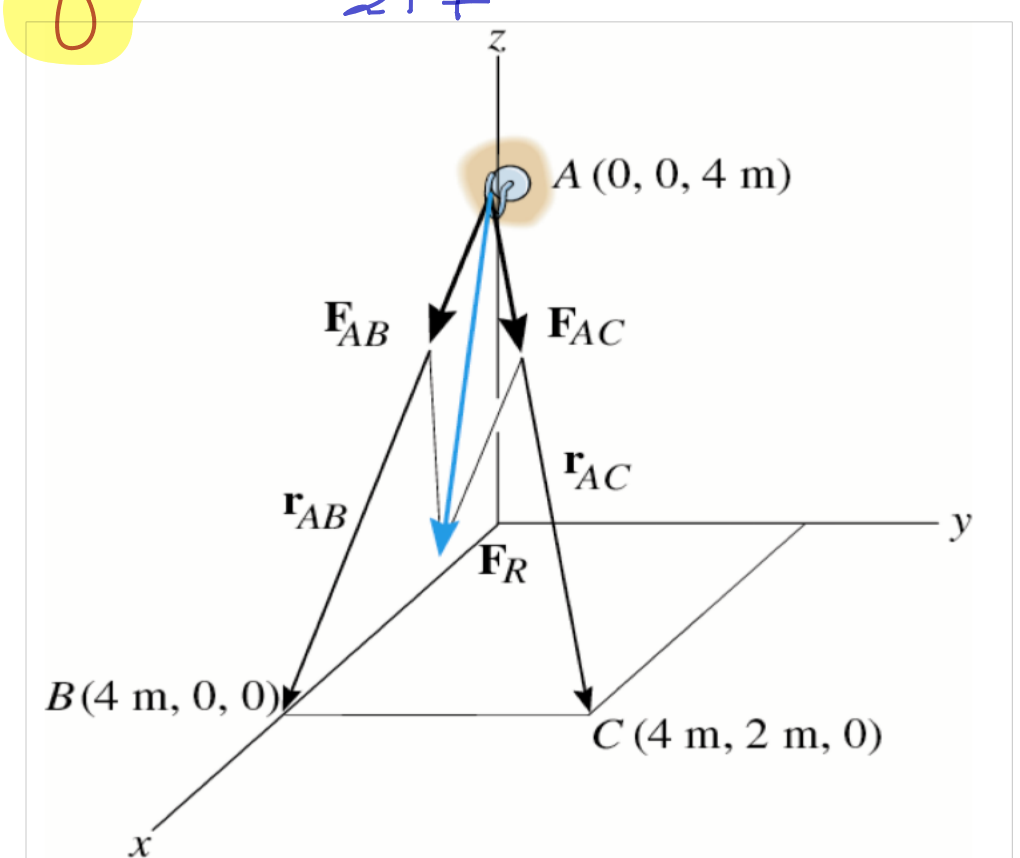
$F_R = \sqrt{(150.7)^2 + (40)^2 + (-150.7)^2}$

$= 217$ (N)

$\cos\alpha = \frac{A_x}{A} = \frac{150.7}{217} \Rightarrow \alpha =$ ✓

$\cos\beta = \frac{A_y}{A} = \frac{40}{217} \rightarrow \beta =$ ✓

$\cos\gamma = \frac{-150.7}{217} \rightarrow \gamma =$ ✓

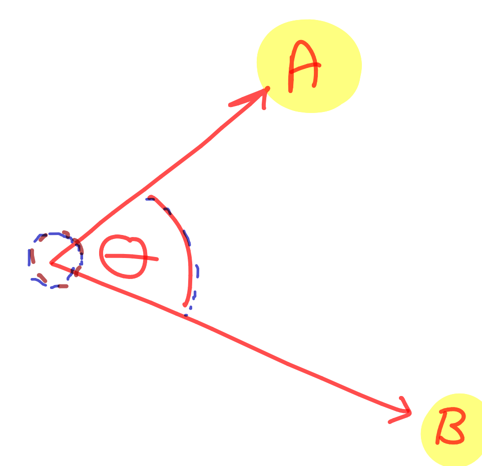


Dot - Product (Scalar) :-

(1) $\vec{A}\cdot\vec{B} = A\,B\,\cos\theta$

$\vec{A} = A_x\,\hat{i} + A_y\,\hat{j} + A_z\,\hat{k}$

$\vec{B} = B_x\,\hat{i} + B_y\,\hat{j} + B_z\,\hat{k}$



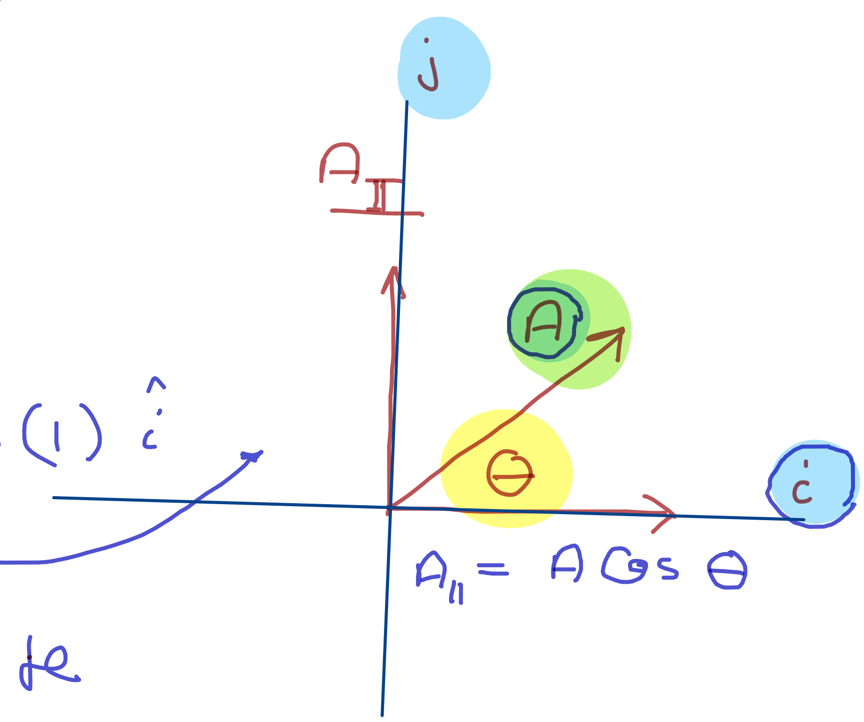
$\vec{A}\cdot\vec{B} = (A_x\,B_x) + (A_y\,B_y) + (A_z\,B_z)$

application (1)

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{A\,B}$$

$$\theta = \cos^{-1}\frac{\vec{A}\cdot\vec{B}}{A\,B}$$

application (2)



$A_{\parallel} = \vec{A}\cdot\hat{i}$

$= (A\cos\theta\,\hat{i} + A\sin\theta\,\hat{j})\cdot(1)\,\hat{i}$

$= A\cos\theta$ Magnitude

$\vec{A}_{\parallel} = A_{\parallel}\,\vec{U}$

$\vec{A} = \vec{A}_{\parallel} + \vec{A}_{\perp}$

$\vec{A}_{\perp} = \vec{A} - \vec{A}_{\parallel}$

EXAMPLE 2.16

The frame shown in Fig. 2–43*a* is subjected to a horizontal force **F** = {300**j**} N. Determine the magnitude of the components of this force parallel and perpendicular to member *AB*.

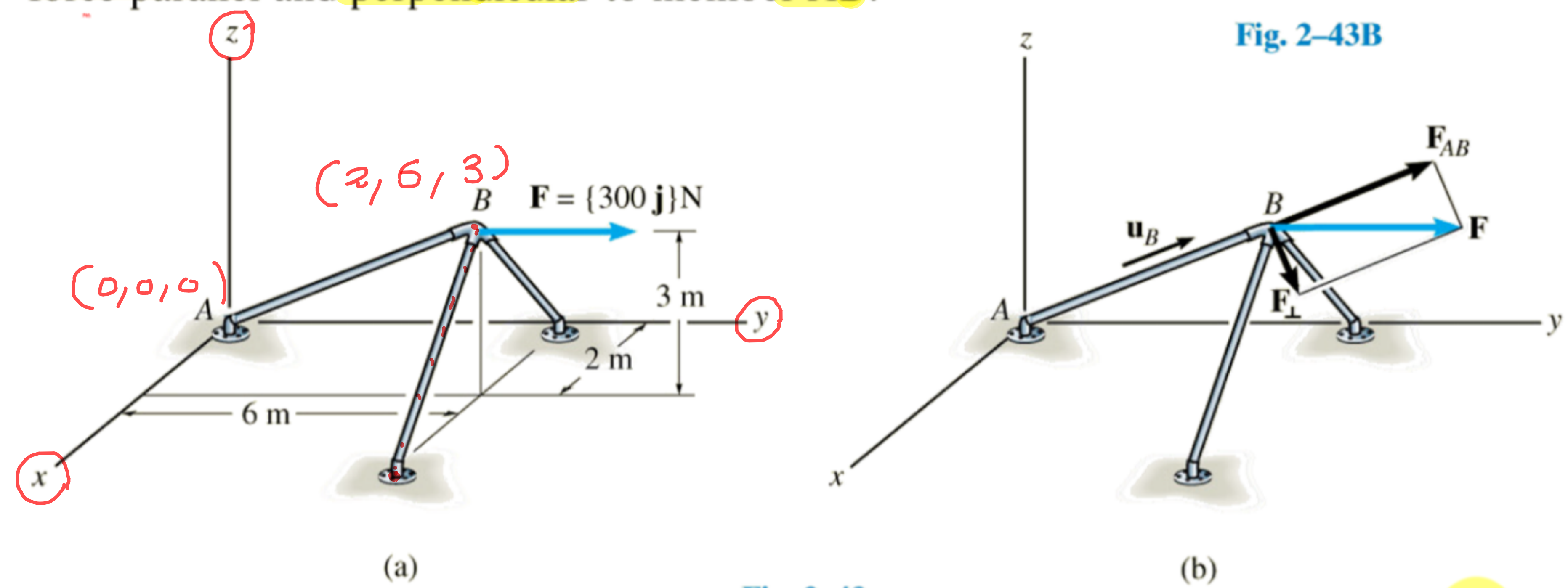


Fig. 2–43

$$\vec{AB} = 2\hat{i} + 6\hat{j} + 3\hat{k}$$

$$|\vec{AB}| = \sqrt{2^2 + 6^2 + 3^2} = 7$$

$$\vec{U}_{AB} = \frac{\vec{AB}}{|\vec{AB}|} = \frac{2\hat{i} + 6\hat{j} + 3\hat{k}}{7}$$

$$= 0.286\hat{i} + 0.857\hat{j} + 0.429\hat{k}$$

$$F_{AB} = \vec{F} \cdot \vec{U}_{AB}$$

$$= (300)\hat{j} \cdot (0.286\hat{i} + 0.857\hat{j} + 0.429\hat{k})$$

$$= 300 * 0.857 = 257.1 \text{ (N)}$$

$$\vec{F}_{AB} = F_{AB}\,\vec{U}_{AB}$$

$$= 257.1\,(0.286\hat{i} + 0.857\hat{j} + 0.429\hat{k})$$

$$\vec{F}_{AB} = 73.5\hat{i} + 220\hat{j} + 110\hat{k}$$

$$\vec{F}_{\perp} = \vec{F} - \vec{F}_{AB}$$

$$= (0\hat{i} + 300\hat{j} + 0\hat{k}) - (73.5\hat{i} + 220\hat{j} + 110\hat{k})$$

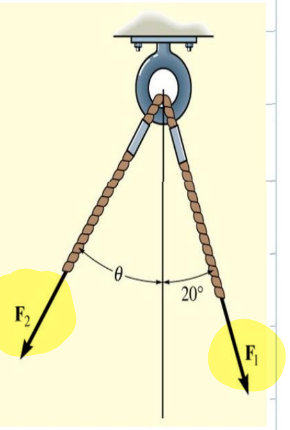
$$= -73.5\hat{i} + 80\hat{j} - 110\hat{k}$$

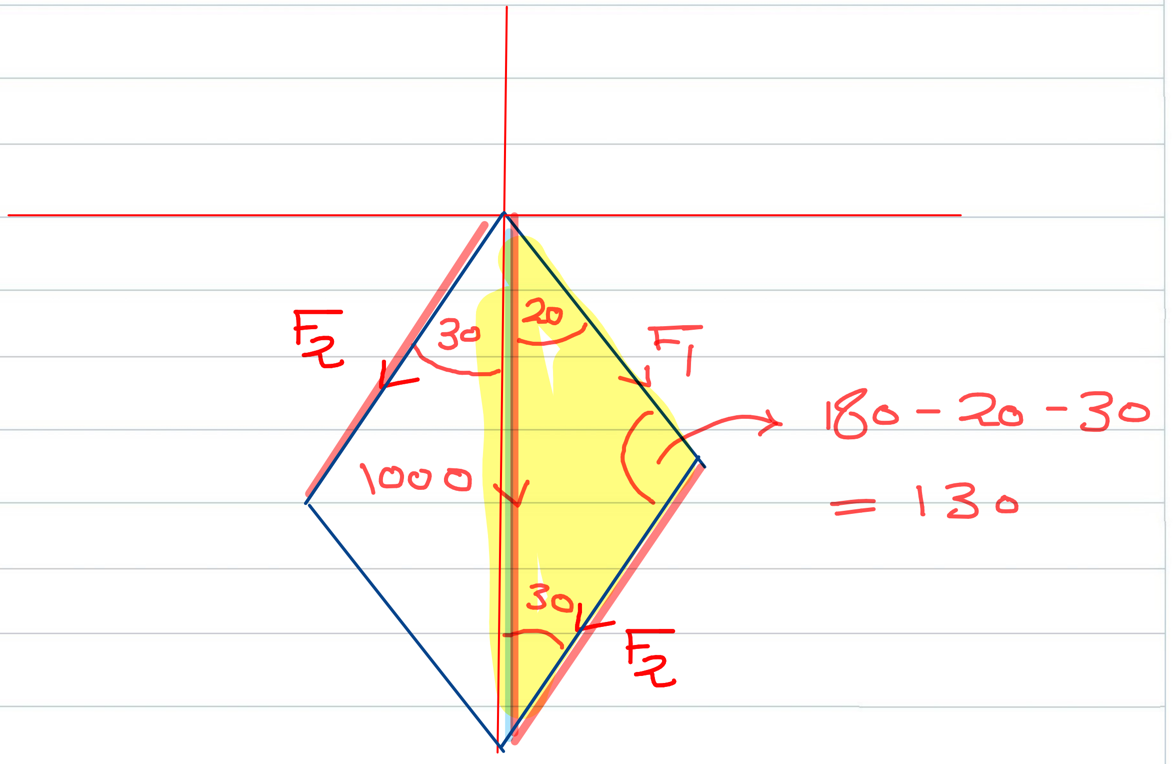
$$F_{\perp} = \sqrt{(-73.5)^2 + (80)^2 + (-110)^2}$$

$$= 155 \text{ (N)}$$

Example 2 :-

The ring below is subjected to F_1 and F_2. If we want a resultant force of 1kN and directed vertically downward, determine the magnitude of F_1 and F_2 if $\theta = 30°$.



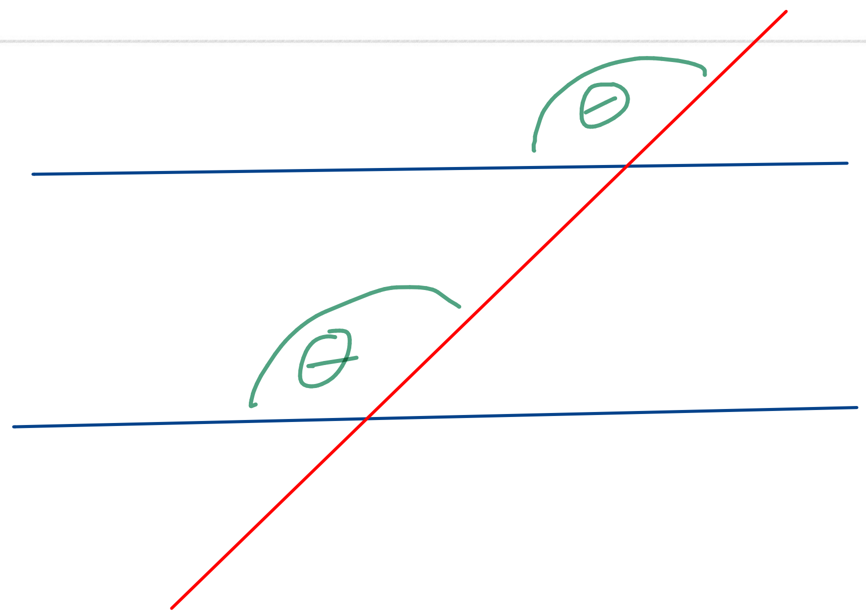


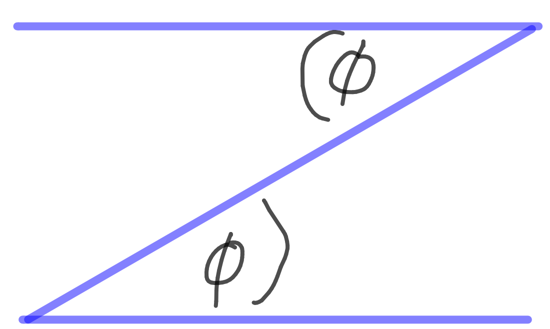
Sin - law

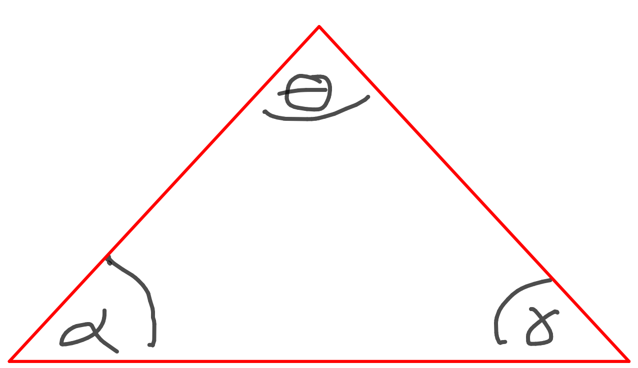
$$\frac{F_1}{\sin 30} = \frac{F_2}{\sin 20} = \frac{1000}{\sin 130}$$

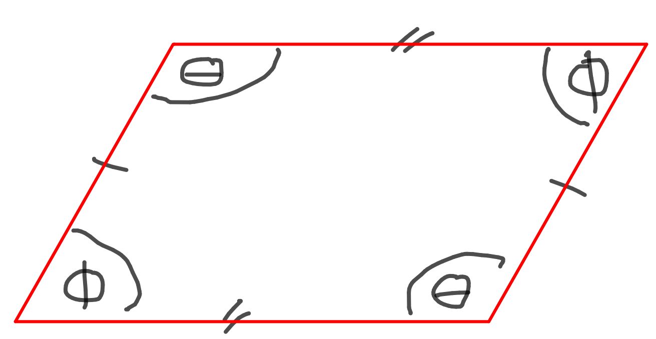
$$F_1 = \frac{1000 \sin 30}{\sin 130} = 658 \text{ N}$$

$$F_2 = \frac{1000 \sin 20}{\sin 130} = 446 \text{ N}$$









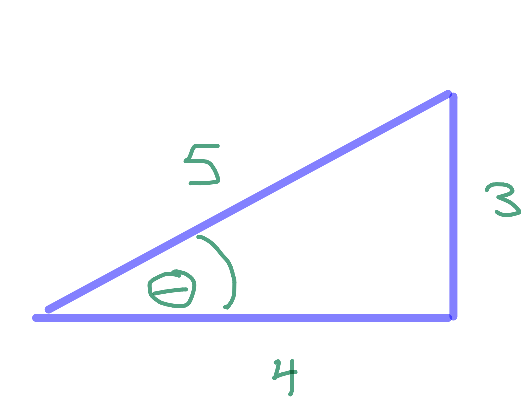
$\theta + \alpha + \gamma = 180$

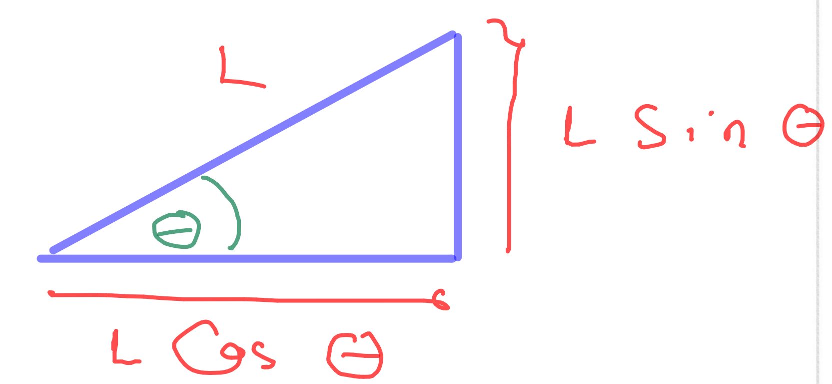
$\theta + \phi = 180^\circ$

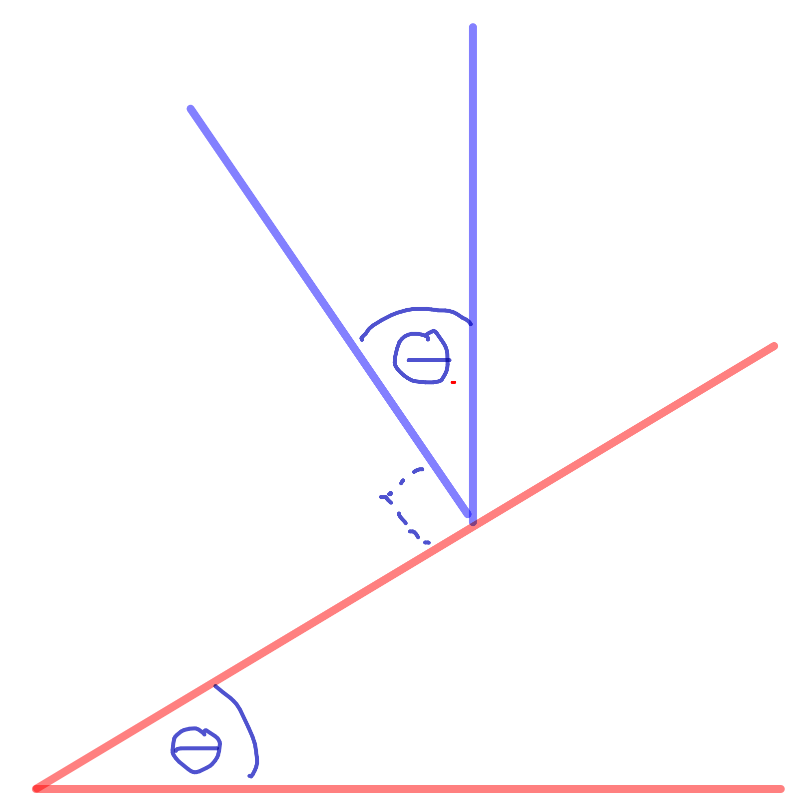
$\sin\theta = 3/5$

$\cos\theta = 4/5$

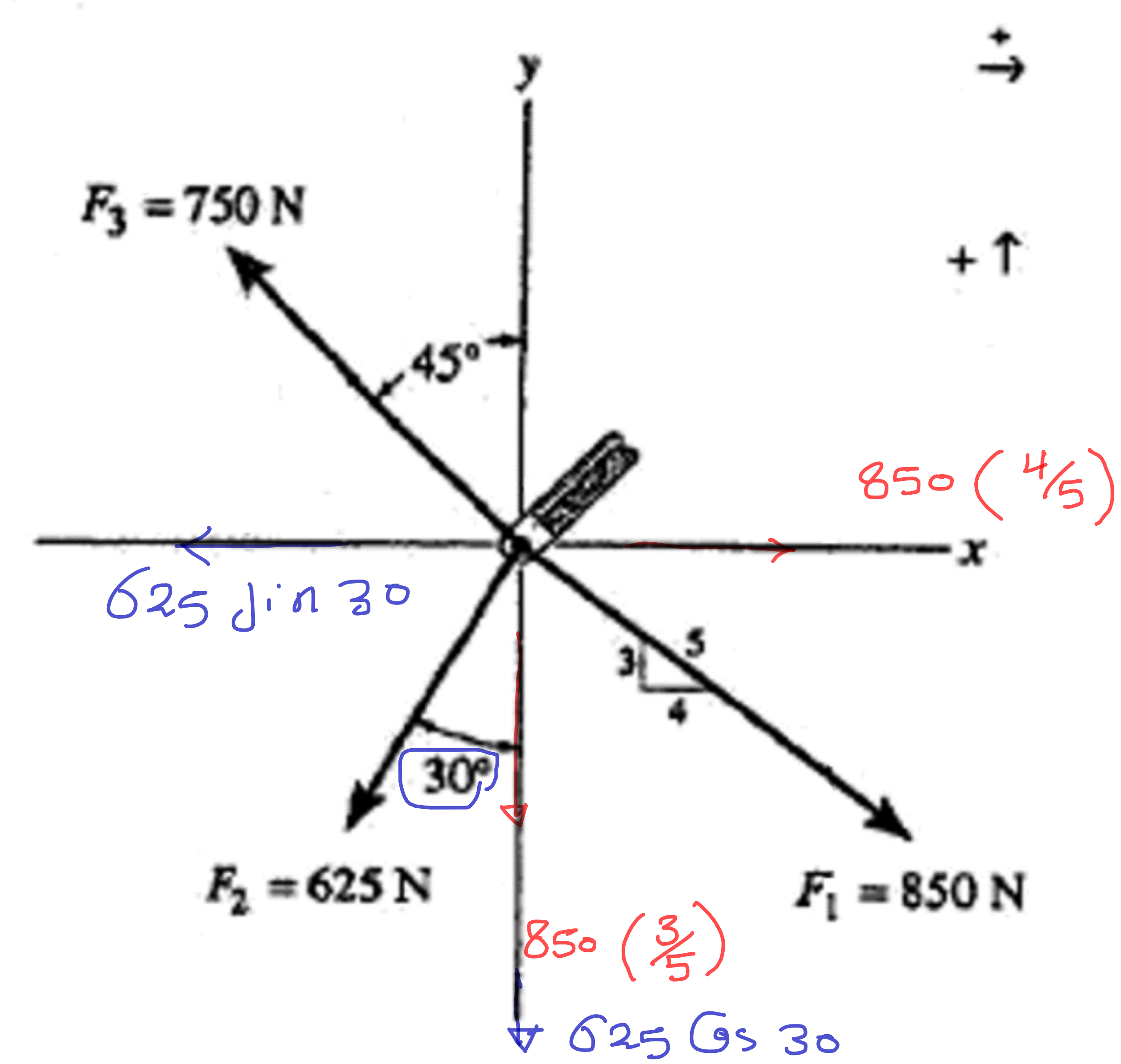
$\tan\theta = 3/4$







2-34. Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



(1) $F_{1x} = 680$

$F_{1y} = -510$

$F_1 = (680)\,i - (510)\,j$

$F_{2x} = -625 \sin 30 = -312.5$ N

$F_{2y} = -625 \cos 30 = -541.3$ N

$F_2 = (-312.5)\,i + (-541.3)\,j$

$F_{3x} = -750 \sin 45 = -530.3$

$F_{3y} = 750 \cos 45 = 530.3$

$F_{Rx} = \Sigma F_x = 680 - 312.5 - 530.3$

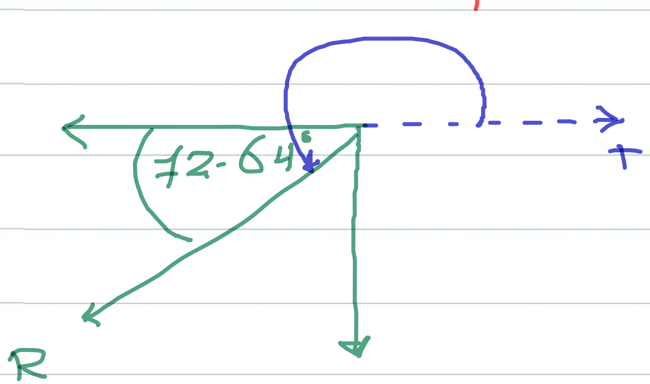
$= -162.8$

$F_{Ry} = \Sigma F_y = -510 - 541.3 + 530.3$

$= -521$

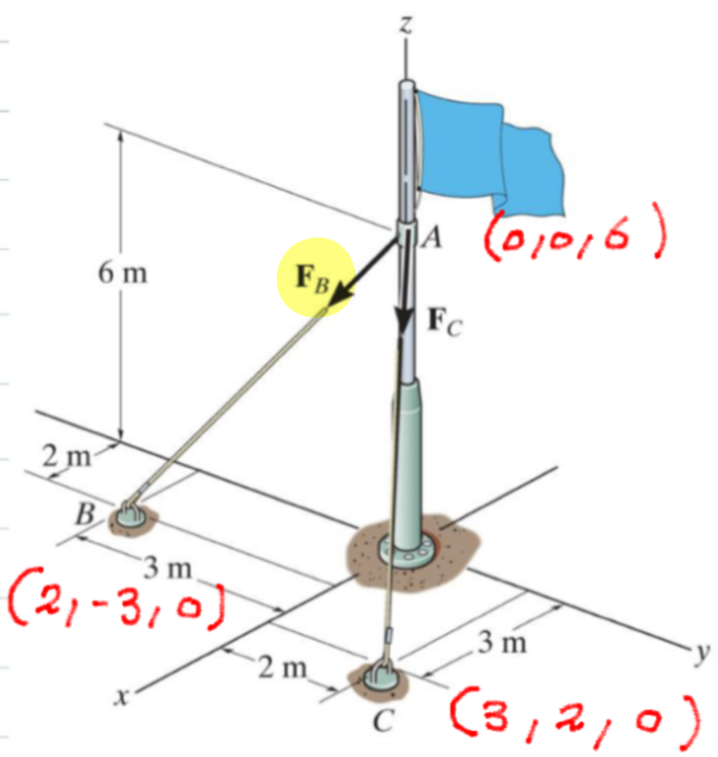
$F_R = \sqrt{(-162.8)^2 + (-521)^2} = 546$

$\theta = \tan^{-1} \dfrac{-521}{-162.8} = 72.64$



angle with (+) x-axis CCW $= 72.64 + 180 = 252.64°$

Try Yourself



Given: Two forces are acting on a flag pole as shown in the figure. F_B = 700 N and F_C = 560 N

Find: The magnitude and the coordinate direction angles of the resultant force.

$$r_A = 0\,i + 0\,j + 6\,k$$

$$r_B = 2\,i - 3\,j + 0\,k$$

$$r_C = 3\,i + 2\,j + 0\,k$$

$$\vec{r}_{AB} = \vec{r}_B - \vec{r}_A = 2i - 3j - 6k$$

$$\vec{U}_{AB} = \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|} = \frac{2i - 3j - 6k}{\sqrt{4+9+36}}$$

$$= \tfrac{2}{7}\,i - \tfrac{3}{7}\,j - \tfrac{6}{7}\,k$$

$$\vec{F}_B = F_B\,\vec{U}_{AB}$$

$$= 700\left(\tfrac{2}{7}\,i - \tfrac{3}{7}\,j - \tfrac{6}{7}\,k\right)$$

$$\vec{F}_B = 200\,\hat{i} - 300\,\hat{j} - 600\,\hat{k}$$

$$\vec{r}_{AC} = \vec{r}_C - \vec{r}_A = 3i + 2j - 6k$$

$$\vec{U}_{AC} = \frac{\vec{r}_{AC}}{|\vec{r}_{AC}|} = \frac{3i + 2j - 6k}{\sqrt{9+4+36}}$$

$$= \tfrac{3}{7}\,i + \tfrac{2}{7}\,j - \tfrac{6}{7}\,k$$

$$\vec{F}_C = F_C\,\vec{U}_{AC}$$

$$= 560\left(\tfrac{3}{7}\,i + \tfrac{2}{7}\,j - \tfrac{6}{7}\,k\right)$$

$$\vec{F}_C = 240\,i + 160\,j - 480\,k$$

$$\vec{F}_R = \vec{F}_B + \vec{F}_C$$

$$= (200+240)\,i + (-300+160)\,j + (-600-480)\,k$$

$$= 440\,\hat{i} - 140\,\hat{j} - 1080\,\hat{k}$$

$$F_R = \sqrt{440^2 + (-140)^2 + (-1080)^2}$$

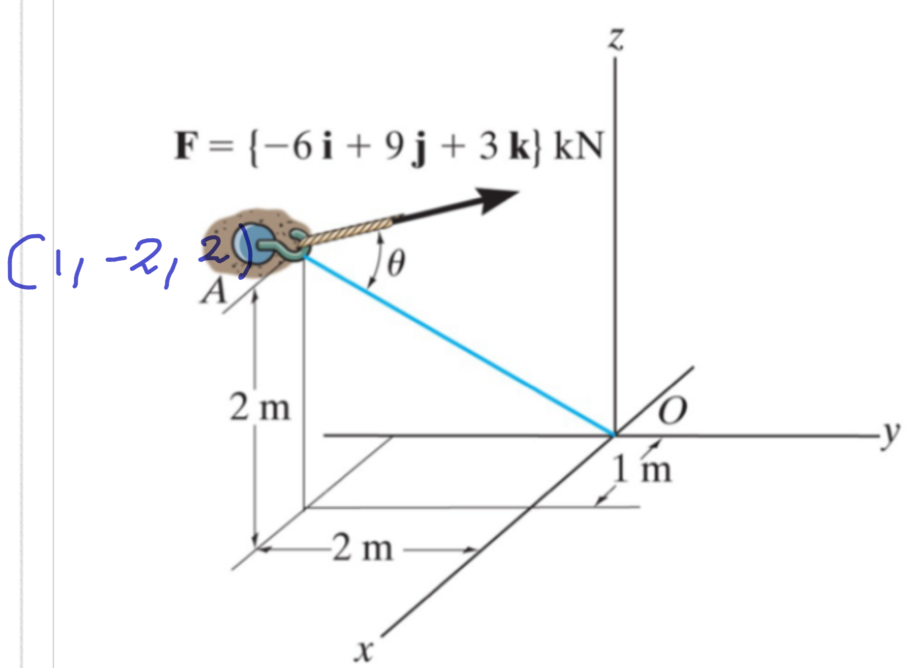
$$= 1174.6 \text{ N}$$

$$\alpha = \cos^{-1} \frac{440}{1174.6} = 68$$

$$\beta = \cos^{-1} \frac{-140}{1174.6} = 96.8$$

$$\gamma = \cos^{-1} \frac{-1080}{1174.6} = 156.8^\circ$$

Try Yourself



Given: The force acting on the hook at point A.

Find: The angle between the force vector and the line AO, and the magnitude of the projection of the force along the line AO.

$\vec{r}_{AO} = -i + 2j - 2k$

$|\vec{r}_{AO}| = r_{AO} = \sqrt{1 + 4 + 4} = 3$

$\vec{F} = -6i + 9j + 3k$

$|\vec{F}| = F = \sqrt{36 + 81 + 9} = 11.22$

$\vec{F} \cdot \vec{r}_{AO} = (-1 * -6) + (2 * 9) + (-2 * 3)$

$= 18 \text{ kN}\cdot\text{m}$

$\theta = \cos^{-1} \dfrac{\vec{F} \cdot \vec{r}_{OA}}{F \; r_{OA}}$

$= \cos^{-1} \dfrac{18}{11.22 * 3} = 57.67^{\circ}$

$F_{AO} = \vec{F} \cdot \vec{U}_{AO}$

$\vec{U}_{AO} = \dfrac{\vec{r}_{AO}}{r_{AO}} = \dfrac{-i + 2j - 2k}{3}$

$= -\frac{1}{3} i + \frac{2}{3} j - \frac{2}{3} k$

$F_{AO} = (-6 * -\frac{1}{3}) + (9 * \frac{2}{3}) + (3 * \frac{-2}{3})$

$= 6 \text{ N}$

or

$F_{AO} = F \cos\theta$

$= 11.22 \cos 57.67$

$= 6 \text{ N}$

(*)

x