QATAR UNIVERSITY

College of Engineering
Department of Civil & Architectural

CVEN 214: STRENGTH OF MATERIALS

Chapter 1: Introduction & basic concepts

Dr Mohammed Elshafie

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CHAPTER 1

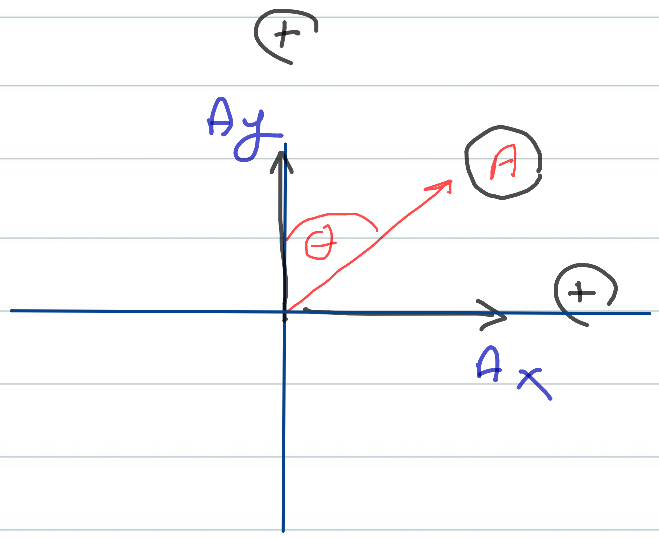
Introduction – Concept of Stress

Rectangular/Cartesian Components Method

$A_x = A \sin\theta$

$A_y = A \cos\theta$

$\vec{A} = A_x\,\hat{i} + A_y\,\hat{j}$

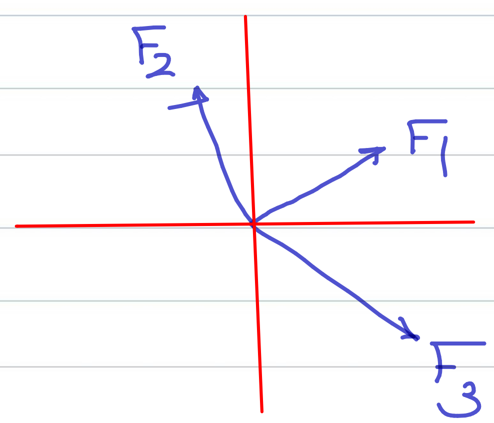


Equilibrium of a Particle

1) Resolve

2) $\Sigma F_x = 0 \quad \xrightarrow{+}$

$\Sigma F_y = 0 \quad \uparrow +$



moment of the force (Vector)

$M = \Sigma F_x * d_{\perp y}$; $F_y * d_x$

ccw (+)

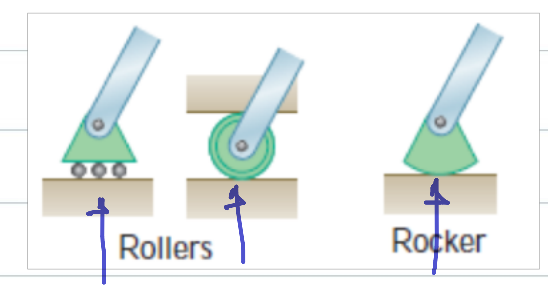
cw (−)

Equilibrium of Rigid Bodies

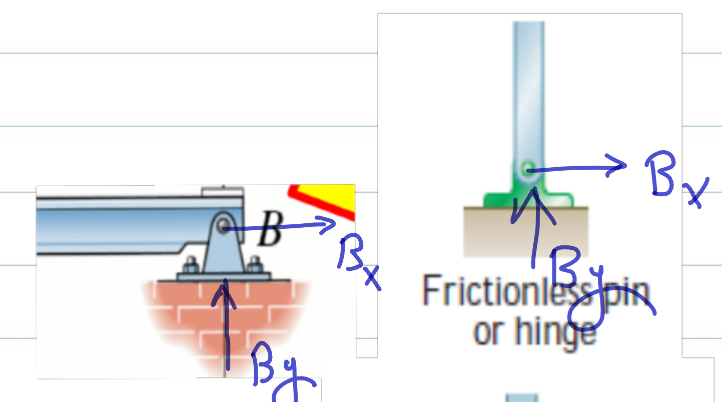
$\Sigma F_x = 0 \quad \xrightarrow{+}$ $\qquad \Sigma F_y = 0 \quad \uparrow +$ $\qquad \Sigma M = 0 \quad \circlearrowleft +$

Support Reactions :-

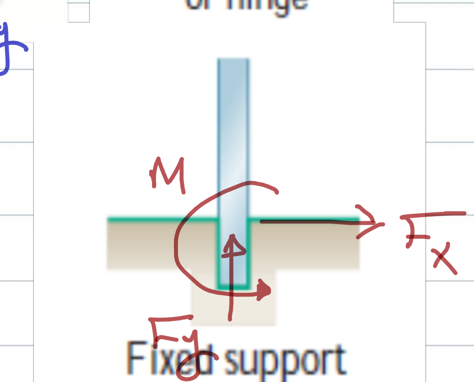
1) Roller — 1-Reaction



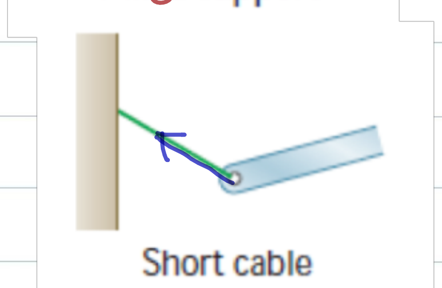
2) Pin — 2-Reaction



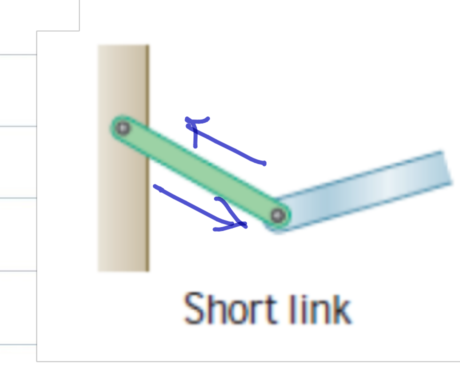
3) Fixed



4) Cable — Tension Force outside



5) Link



EXAMPLE 1.2

Determine the resultant internal loadings acting on the cross section at C of the machine shaft shown in Fig. 1–5a. The shaft is supported by bearings at A and B, which exert only vertical forces on the shaft.

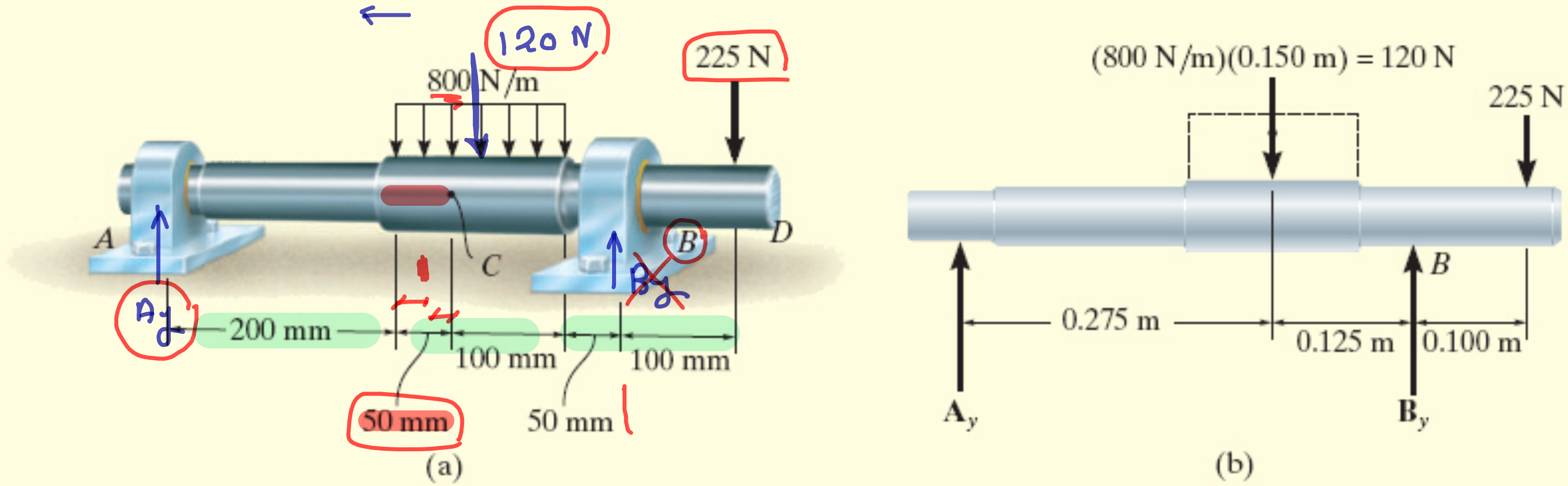


Fig. 1–5

SOLUTION

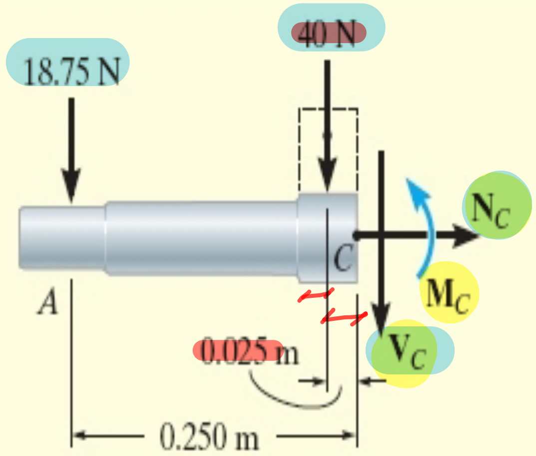
We will solve this problem using segment AC of the shaft.

Support Reactions. A free-body diagram of the entire shaft is shown in Fig. 1–5b. Since segment AC is to be considered, only the reaction at A has to be determined. Why?

$$\circlearrowleft + \Sigma M_B = 0; \quad -A_y(0.400\text{ m}) + 120\text{ N}(0.125\text{ m}) - 225\text{ N}(0.100\text{ m}) = 0$$

$$A_y = -18.75\text{ N}$$

EXAMPLE 1.2 *(Continued)*



(c)

The negative sign for A_y indicates that $\mathbf{A}_y$ acts in the *opposite sense* to that shown on the free-body diagram.

Free-Body Diagram. Passing an imaginary section perpendicular to the axis of the shaft through *C* yields the free-body diagram of segment *AC* shown in Fig. 1–5*c*.

Equations of Equilibrium.

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad N_C = 0 \qquad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad -18.75\text{ N} - 40\text{ N} - V_C = 0$$

$$V_C = -58.8\text{ N} \qquad \textit{Ans.}$$

$$\circlearrowleft + \Sigma M_C = 0; \quad M_C + 40\text{ N}(0.025\text{ m}) + 18.75\text{ N}(0.250\text{ m}) = 0$$

$$M_C = -5.69\text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

NOTE: The negative signs for V_C and M_C indicate they act in the opposite directions on the free-body diagram. As an exercise, calculate the reaction at *B* and try to obtain the same results using segment *CBD* of the shaft.

H.W

EXAMPLE 1.1

Determine the resultant internal loadings acting on the cross section at C of the cantilevered beam shown in Fig. 1–4a.

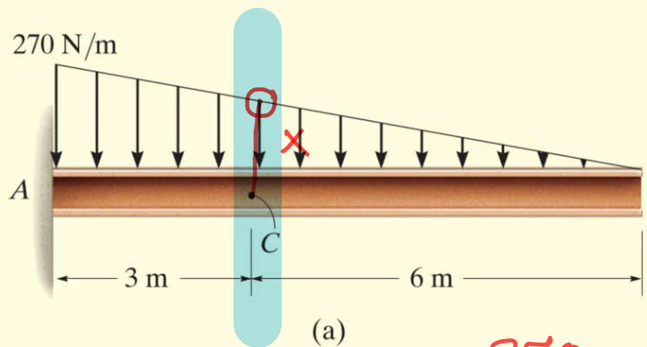
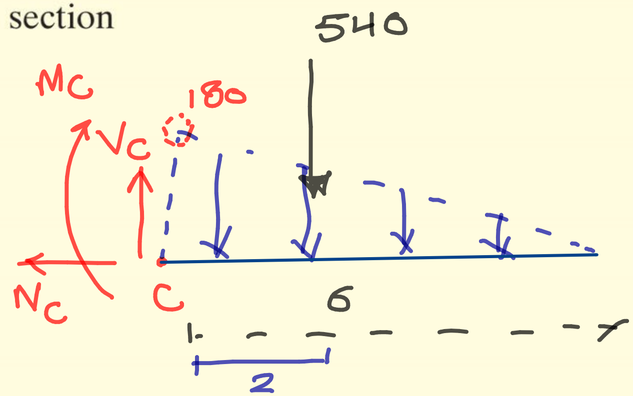


Fig. 1–4

$$\frac{270}{9} = \frac{x}{6}$$

$$x = 180$$



$\Sigma F_x = 0 \qquad N_C = 0$ ✓

$\Sigma F_y = 0 \qquad V_C - 540 = 0$

$V_C = 540$ N

$\Sigma M_C = 0 \quad \circlearrowleft +$

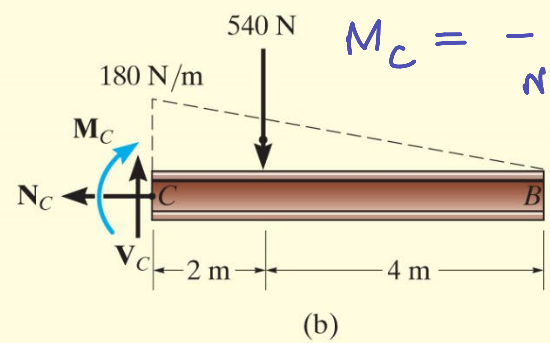
$-M_C - 540 \times 2 = 0$

$M_C = -1080$ N·m

SOLUTION

Support Reactions. The support reactions at A do not have to be determined if segment CB is considered.

Free-Body Diagram. The free-body diagram of segment CB is shown in Fig. 1–4b. It is important to keep the distributed loading on the segment until *after* the section is made. Only then should this loading be replaced by a single resultant force. Notice that the intensity of the distributed loading at C is found by proportion, i.e., from Fig. 1–4a, $w/6\text{ m} = (270\text{ N/m})/9\text{ m}$, $w = 180\text{ N/m}$. The magnitude of the resultant of the distributed load is equal to the area under the loading curve (triangle) and acts through the centroid of this area. Thus, $F = \frac{1}{2}(180\text{ N/m})(6\text{ m}) = 540\text{ N}$, which acts $\frac{1}{3}(6\text{ m}) = 2\text{ m}$ from C as shown in Fig. 1–4b.



EXAMPLE 1.1 CONTINUED

Equations of Equilibrium. Applying the equations of equilibrium we have

$$\xrightarrow{+} \Sigma F_x = 0; \qquad -N_C = 0$$

$$N_C = 0 \qquad \textit{Ans.}$$

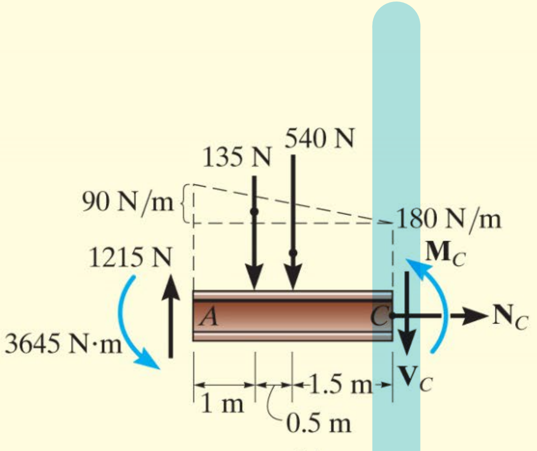
$$+\uparrow \Sigma F_y = 0; \qquad V_C - 540\text{ N} = 0$$

$$V_C = 540\text{ N} \qquad \textit{Ans.}$$

$$\circlearrowleft + \Sigma M_C = 0; \qquad -M_C - 540\text{ N}(2\text{ m}) = 0$$

$$M_C = -1080\text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

NOTE: The negative sign indicates that $\mathbf{M}_C$ acts in the opposite direction to that shown on the free-body diagram. Try solving this problem using segment AC, by first obtaining the support reactions at A, which are given in Fig. 1–4c.



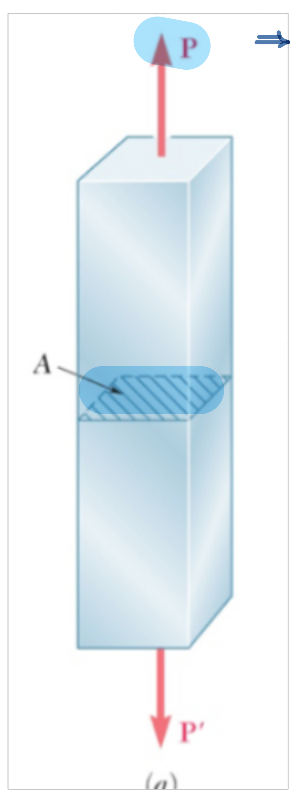
(c)

Concept of Stress

$$\text{Stress} = \frac{\text{Force}}{\text{Area}} \quad \left(N/m^2 = Pa \right)$$

Normal stress

$P \perp A$



→ axial Force

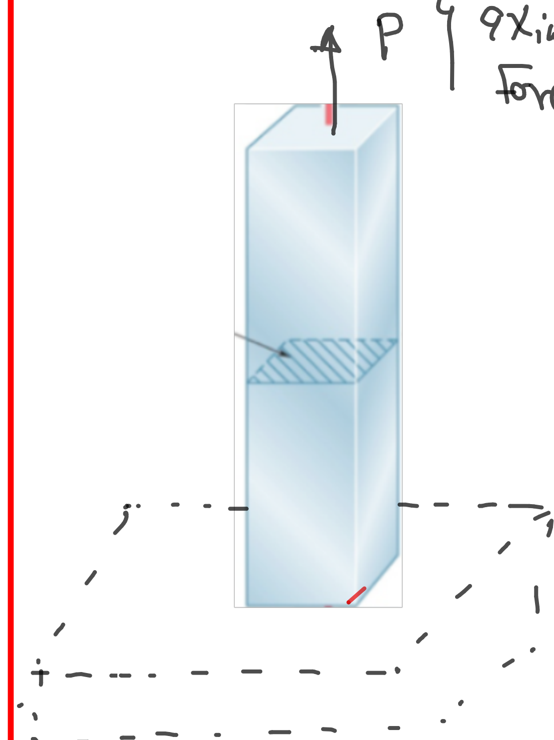
$$\sigma_{av} = P/A$$

Tension Force (+) outside

Compression Force (−) inside

Bearing stress

P } axial Force



Contacting surface

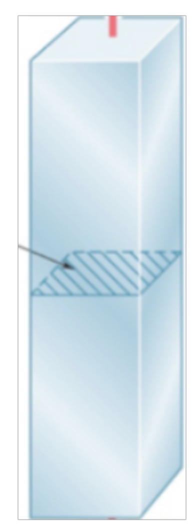
$$\sigma_b = P/A$$

Shear stress

$P \parallel A$

P

Shear Force



$$\tau_{av} = P/A$$

single shear

double shear

Shearing Stress Examples

Single Shear

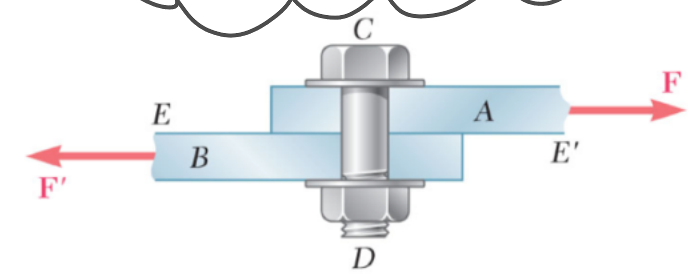
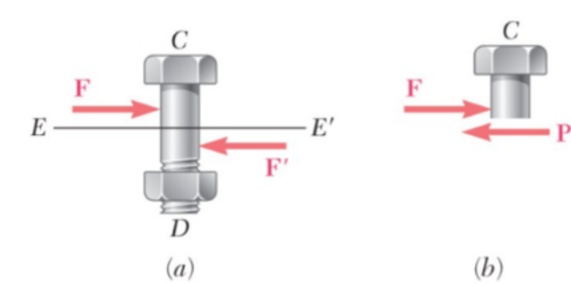


Fig. 1.16 Bolt subject to single shear.



Double Shear

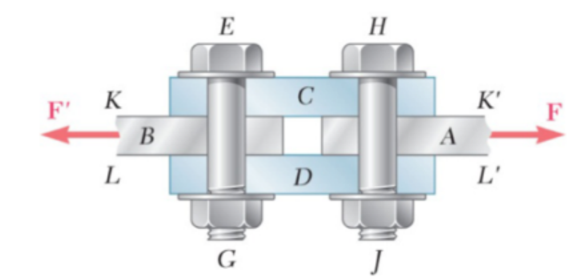
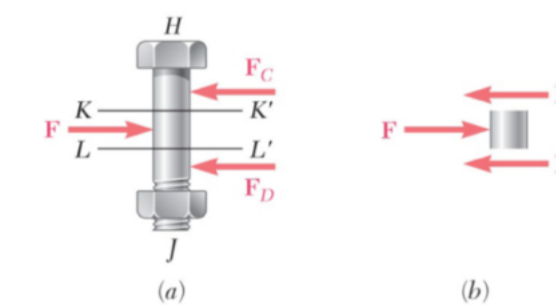
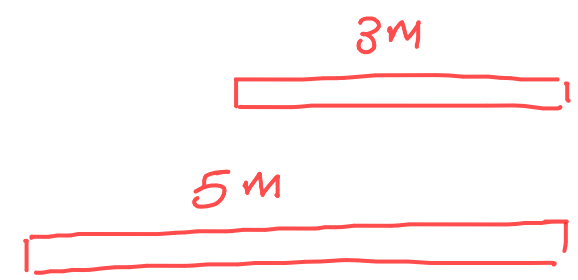
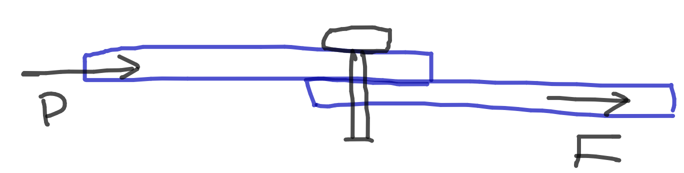


Fig. 1.18 Bolt subject to double shear.



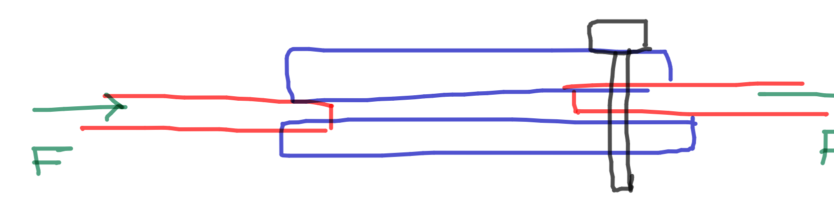




2- Member

1 – Cut

$$\tau_{av} = F/A_{bolt}$$



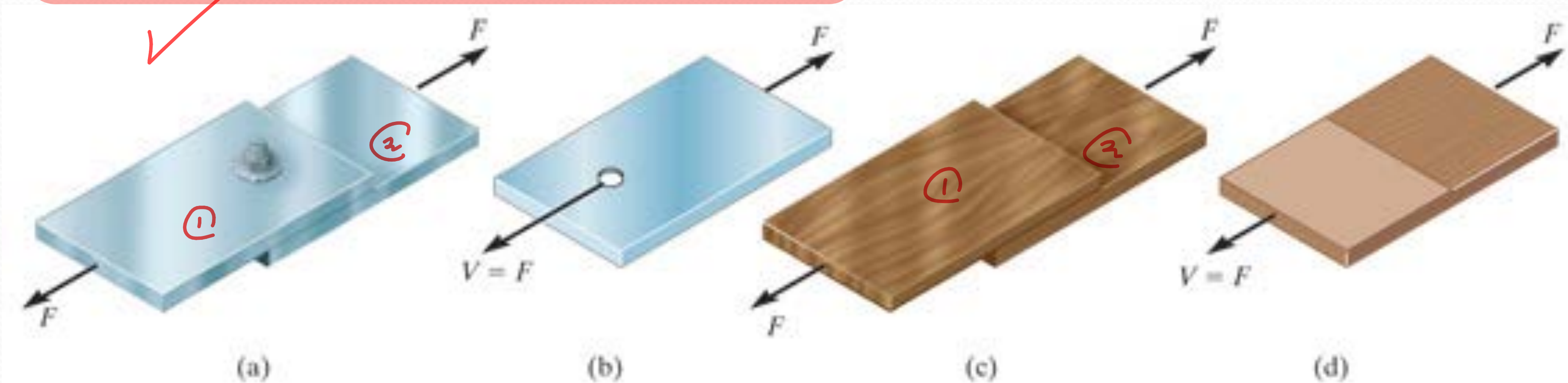
3-Member

2-Cut

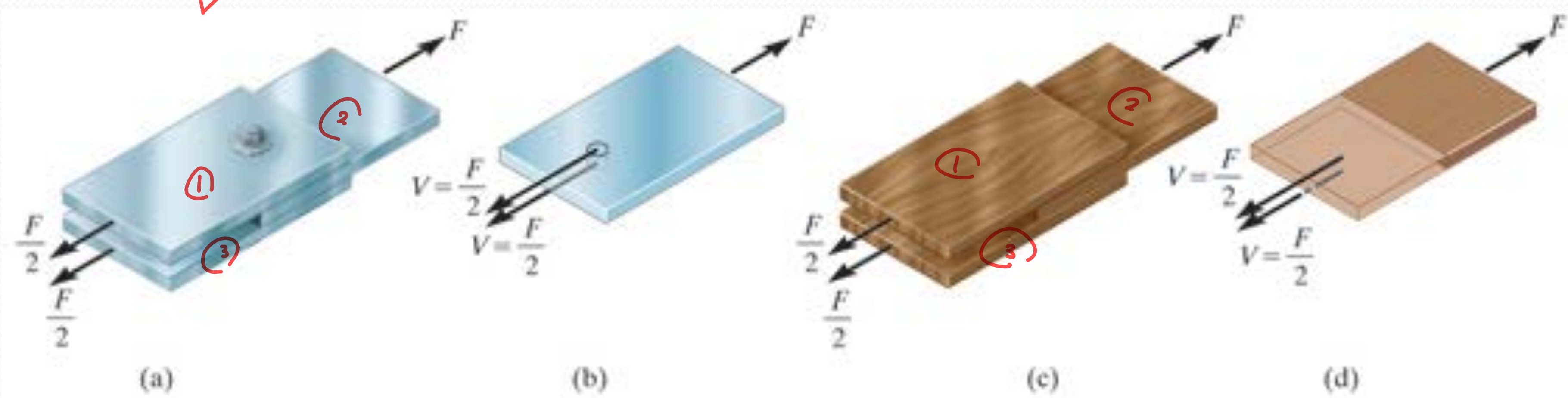
$$\tau_{av} = \frac{F}{2 A_{bolt}}$$

Two Different types of shear

a) Single Shear connections: e.g lap joints

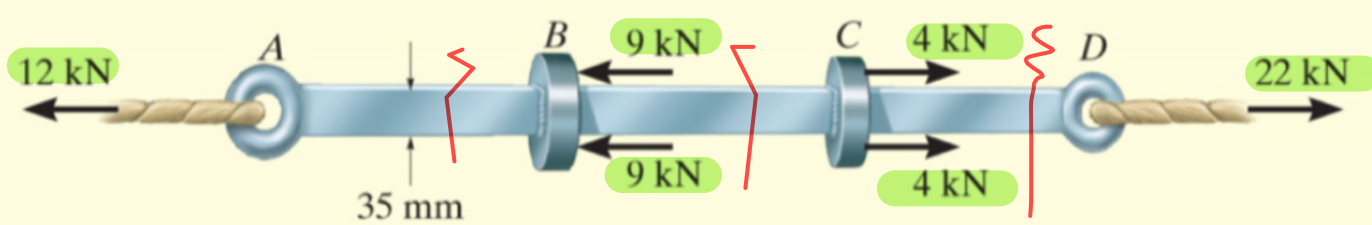


b) Double Shear-surfaces: eg double lap-joints



EXAMPLE 1.5

The bar in Fig. 1–15*a* has a constant width of 35 mm and a thickness of 10 mm. Determine the maximum average normal stress in the bar when it is subjected to the loading shown.



$$\boxed{\sigma = \frac{P}{A}}$$

Section ⇒ Change Force

35 mm

10 mm

$$P_{AB} = 12 \text{ kN} \quad (\text{tension})$$

$$P_{BC} = 12 + 9 + 9 = 30 \text{ kN} \quad (\text{tension})$$

$$P_{CD} = 22 \text{ kN} \quad (\text{tension})$$

$$\sigma_{Max} = \frac{P_{Max}}{A} = \frac{P_{BC}}{A}$$

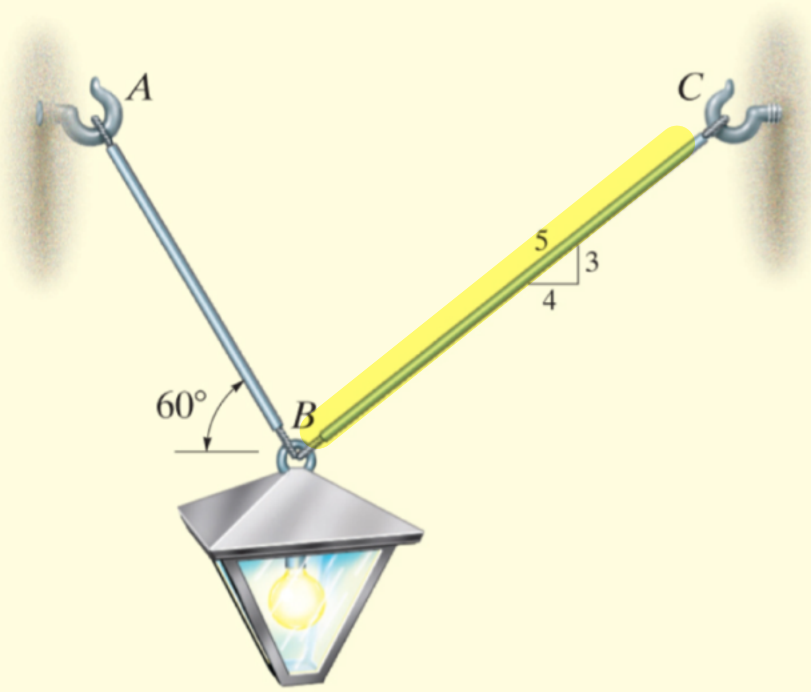
$$= \frac{30 \times 10^{3}}{0.035 \times 0.01}$$

$$\sigma_{Max} = 85.7 \text{ MPa}$$

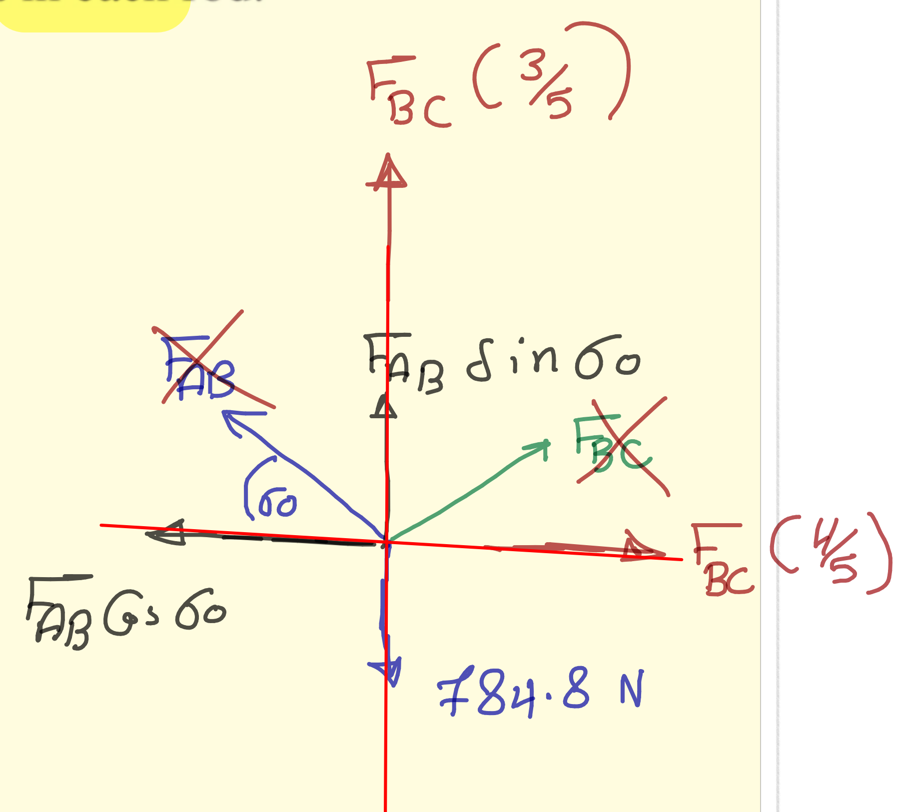
Tensile Stress

EXAMPLE 1.6

The 80-kg lamp is supported by two rods *AB* and *BC* as shown in Fig. 1–16*a*. If *AB* has a diameter of 10 mm and *BC* has a diameter of 8 mm, determine the average normal stress in each rod.



(a)



$$W = Mg = 80 * 9.81 = 784.8 \text{ N}$$

$$\Sigma F_x = 0 \quad \overset{+}{\rightarrow}$$

$$\frac{4}{5} F_{BC} - F_{AB}\cos 60 = 0 \quad \Rightarrow (1)$$

$$\Sigma F_y = 0 \quad \uparrow +$$

$$\frac{3}{5} F_{BC} + F_{AB}\sin 60 = 784.8 \quad \Rightarrow (2)$$

By Calculator

$$F_{BC} = 395.2 \text{ N}$$

$$F_{AB} = 632.4 \text{ N}$$

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}}$$

$$= \frac{395.2}{\frac{\pi}{4}\left(\frac{8}{1000}\right)^2} = 7.86 * 10^6 \text{ Pa}$$

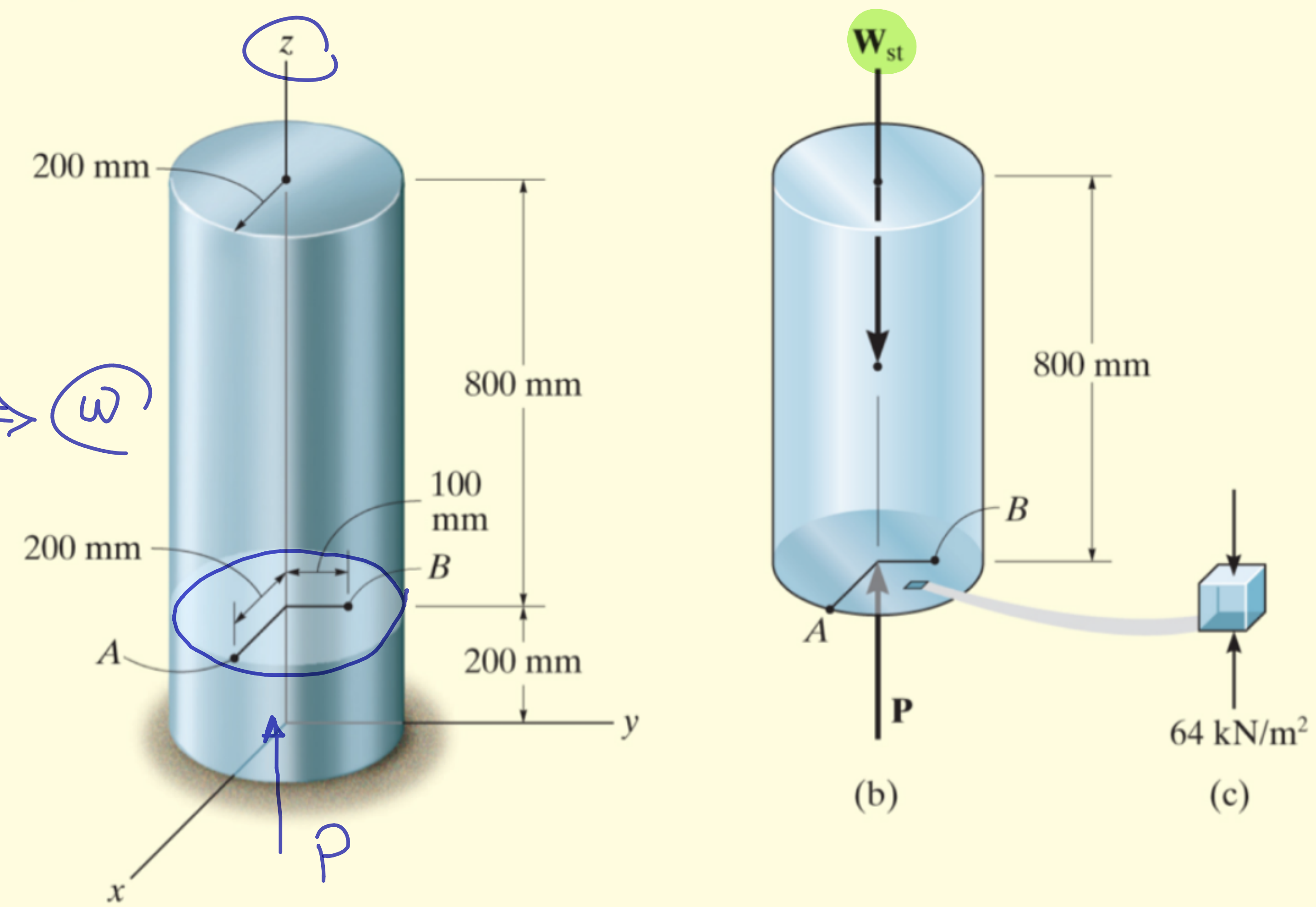
$$\sigma_{BC} = 7.86 \text{ MPa}$$

$$\sigma_{BA} = \frac{F_{BA}}{A_{BA}} = \frac{632.4}{\frac{\pi}{4}\left(\frac{10}{1000}\right)^2}$$

$$= 8.05 \text{ MPa}$$

EXAMPLE 1.7

The casting shown in Fig. 1–17*a* is made of steel having a specific weight of 80 kN/m³. Determine the average compressive stress acting at points *A* and *B*.



$\gamma = 80 \ kN/m^2 \Rightarrow w$

$\gamma = w / V$

$w = \gamma V$

$\Sigma F_z = 0 \quad \uparrow +$

$P - w_{st} = 0$

$P - 80 \left(\pi (0.2)^2 * 0.8 \right) = 0$

$P = 8.042 \ kN$

$A = \pi r^2$

$= \pi (0.2)^2$

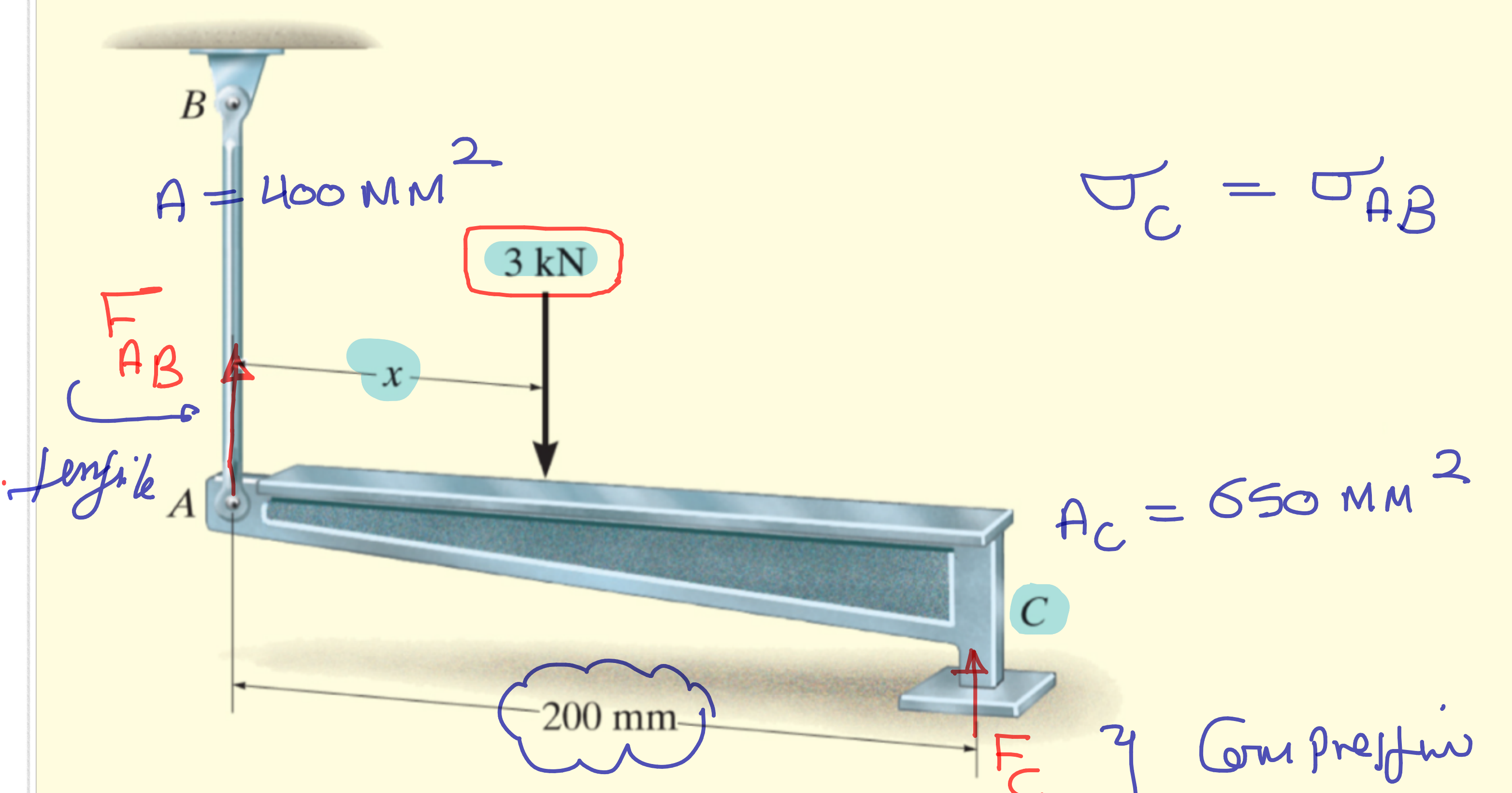
$\Rightarrow \sigma = P / A$

$= \dfrac{8.042 * 10^3}{\pi (0.2)^2}$

$= 64 * 10^3 \ N/m^2$

EXAMPLE 1.8

Member *AC* shown in Fig. 1–18*a* is subjected to a vertical force of 3 kN. Determine the position *x* of this force so that the average compressive stress at the smooth support *C* is equal to the average tensile stress in the tie rod *AB*. The rod has a cross-sectional area of 400 mm² and the contact area at *C* is 650 mm².



(a)

$0 < X < 200$

Static

$\Sigma F_y = 0 \quad \uparrow +$

$F_{AB} + F_C - 3000 = 0 \quad (1)$

$\Sigma M_A = 0 \quad \circlearrowleft +$

$-3000\,X + F_C * 200 = 0 \Rightarrow (2)$

Stress : —

$\sigma_{AB} = \sigma_C$

$\frac{F_{AB}}{400} = \frac{F_C}{650}$

$F_C = 1.625\,F_{AB} \Rightarrow (3)$

(3) in Eq (1)

$F_{AB} + 1.625\,F_{AB} - 3000 = 0$

$F_{AB} = 1143\ N$

in Eq (3) $\quad F_C = 1.625 * 1143$

$= 1857\ N$

in Eq (2)

$-3000\,X + 1857 * 200 = 0$

$X = 124\ MM$

Questions

1) What is the normal stress in the bar if P=10 kN and 500mm²?

$1 \text{ MPa} = \text{N}/\text{mm}^2$

a) 0.02 kPa

b) 20 Pa

c) 20 kPa

d) 200 N/mm²

e) 20 MPa

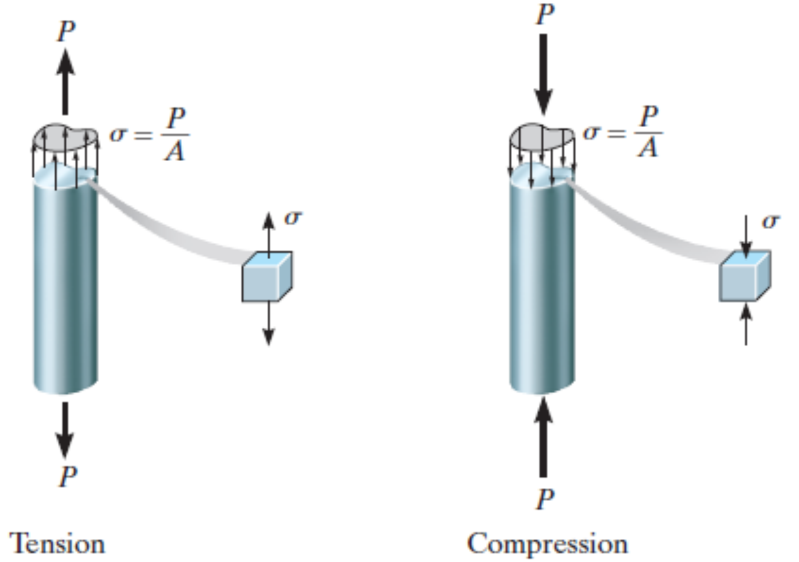


Fig. 1–15

$$\sigma = P/A = \frac{10000 \text{ N}}{500 * 10^{-6} \text{ m}^2} = \underline{20 * 10^6 \text{ Pa}}$$

$$= 20 \text{ MPa}$$

$$= 20 \text{ N}/\text{mm}^2$$

2) The thrust bearing is subjected to the loads as shown. Determine the order of average normal stress developed on cross section through BC and D.

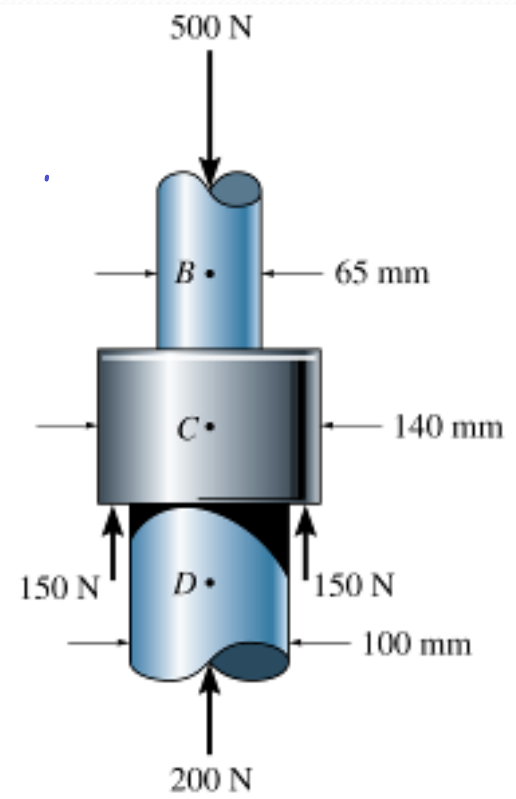
a) C > B > D

b) C > D > B

c) B > C > D

d) D > B > C

B > C > D



$$\sigma_B = \frac{500}{\frac{\pi}{4}(65)^2} = 0.15$$

$$\sigma_C = \frac{500}{\frac{\pi}{4}(140)^2} = 0.032$$

$$\sigma_D = \frac{200}{\frac{\pi}{4}(100)^2} = 0.025$$

2. **What is the average shear stress in the internal vertical surface AB (or CD), if F=20kN, and $A_{AB}=A_{CD}=1000mm^2$?**

a) 20 N/mm²

b) 10 N/mm²

c) 10 kPa

d) 200 kN/m²

e) 20 MPa

F = 20 kN

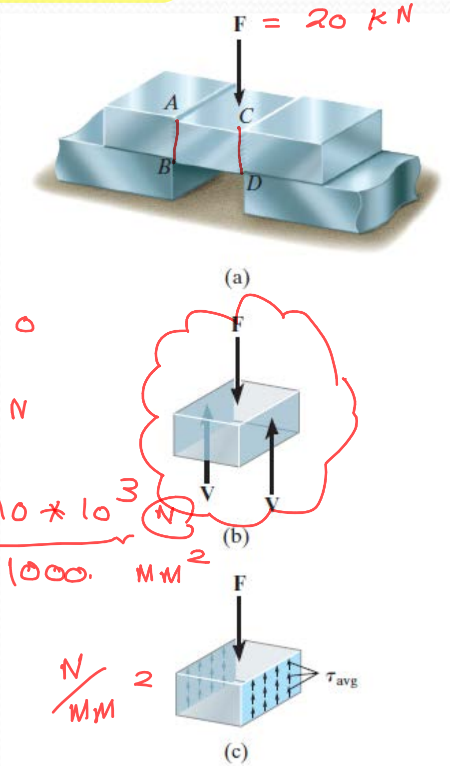


Fig. 1–20

$\Sigma F_y = 0$

$2V - 20 = 0$

$V = 10 \text{ kN}$

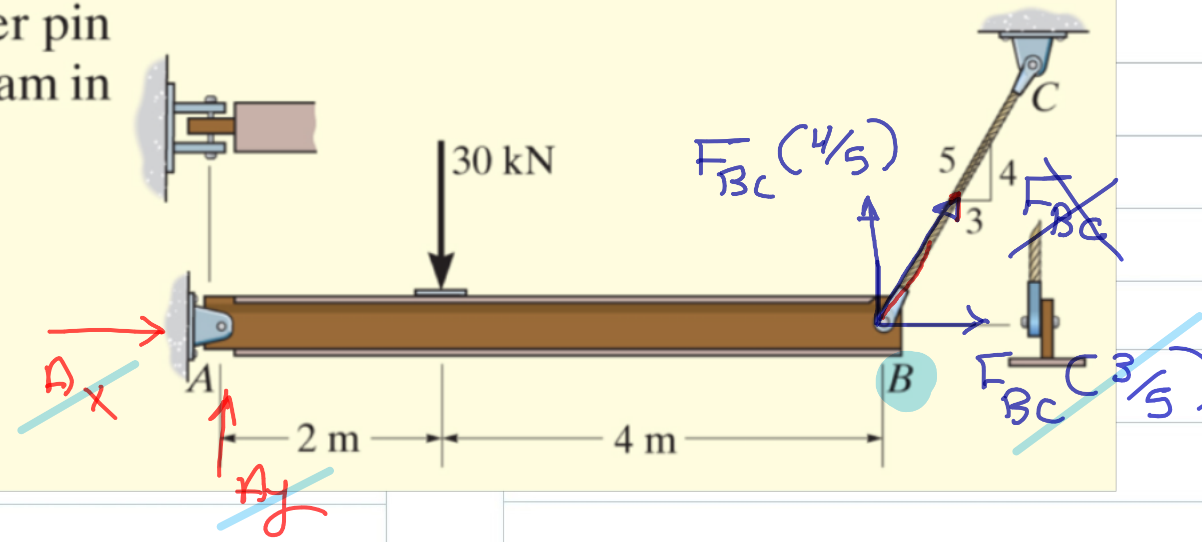
$\tau_{av} = V/A = \dfrac{10 * 10^3 \text{ N}}{1000 \text{ mm}^2}$

$= 10 \text{ N/mm}^2$

EXAMPLE 1.9

Determine the average shear stress in the 20-mm-diameter pin at A and the 30-mm-diameter pin at B that support the beam in Fig. 1–21a.

SOLUTION



* $\sum M_A = 0 \quad \circlearrowleft +$

$$-30 * 2 + F_{BC}(4/5) * 6 = 0$$

$$F_{BC} = 12.5 \text{ kN}$$

* $\sum F_x = 0 \quad \overset{+}{\rightarrow}$

$$A_x + 12.5(3/5) = 0 \qquad A_x = -7.5 \text{ kN}$$

* $\sum F_y = 0 \quad +\uparrow$

$$A_y - 30 + 12.5(4/5) = 0$$

$$A_y = 20 \text{ kN}$$

$$F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{7.5^2 + 20^2}$$

$$F_A = 21.36 \text{ kN}$$

* Double shear @ A

$$V_A = \frac{F_A}{2} = \frac{21.36}{2} = 10.68 \text{ kN}$$

* single shear @ B :-

$$V_B = F_{BC} = 12.5 \text{ kN}$$

$$(\tau_A)_{av} = \frac{V_A}{A_A} = \frac{10.68 * 10^3}{\frac{\pi}{4}(0.02)^2} = 34 \text{ MPa}$$

$$(\tau_B)_{av} = \frac{V_B}{A_B} = \frac{12.5 * 10^3}{\frac{\pi}{4}(0.03)^2} = 17.7 \text{ MPa}$$

Factor of Safety

$$F.S = \frac{\sigma_{ult}}{\sigma_{all}}$$

$\sigma_{ult} \rightarrow$ Fail

$\sigma_{all} \rightarrow$ allowable stress

used For Design

$$\sigma_{ult} = \frac{P_{ult}}{A}$$

$$F.S = \frac{\tau_{ult}}{\tau_{all}}$$

$\tau_{ult} \rightarrow$ Fail

$$F.S > 1$$

100 MPa

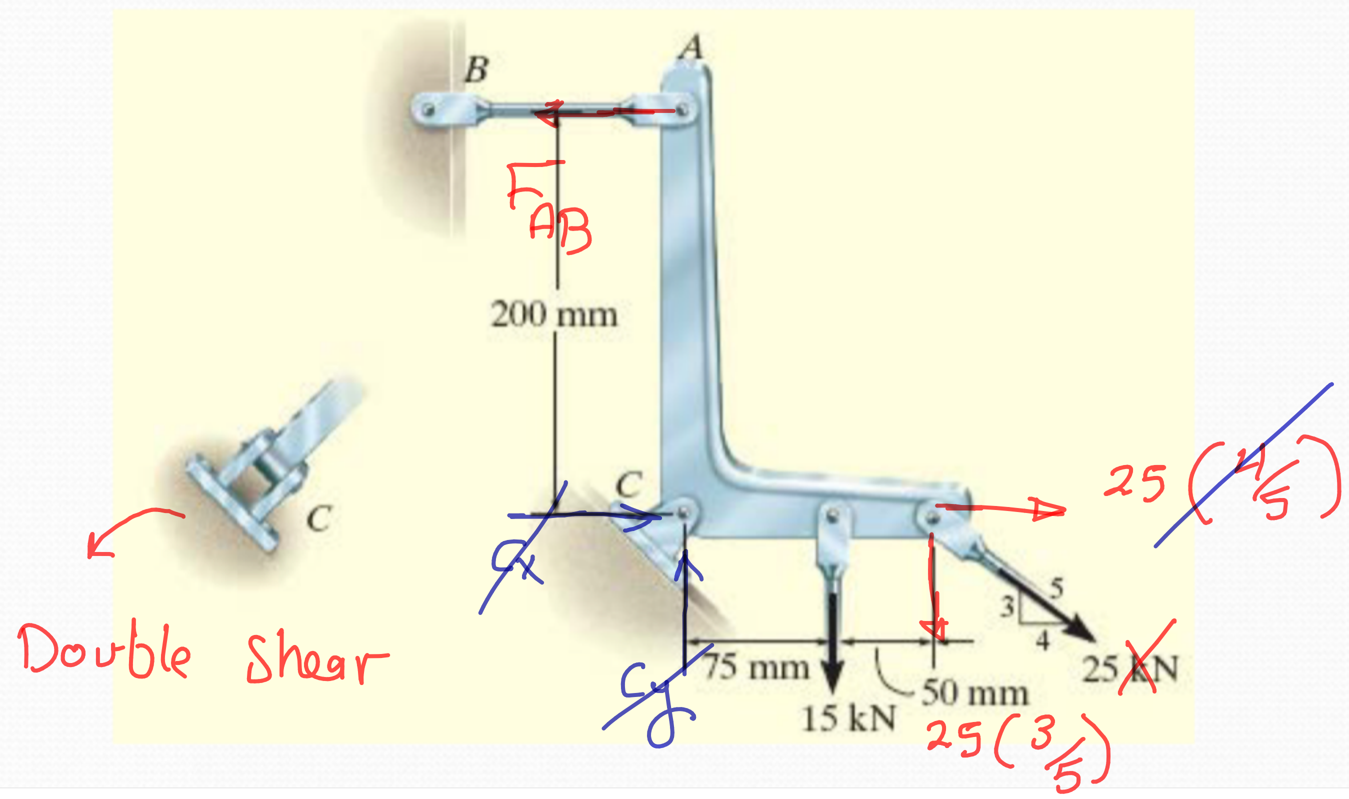
80 MPa

F.C

$$F.S = \frac{P_{ult}}{P_{all}}$$

Example 1.11

The control arm is subjected to the loading. Determine to the nearest 5 mm the required diameter of the steel pin at C if the allowable shear stress for the steel is $\tau_{allowable} = 55\ \text{MPa}$. Note in the figure that the pin is subjected to double shear.



$$\sum M_C = 0 \quad \circlearrowleft +$$

$$F_{AB} * 0.2 - 15 * 0.075 - 25\left(\tfrac{3}{5}\right) * 0.125 = 0$$

$$F_{AB} = 15\ \text{KN}$$

$$\sum F_x = 0 \quad \xrightarrow{+}$$

$$-15 + C_x + 25\left(\tfrac{4}{5}\right) = 0$$

$$C_x = -5\ \text{kN}$$

$$\sum F_y = 0 \quad \uparrow +$$

$$C_y - 15 - 25\left(\tfrac{3}{5}\right) = 0$$

$$C_y = 30\ \text{KN}$$

$$F_C = \sqrt{5^2 + 30^2} = 30.41\ \text{kN}$$

$$V = \frac{F_C}{2} = \frac{30.41}{2} = 15.205\ \text{kN}$$

$$\tau = \frac{V}{A} = \frac{4V}{\pi D^2}$$

$$D = \sqrt{\frac{4V}{\pi \tau}}$$

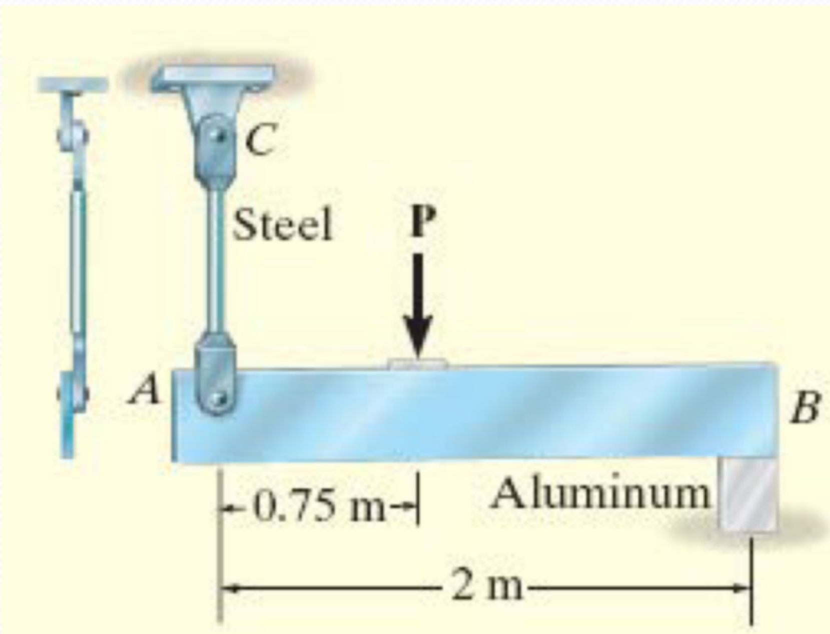
$$D = \sqrt{\frac{4 * 15.205 * 1000}{\pi * 55 * 10^6}}$$

$$= 0.0188\ \text{M} = 18.8\ \text{MM} \approx 20\ \text{mm}$$

Example 1.12

$r = 10$ mm

The rigid bar *AB* supported by a steel rod *AC* having a diameter of 20 mm and an aluminum block having a cross sectional area of 1800 mm^2. The 18-mm-diameter pins at *A* and *C* are subjected to *single shear*. If the failure stress for the steel and aluminum is $(\sigma_{st})_{fail} = 680$ MPa & $(\sigma_{al})_{fail} = 70$ MPa respectively, and the failure shear stress for each pin is $\tau_{fail} = 900$ MPa, determine the largest load *P* that can be applied to the bar. Apply a factor of safety of F.S. = 2.

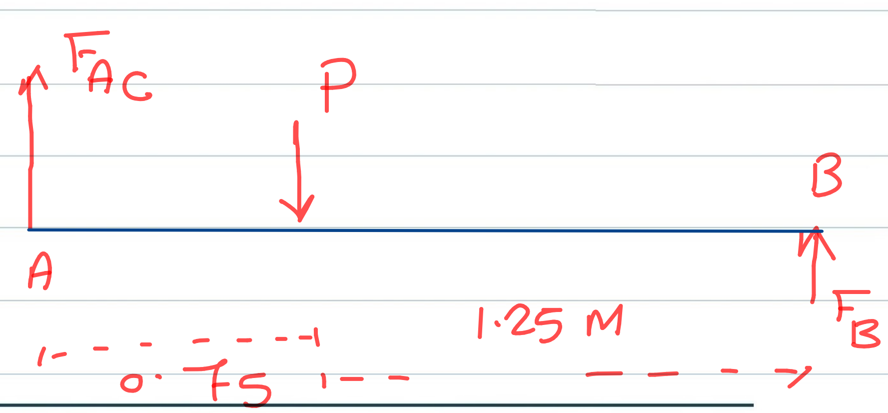


① $(\sigma_{st})_{all} = \dfrac{(\sigma_{st})_{Fail}}{F.S} = \dfrac{680}{2} = 340$ MPa

$(\sigma_{aL})_{aLL} = \dfrac{(\sigma_{aL})_{Fail}}{F.S} = \dfrac{70}{2} = 35$ MPa

$\tau_{all} = \dfrac{\tau_{FaiL}}{F.S} = \dfrac{900}{2} = 450$ MPa

② static



$\sum M_B = 0$ ↺ +

$P * 1.25 - F_{AC} * 2 = 0$

$P = 1.6 F_{AC} \Rightarrow$ ①

$\sum M_A = 0$ ↺ +

$-P * 0.75 + F_B * 2 = 0$

$P = 2.67 F_B \Rightarrow$ ②

For Rod AC :- $\sigma_{all} = F/A$

$F_{AC} = (\sigma_{St})_{all} \; A_{AC} = 340 * 10^6 * \pi (0.01)^2$

$= 106.8$ kN

in Eq ① $P = 1.6 * 106.8 = 171$ kN

For Block B

$F_B = (\sigma_{aL})_{all} * A_B = 35 * 10^6 * 1800 * 10^{-6}$

$= 63$ kN

in Eq ② $P = 2.67 * 63 = 168$ kN

For Pin A or C :-

$$V = F_{AC} = \tau_{all} \, A_{volt}$$

$$= 450 * 10^6 * \pi (0.009)^2$$

$$= 114.5$$

in Eq^n ①

$$P = 1.6 * 114.5 = 183 \text{ kN}$$

Smallest Value Control :-

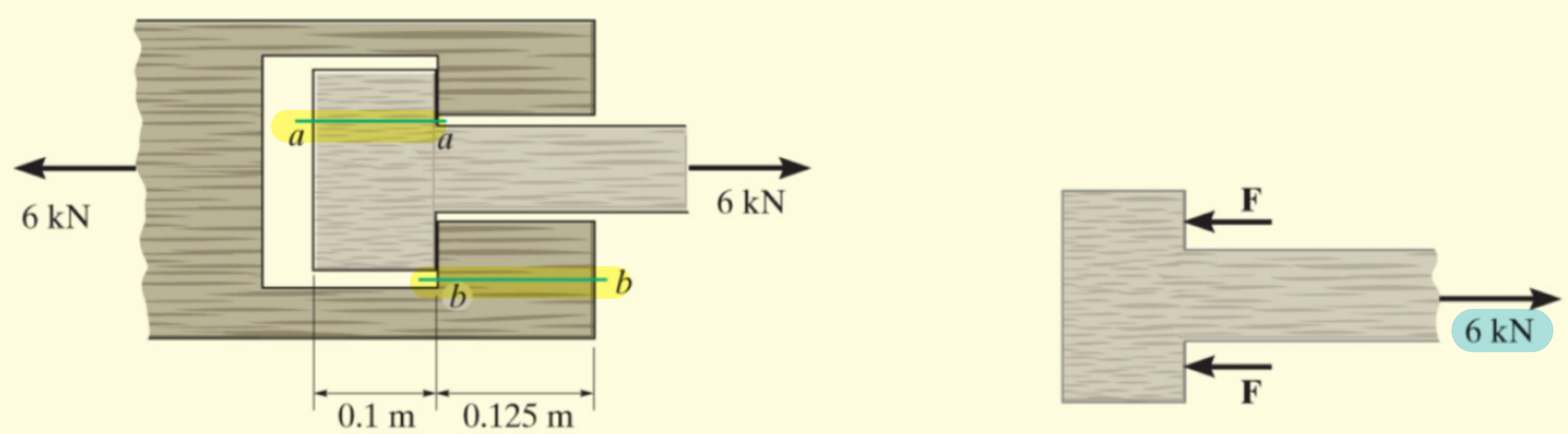
$$\boxed{P = 168 \text{ kN}}$$

(*)

lec ④ Finished

EXAMPLE 1.10

If the wood joint in Fig. 1–22*a* has a width of 150 mm, determine the average shear stress developed along shear planes *a–a* and *b–b*. For each plane, represent the state of stress on an element of the material.



$\Sigma F_x = 0 \quad \xrightarrow{+}$

$6 - 2F = 0 \qquad F = 3\ kN$

Section a – a

$\Sigma F_x = 0 \quad \xrightarrow{+}$

3 kN

V_a

$V_a = 3\ kN$

$$\tau_a = V_a / A_a = \frac{3 * 10^3}{0.1 * 0.15} = 200\ kPa$$

Section b – b

3 kN

V_b

$\Sigma F_x = 0 \quad \xrightarrow{+}$

$3 - V_b = 0$

$\boxed{V_b = 3\ kN}$

$$\tau_b = \frac{V_b}{A_b} = \frac{3 * 10^3}{0.125 * 0.15}$$

$$= 160\ kPa$$