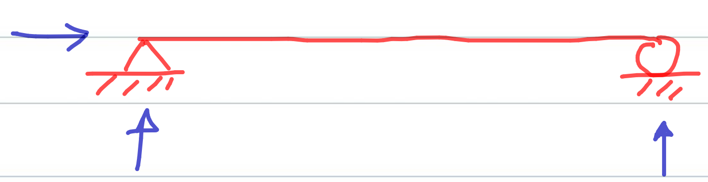
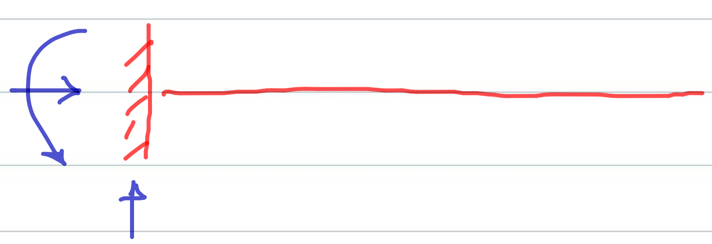
Review

* Beam } to resist M
* Column } to resist axial compressive Force

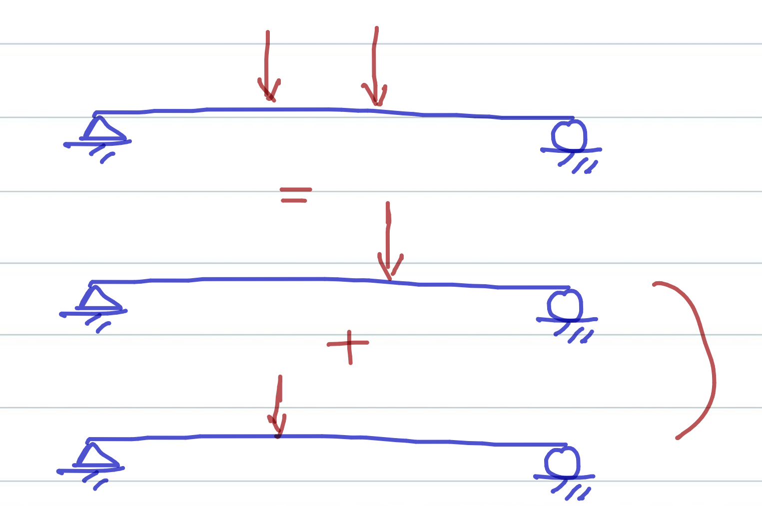
Simply supported beam

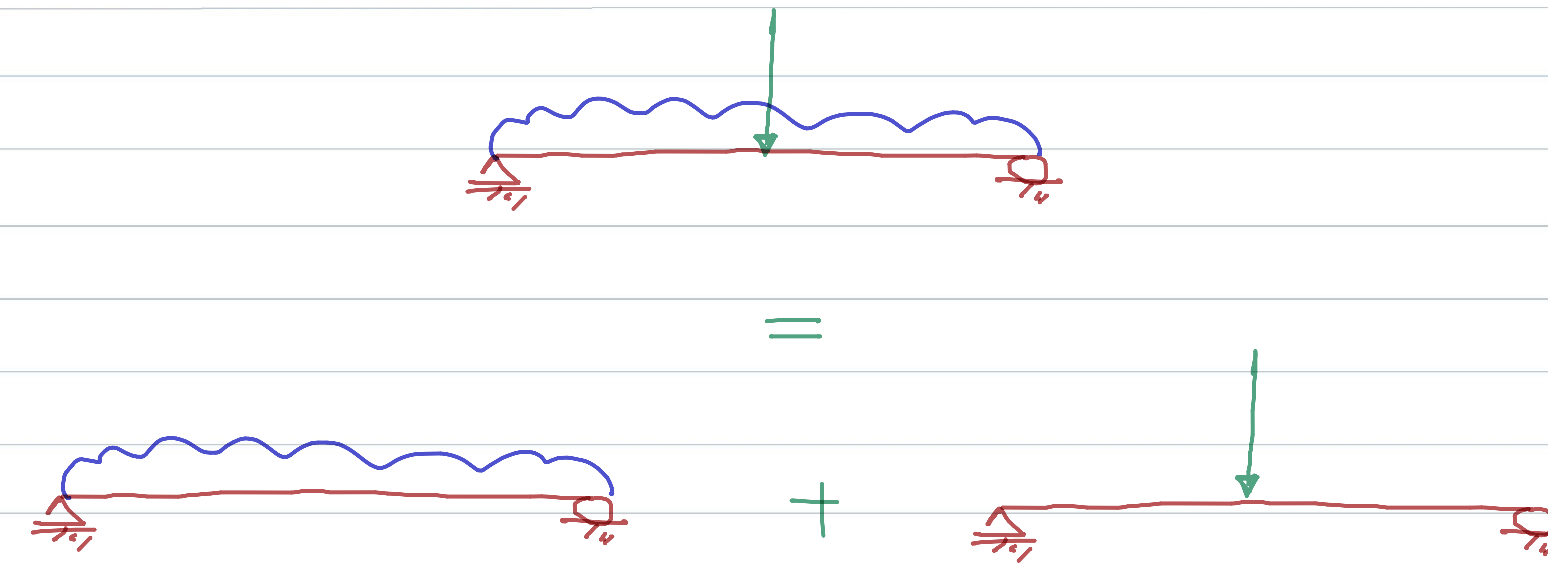


Cantilever beam



(*) Principal of Superposition :-





* $\sum F_x = 0$ $\overset{+}{\rightarrow}$

$\sum F_y = 0$ $\uparrow +$

$\sum M = 0$ ↺ +

} Equilibrium Eq's

Beam } n members
r reactions

$r = 3n$ } Statically determinate

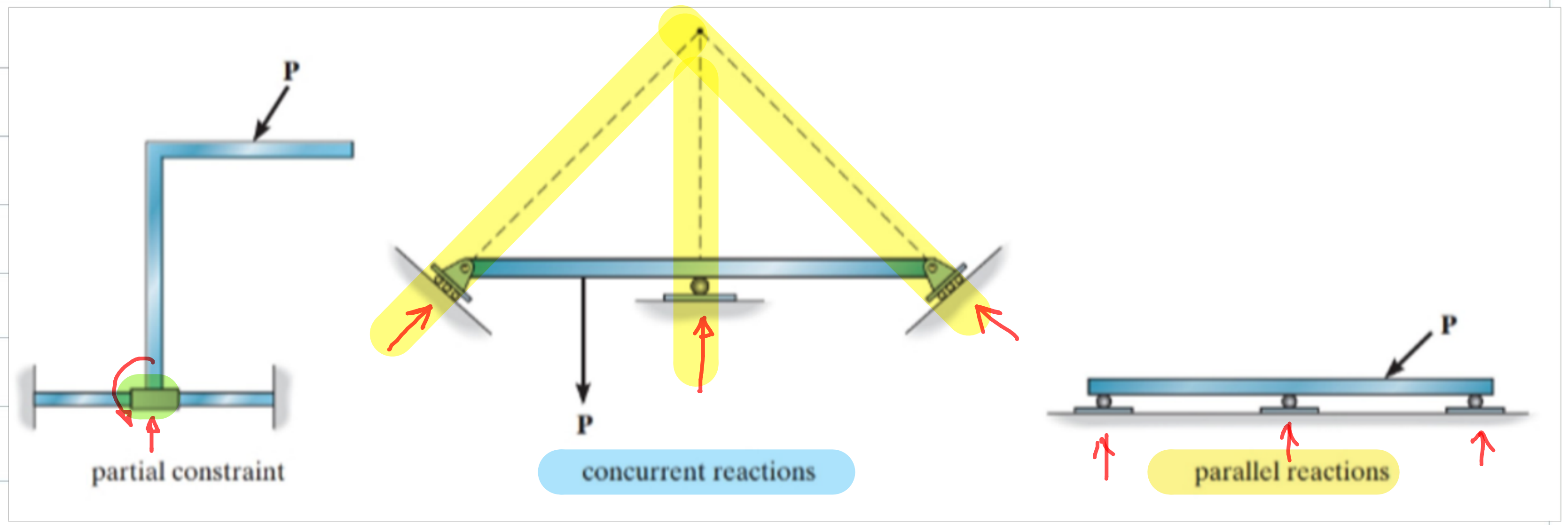
\# unknowns ≤ 3

$r > 3n$ } Statically indeterminate

\# unknowns > 3

more stable

Stability



Reaction < Eq^{u}/s

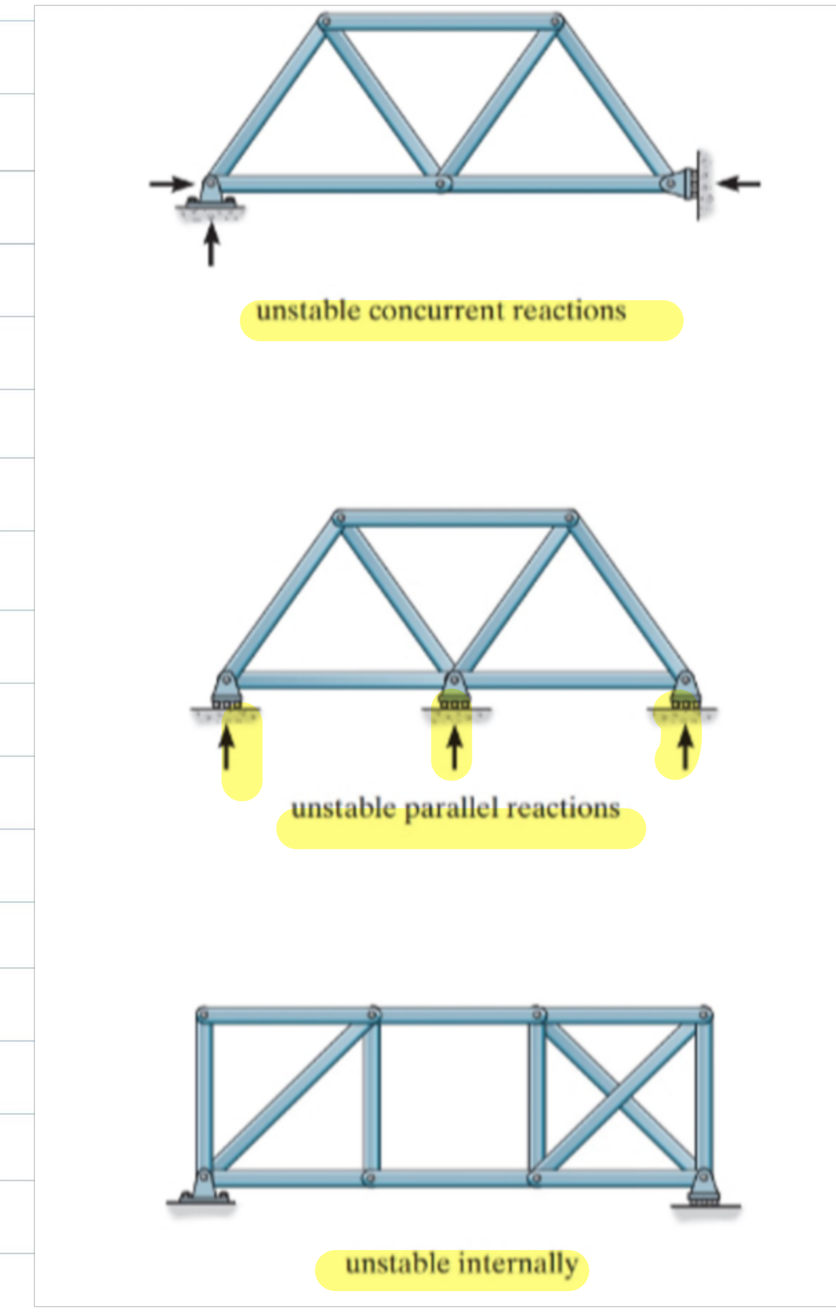
unstable

trusses

- \# members = b
- \# reaction = r
- \# Joints = J

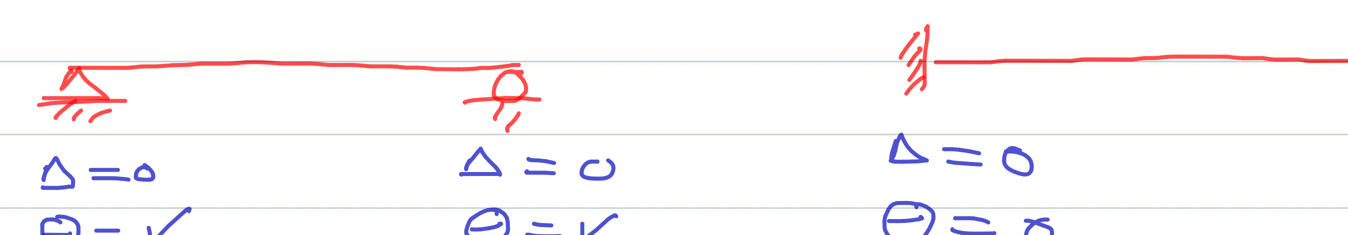
$b + r = 2j$ } statically determinate

$b + r > 2j$ } statically indeterminate



$b + r < 2j$ } unstable

$$\frac{d^2 v}{dx^2} = \frac{M}{EI}$$ } Deflection



Methods of analysis

Equilibrium

$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma M = 0$

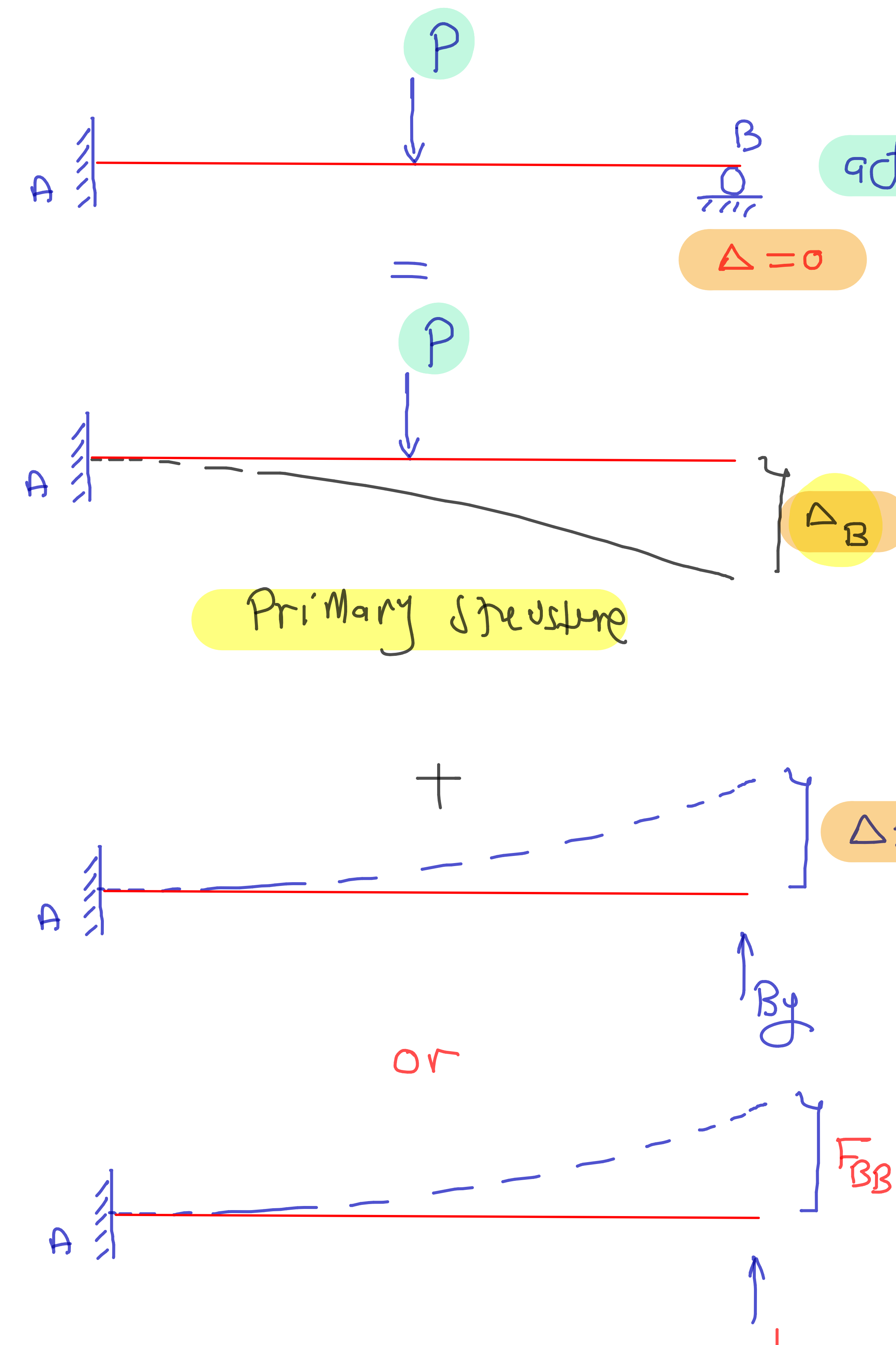
$\Big\}$ 3-Eq's

Compatibility

(1) Force Method

(2) Displacement Method

Ex



4-unknowns

3-Eq's

1st degree

⇓

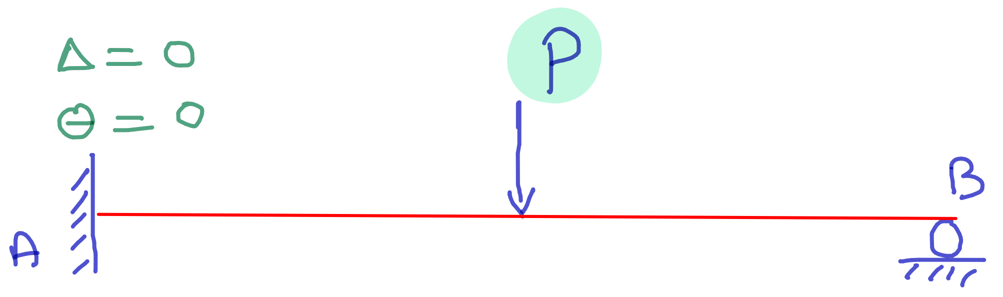
1-redundant @ B

$0 = -\Delta_B + \Delta_B'$

$0 = -\Delta_B + B_y f_{B1}$

$\Delta = 0$

$\theta = 0$



4 - unknowns

3 - Equ.

1st degree

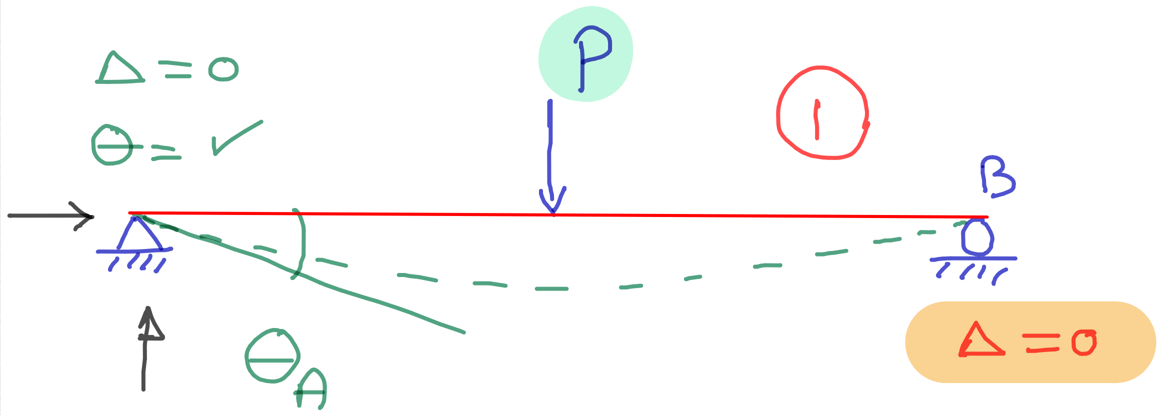
⇓

1 - redundant

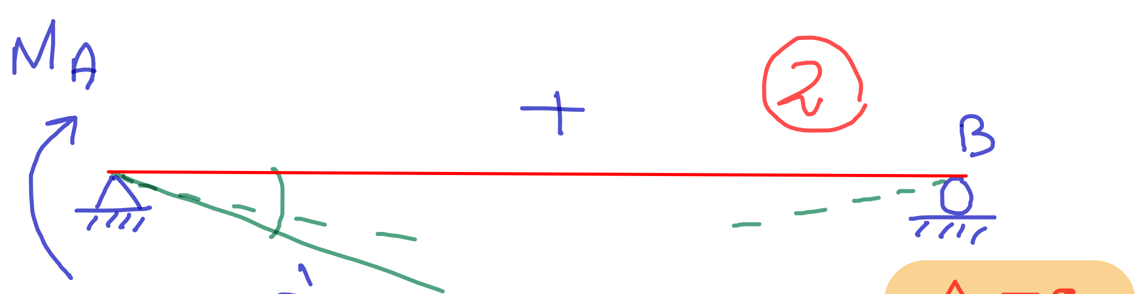
@ B

$\Delta = 0$

$\theta = \checkmark$

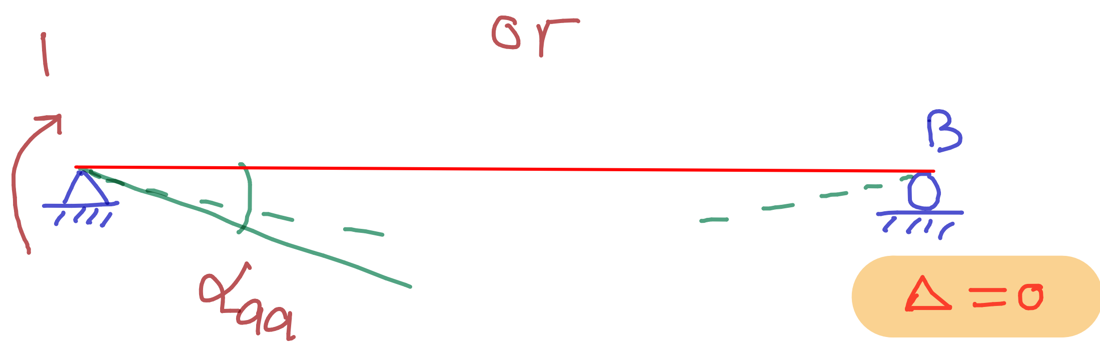


Primary structure



$\theta'_A = M_A \, \alpha_{AA}$

or



$$0 = \theta_A + \theta'_A$$

$$0 = \theta_A + M_A \, \alpha_{AA}$$

⇓

to get Moment M_A

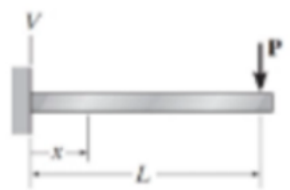
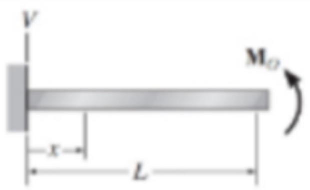
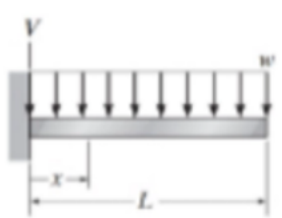
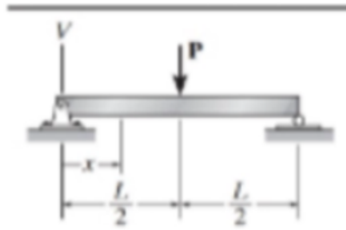
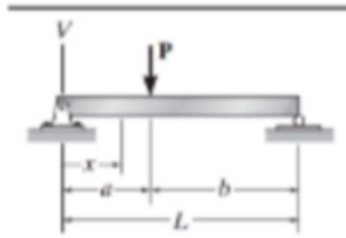
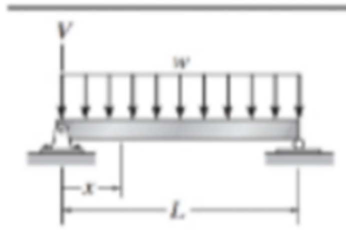
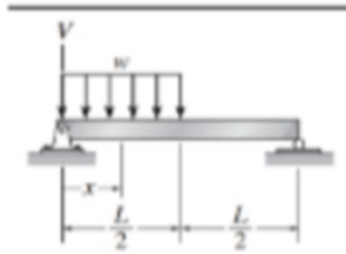
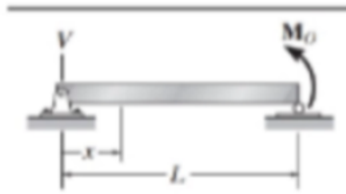
Then we

$\sum F_x = 0$ $\xrightarrow{+}$

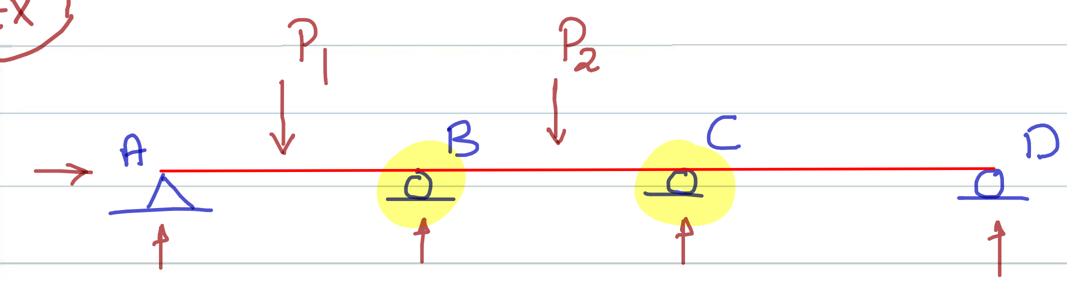
$\sum F_y = 0$ $\uparrow +$

$\sum M = 0$ $\circlearrowleft +$

Beam Deflections and Slopes

Loading	v + ↑	θ + ↶	Equation + ↑ + ↶
	$v_{max} = -\frac{PL^3}{3EI}$ at $x = L$	$\theta_{max} = -\frac{PL^2}{2EI}$ at $x = L$	$v = \frac{P}{6EI}(x^3 - 3Lx^2)$
	$v_{max} = \frac{M_O L^2}{2EI}$ at $x = L$	$\theta_{max} = \frac{M_O L}{EI}$ at $x = L$	$v = \frac{M_O}{2EI}x^2$
	$v_{max} = -\frac{wL^4}{8EI}$ at $x = L$	$\theta_{max} = -\frac{wL^3}{6EI}$ at $x = L$	$v = -\frac{w}{24EI}(x^4 - 4Lx^3 + 6L^2x^2)$
	$v_{max} = -\frac{PL^3}{48EI}$ at $x = L/2$	$\theta_{max} = \pm\frac{PL^2}{16EI}$ at $x = 0$ or $x = L$	$v = \frac{P}{48EI}(4x^3 - 3L^2x)$, $0 \le x \le L/2$
		$\theta_L = -\frac{Pab(L+b)}{6LEI}$ $\theta_R = \frac{Pab(L+a)}{6LEI}$	$v = -\frac{Pbx}{6LEI}(L^2 - b^2 - x^2)$ $0 \le x \le a$
	$v_{max} = -\frac{5wL^4}{384EI}$ at $x = \frac{L}{2}$	$\theta_{max} = \pm\frac{wL^3}{24EI}$	$v = -\frac{wx}{24EI}(x^3 - 2Lx^2 + L^3)$
		$\theta_L = -\frac{3wL^3}{128EI}$ $\theta_R = \frac{7wL^3}{384EI}$	$v = -\frac{wx}{384EI}(16x^3 - 24Lx^2 + 9L^3)$ $0 \le x \le L/2$ $v = -\frac{wL}{384EI}(8x^3 - 24Lx^2 + 17L^2x - L^3)$ $L/2 \le x \le L$
	$v_{max} = -\frac{M_O L^2}{9\sqrt{3}EI}$	$\theta_L = -\frac{M_O L}{6EI}$ $\theta_R = \frac{M_O L}{3EI}$	$v = -\frac{M_O x}{6EIL}(L^2 - x^2)$

Ex



actual beam

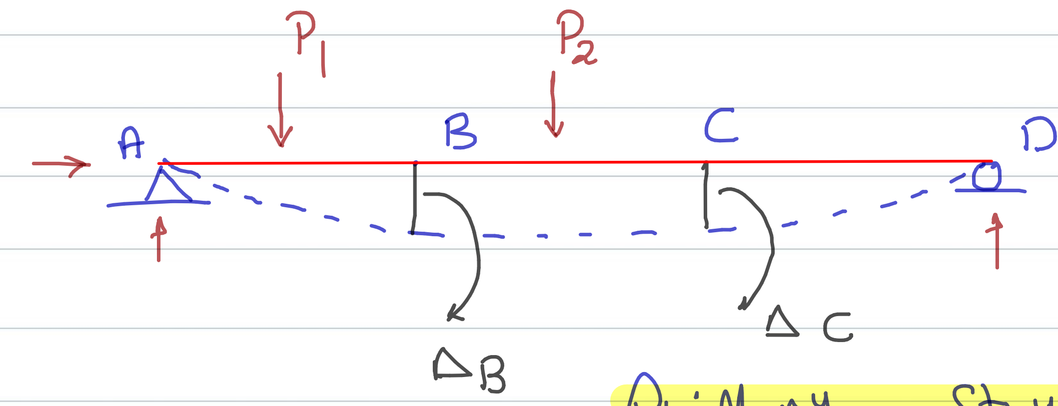
=

5-unknowns

3-Reaction

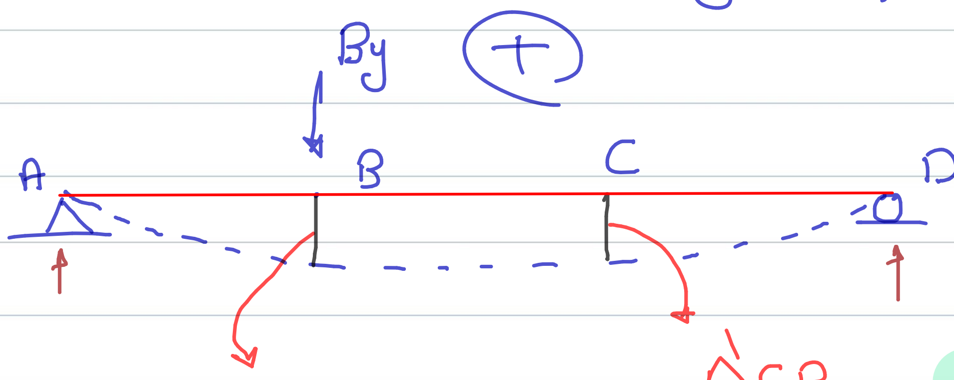
2nd-degree

2 redundant



Primary Structure

+



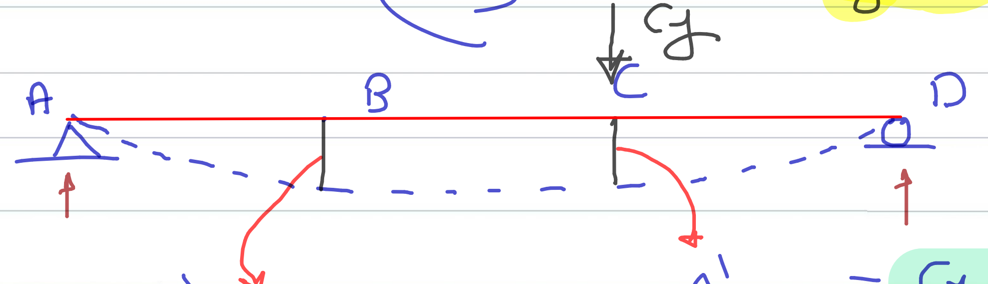
$\Delta'_{BB} = B_y F_{BB}$

$\Delta'_{CB} = B_y F_{CB}$

$\Delta'_{CB} = \Delta'_{BC}$

redundant By Cy applied

+



$\Delta'_{BC} = C_y F_{BC}$

$\Delta'_{CC} = C_y F_{CC}$

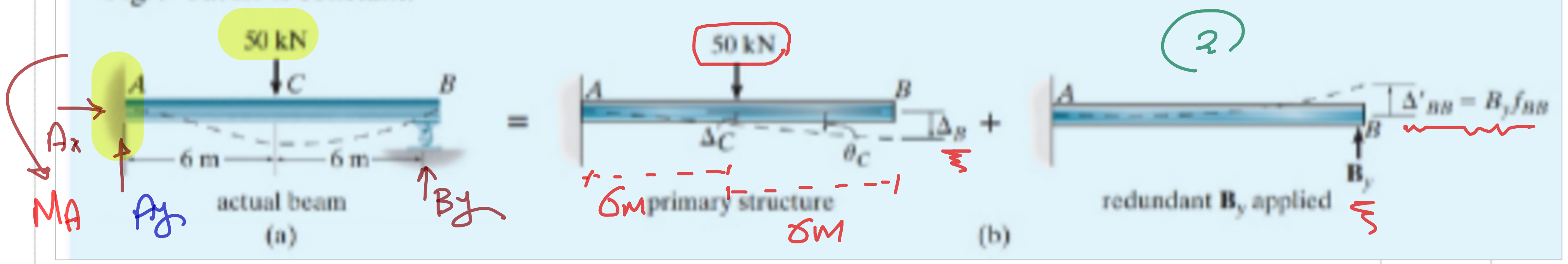
$\Delta_B - \Delta'_{BB} - \Delta'_{BC} = 0 \Rightarrow (1)$

$\Delta_C - \Delta'_{CB} - \Delta'_{CC} = 0 \Rightarrow (2)$

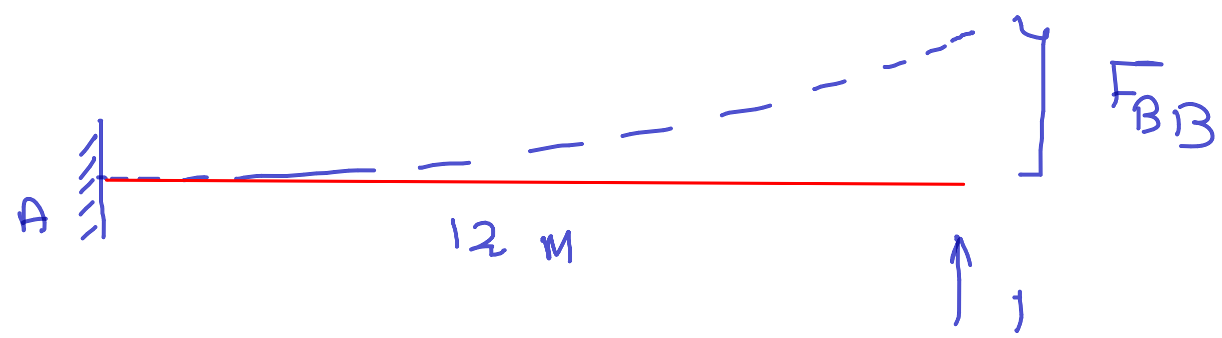
Then Eq of Equilibrium

EXAMPLE 9.1

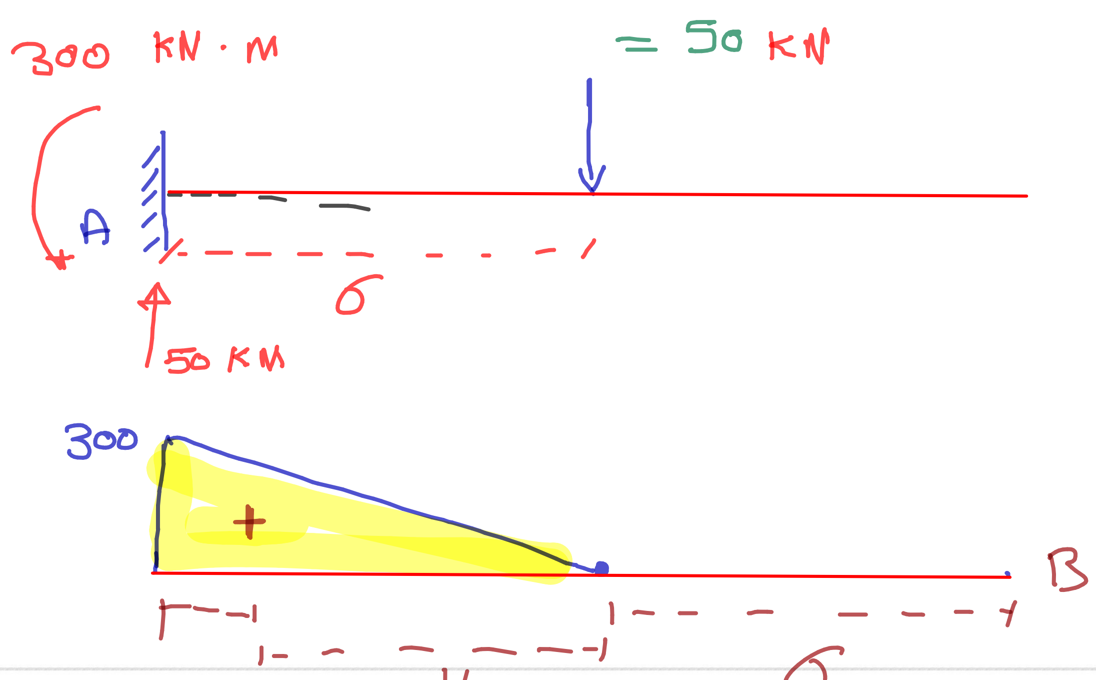
Determine the reaction at the roller support *B* of the beam shown in Fig. 9–9*a*. *EI* is constant.



$$0 = -\Delta_B + B_y F_{BB} \quad (1)$$



$$F_{BB} = \frac{PL^3}{3EI} = \frac{1 * 12^3}{3EI} = \frac{576}{EI}$$



$$\Delta_B = \frac{1}{EI}\left(\tfrac{1}{2} * 6 * 300 * 10\right)$$

$$= \frac{9000}{EI}$$

$$0 = -\frac{9000}{\cancel{EI}} + B_y\left(\frac{576}{\cancel{EI}}\right)$$

$$B_y = 15.6 \text{ kN}$$

* $\Sigma F_y = 0$ ↑+

$$A_y - 50 + 15.6 = 0$$

$A_y = 34.4$ kN

* $\Sigma M_A = 0$ ↺+

$$M_A - 50 * 6 + 15.6 * 12 = 0$$

$$M_A = 112.8 \text{ kN·M}$$