CHAPTER 1

Introduction – Concept of Stress

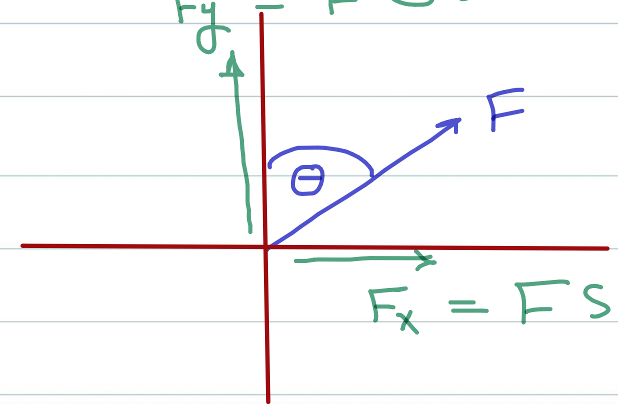
σ_{av}, τ_{av}

Rectangular/Cartesian Components Method

$$\vec{F} = (F_x)\hat{i} + (F_y)\hat{j}$$

$$F = \sqrt{F_x^2 + F_y^2}$$

$F_y = F \cos\theta$



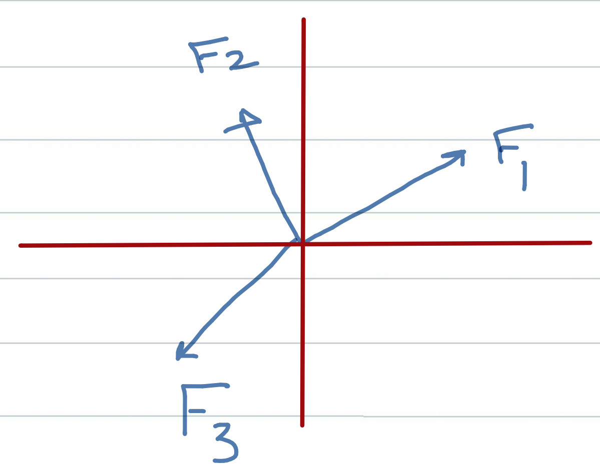
$F_x = F \sin\theta$

Equilibrium of a Particle

} @ rest

1. Resolve
2. $\Sigma F_x = 0$ $\overset{+}{\rightarrow}$

   $\Sigma F_y = 0$ $\uparrow +$



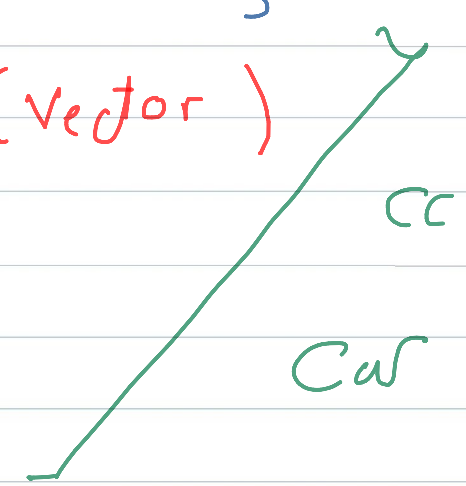
moment of the force

(vector)

$$M = \Sigma F * d_{\perp}$$

x y

y x



ccw +

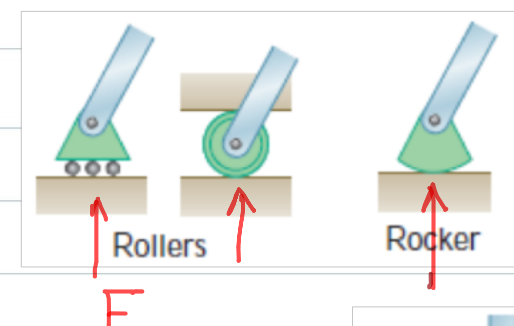
cw (−)

Equilibrium of Rigid Bodies

$\Sigma F_x = 0$ $\overset{+}{\rightarrow}$ $\Sigma F_y = 0$ $\uparrow +$ $\Sigma M = 0$ ↺ +

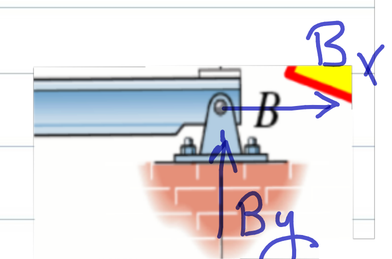
Support Reactions :-

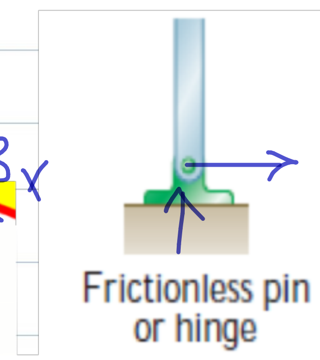
1. Roller



1 – Reaction

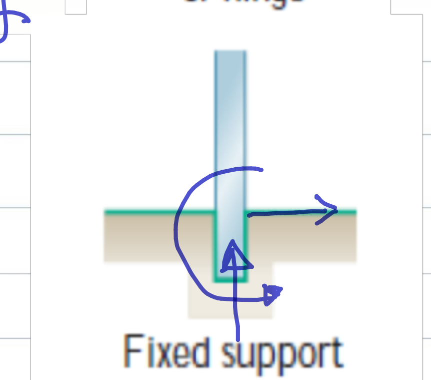
2. Pin or hinge





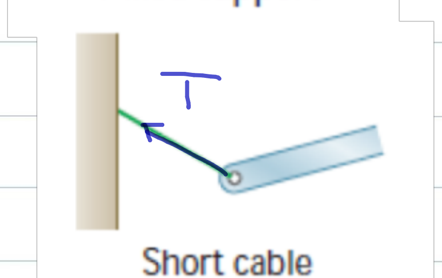
2 - Reaction

3. Fixed



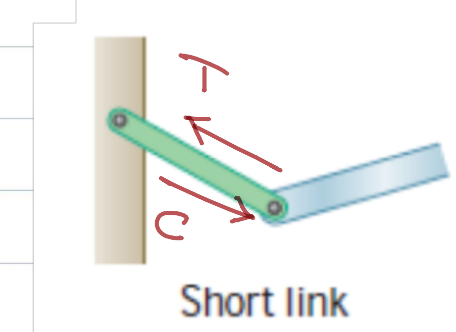
3 – Reaction

4. Cable



only tension outside

5. link



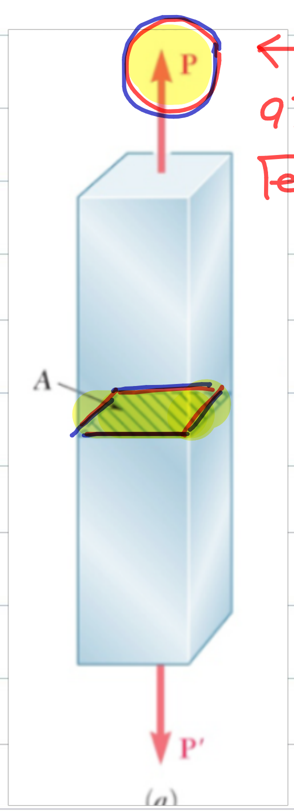
tension or compression

Concept of Stress

$$\text{Stress} = \frac{\text{Force}}{\text{Area}} \quad \left(\frac{N}{m^2} = Pa \right)$$

Normal stress

$P \perp A$



← axial Force

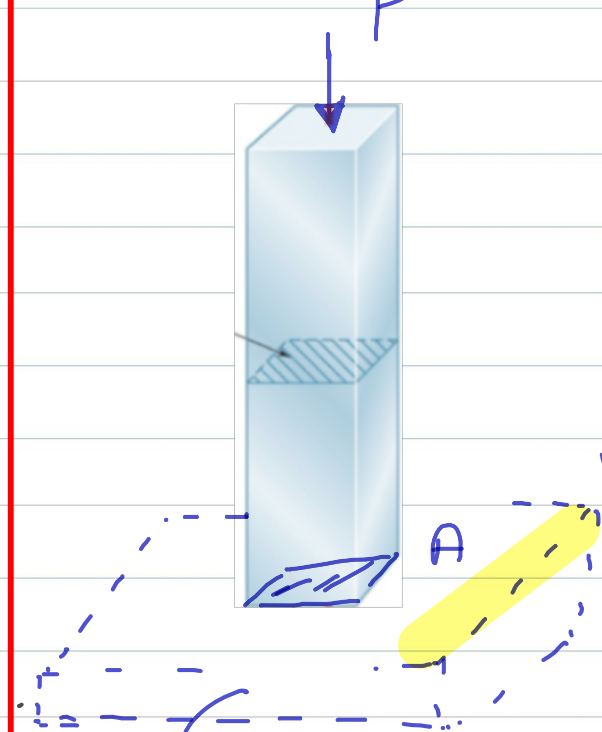
$\sigma_{av} = \frac{P}{A}$

(+) tensile stress (outside)

(−) Compression Force (inside)

Bearing stress

$P \perp A$

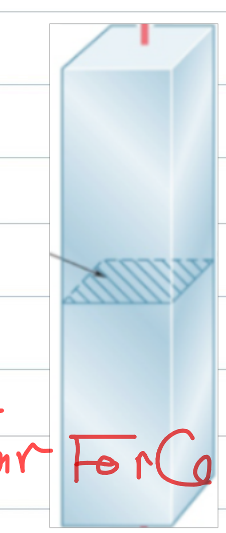


Contact Surface

$$\sigma_b = \frac{P}{A}$$

Shear stress

$P \parallel A$



P Shear Force

$\tau_{av} = \frac{P}{A}$

Single shear — Double shear

Shearing Stress Examples

Single Shear

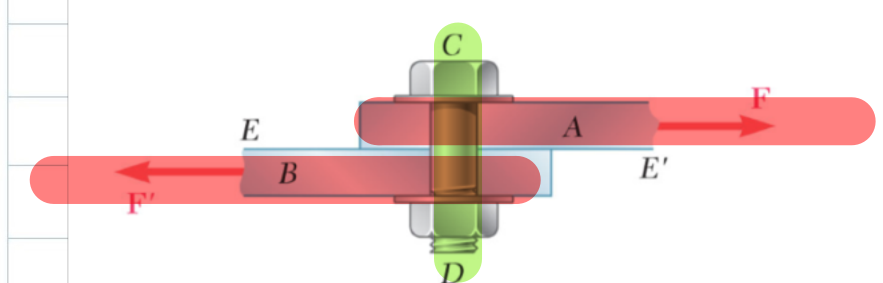
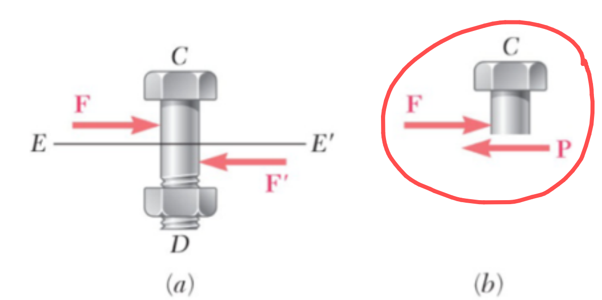


Fig. 1.16 Bolt subject to single shear.



Double Shear

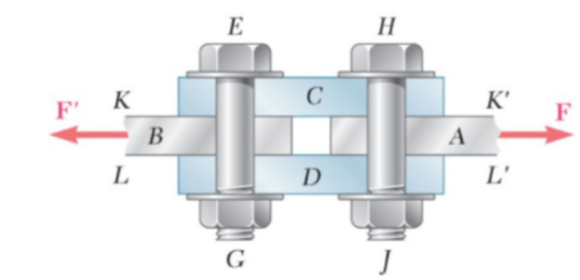
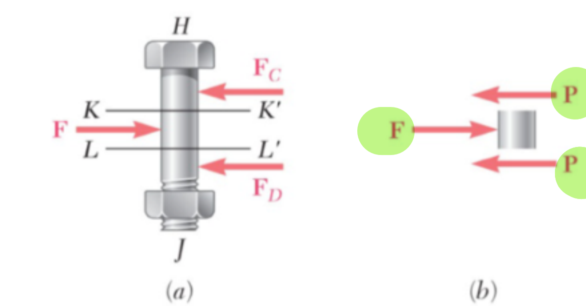
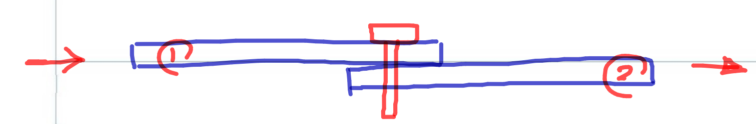


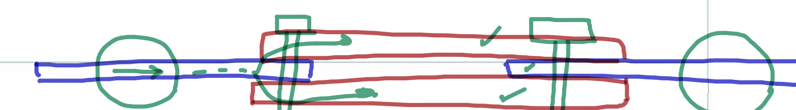
Fig. 1.18 Bolt subject to double shear.





2-Member & 1-Cut

$\tau_{av} = \frac{F}{A}$



3-Member & 2-Cut

$\tau_{av} = \frac{F}{2A_{bolt}}$

Important Note :-

- In tension member while calculate Normal Stress we take only A_{net}

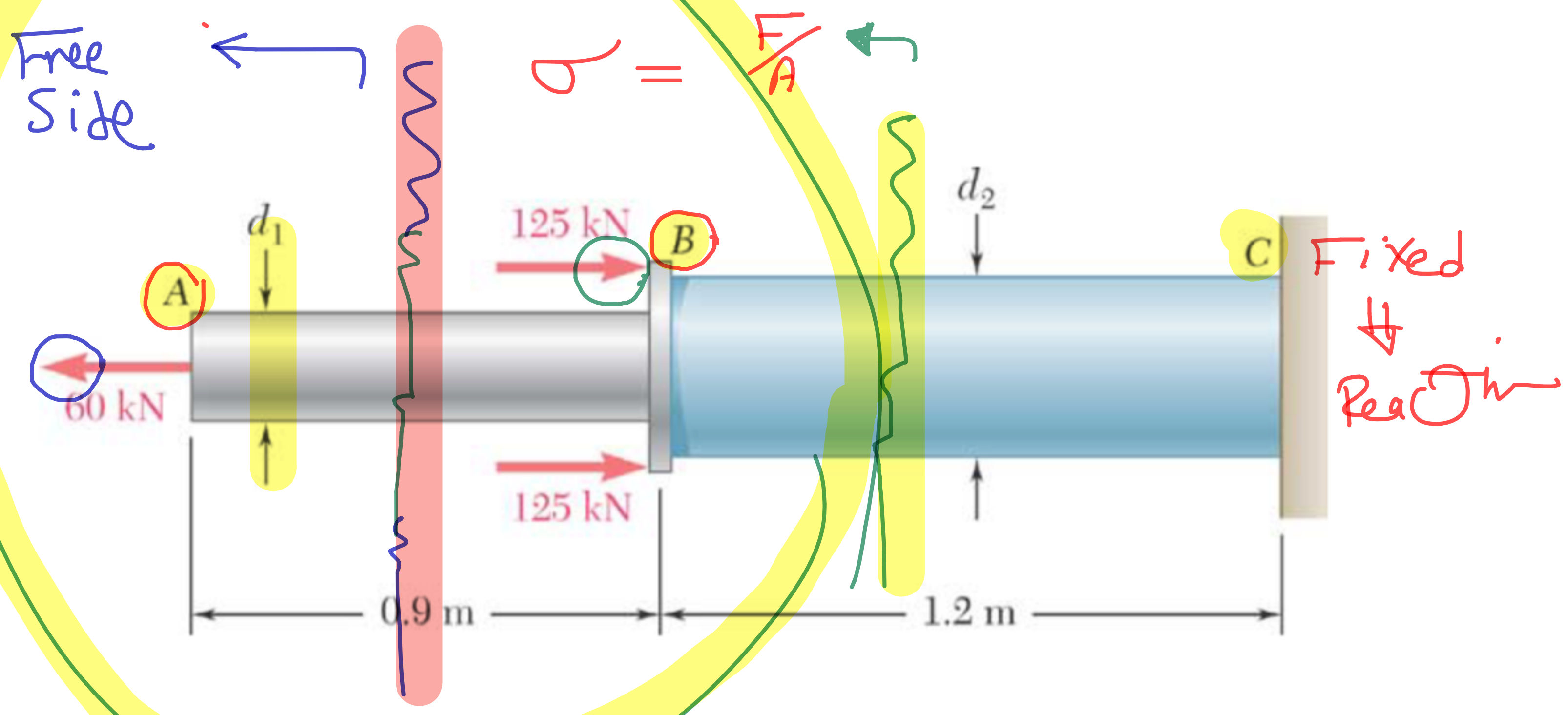
$A_{net} = (\text{Member Width (B)} - \text{Bolt Diameter (D)}) \times \text{Member thickness (t)}$

- In compression member take $A_{gross} = B \times t$

$A_{circle} = \frac{\pi d^2}{4}$ $A_{\square} = b\,h$

Problem # 1

Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Knowing that $d1 = 30$ mm and $d2 = 50$ mm, find the average normal stress at the midsection of (*a*) rod *AB*, (*b*) rod *BC*.



(a) rod AB :-

$P_{AB} = 60$ kN (tension)

$A_{AB} = \frac{\pi d_1^2}{4} = \frac{\pi}{4}\left(\frac{30}{1000}\right)^2$

$= 706.9 \times 10^{-6}$ m^2

$\sigma_{AB} = \frac{60 \times 10^3}{706.9 \times 10^{-6}} = 84.9 \times 10^6$ Pa

$= 84.9$ MPa

rod BC :-

$P_{BC} = -125 - 125 + 60 = -190$ kN

$A = \frac{\pi d_2^2}{4} = \frac{\pi}{4}\left(\frac{50}{1000}\right)^2 = 1.96 \times 10^{-3}$ m^2

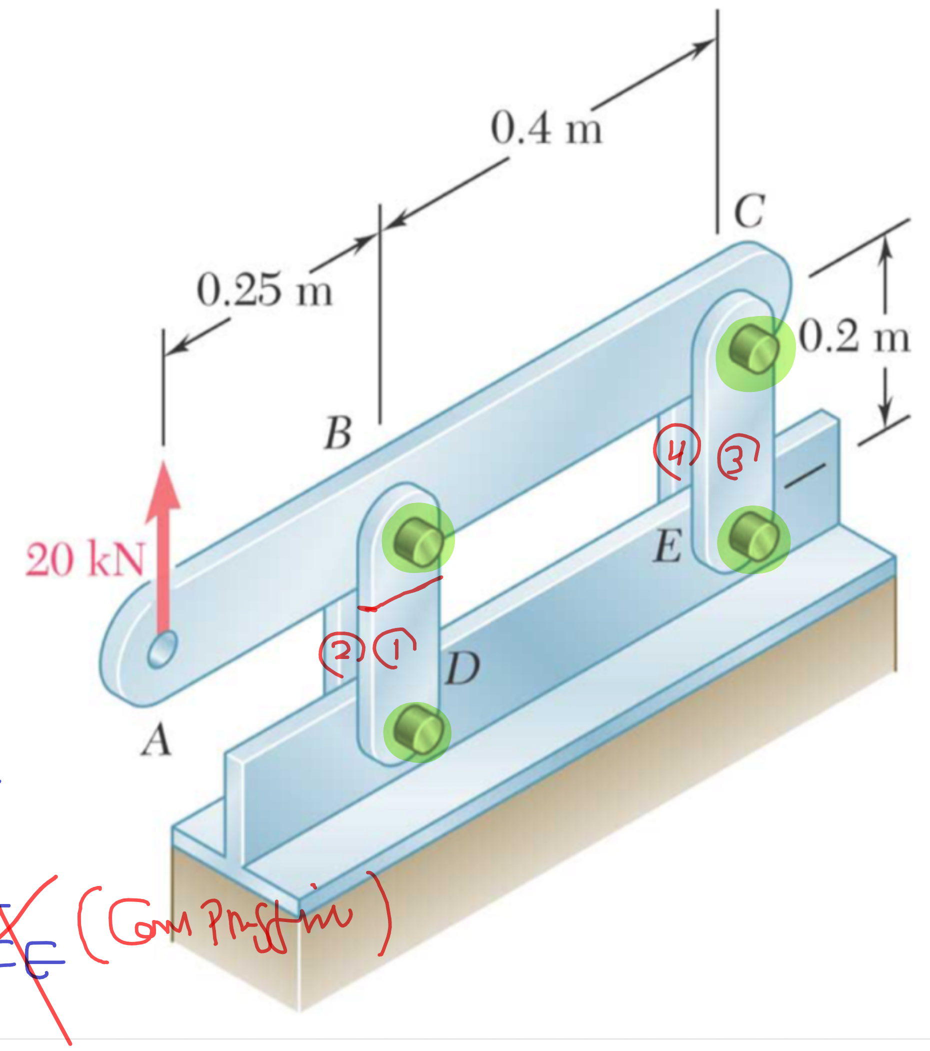
$\sigma_{BC} = \frac{P_{BC}}{A} = \frac{-190 \times 10^3}{1.963 \times 10^{-3}}$

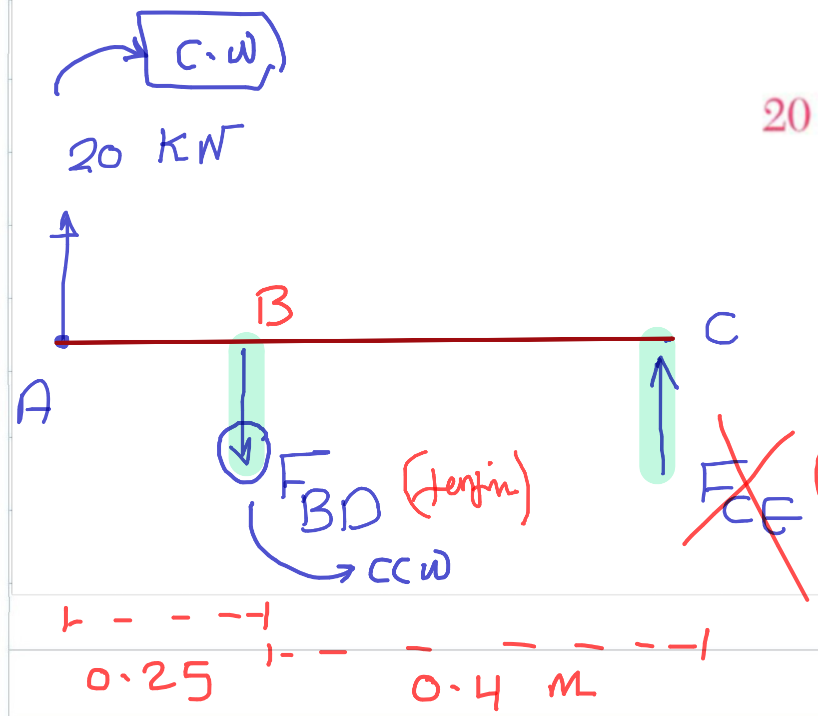
$= -96.8 \times 10^6$ Pa

$= -96.8$ MPa

Problem # 2

1.7 Each of the four vertical links has an 8 × 36-mm uniform rectangular cross section, and each of the four pins has a 16-mm diameter. Determine the maximum value of the average normal stress in the links connecting (*a*) points *B* and *D*, (*b*) points *C* and *E*.





$\Sigma M_C = 0$ ↺ +

$F_{BD} * 0.4 - 20 * (0.25 + 0.4) = 0$

$F_{BD} = 32.5$ kN } tension → A_{net}

$\Sigma F_y = 0$ ↑ +

$20 - 32.5 + F_{CE} = 0$

$F_{CE} = 12.5$ kN } Compression → A_{gross}

(a) Link BD :-

$A_{net} = 2\,(0.036 - 0.016) * 0.008$ (2 → For two links)

$= 320 * 10^{-6}$ m^2

$\sigma_{BD} = \frac{F_{BD}}{A_{net}} = \frac{32.5 * 10^3}{320 * 10^{-6}}$

$= 101.6 * 10^6$ Pa $= 101.6$ MPa

(b)

Link CE :-

$A_{gross} = 2\,(0.036)(0.008)$

$= 576 * 10^{-6}$ m^2

$\sigma_{CE} = \frac{-12.5 * 10^3}{576 * 10^{-6}} = -21.7 * 10^6$

$= -21.7$ MPa

Problem 1

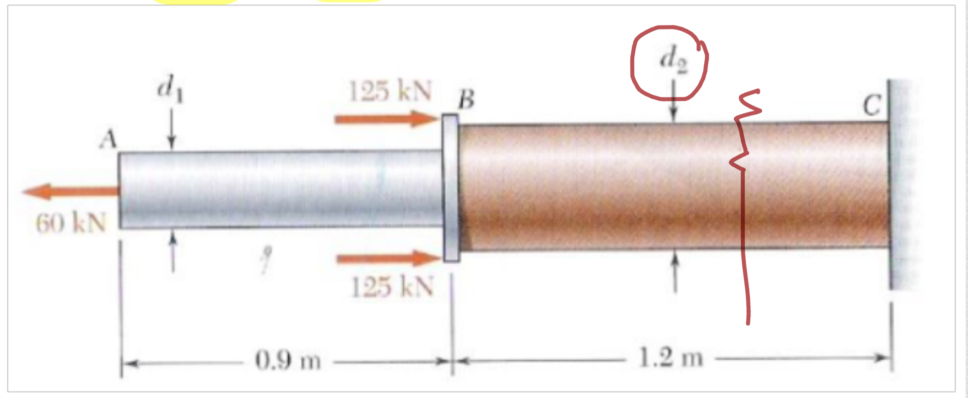
Two solid cylindrical rods **AB** and **BC** are welded together at **B** and loaded as shown

Knowing that the average normal stress must not exceed

50 MPa in either rod. $\sigma_{all} = 50\ MPa$

Determine:

The smallest allowable values of the diameter **d1** and **d2**



$$\sigma_{all} = \frac{P}{A}$$

$$\sigma_{all} = \frac{4P}{\pi D^2}$$

$$D = \sqrt{\frac{4P}{\pi \sigma_{all}}}$$

Rod AB :-

$P_{AB} = 60\ KN$ $\sigma_{all} = 50\ MPa$

$$d_1 = \sqrt{\frac{4 * 60 * 10^3}{\pi * 50 * 10^6}} = 39.1 * 10^{-3}\ M$$

$= 39.1\ MM$

rod BC :-

$$P_{BC} = -125 - 125 + 60 = -190\ kN$$

$$\sigma_{all} = -50\ MPa$$

$$d_2 = \sqrt{\frac{4 * -190 * 10^3}{\pi * -50 * 10^6}}$$

$= 69.56\ MM$

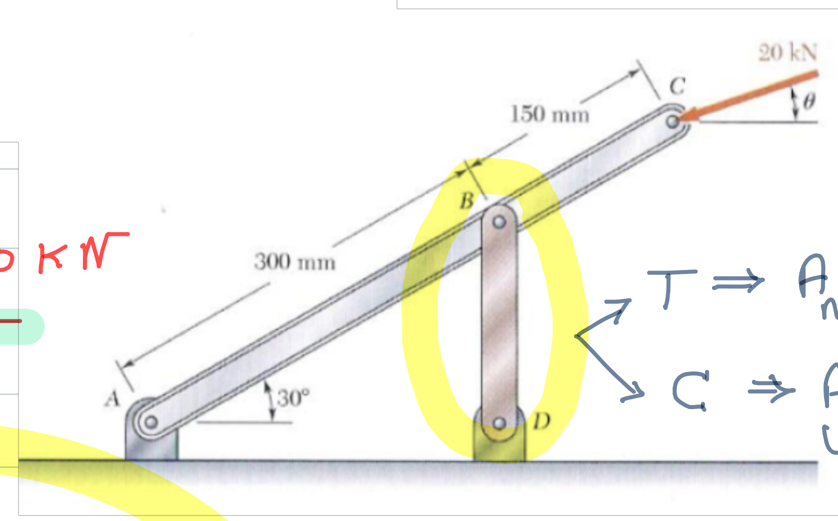
Problem 2

Link **BD** consist of a single bar 30 mm wide and 12 mm thick. Knowing that each pin has a 10 mm diameter

Determine: the maximum value of the average normal stress in link **BD** if (a) $\theta = 0$

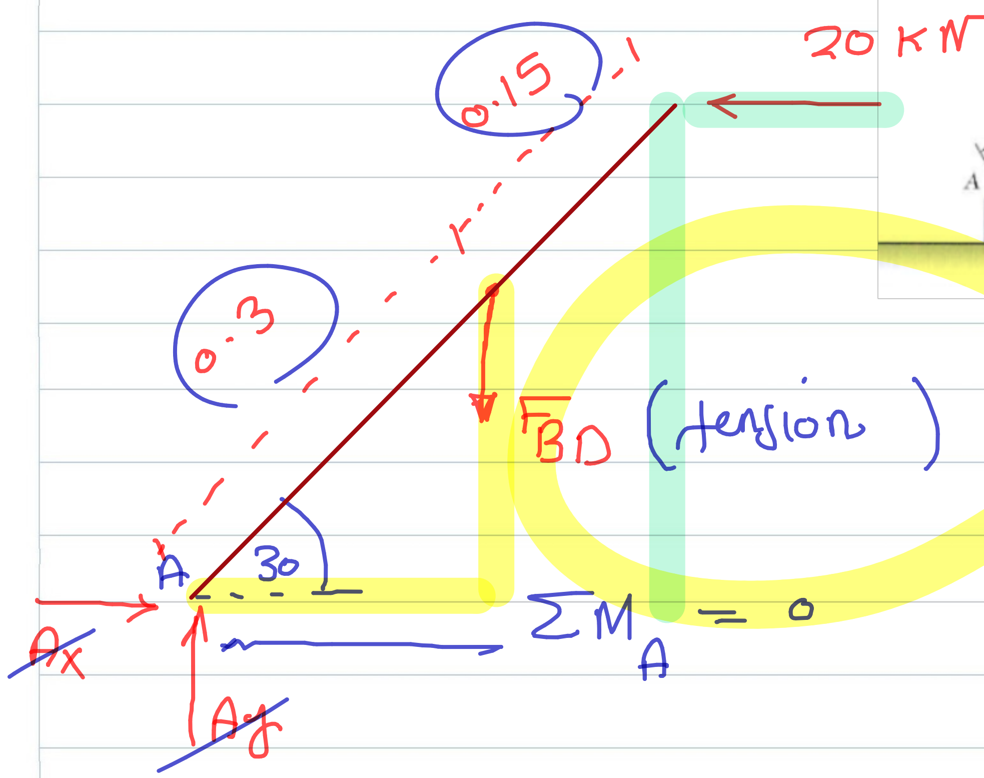
(b) $\theta = 90$

(30 mm wide and 12 mm thick → A_{gross})



T ⇒ A_{net}

C ⇒ A_{Gross}



$\Sigma M_A = 0$ ↺ +

$$-F_{BD} * 0.3 \cos 30 + 20 * 0.45 \sin 30 = 0$$

$$F_{BD} = 17.3 \text{ kN (tension)} \Rightarrow A_{net}$$

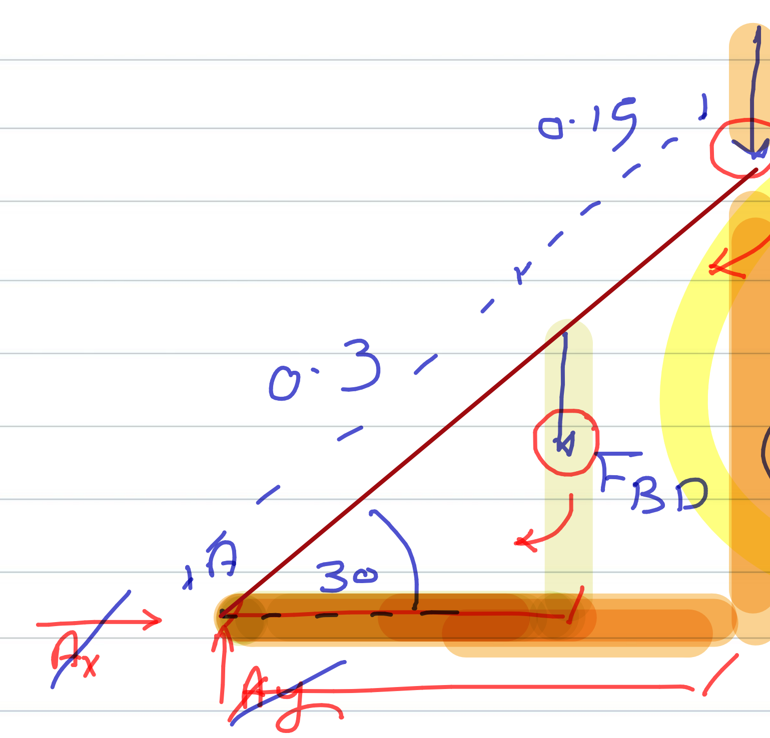
$$A_{net} = (0.03 - 0.01) * 0.012$$

$$= 240 * 10^{-6} \text{ m}^2$$

$$\sigma_{BD} = \frac{F_{BD}}{A_{net}} = \frac{17.3 * 10^3}{240 * 10^{-6}}$$

$$= 72.2 \text{ MPa}$$

(b)



$\Sigma M_A = 0$ ↺ +

$$-F_{BD} * 0.3 \cos 30 - 20 * 0.45 \cos 30 = 0$$

$$F_{BD} = -30 \text{ kN (Compression)}$$

⇓

A_{gross}

$$A_{gross} = 0.03 * 0.012 = 360 * 10^{-6} \text{ m}^2$$

$$\sigma_{BD} = \frac{-30 * 10^3}{360 * 10^{-6}} = -83.3 \text{ MPa}$$

Problem 3

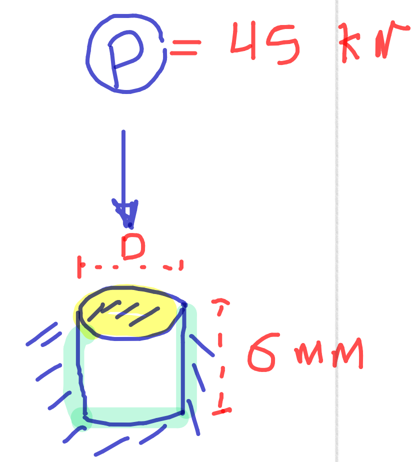
Determine: The diameter of the largest circular hole that can be punched into a sheet of polystyrene 6 mm thick, knowing that the force exerted by the punch is 45 KN and that 55 MPa average shearing stress is required to cause the material to fail.

$D = ??$

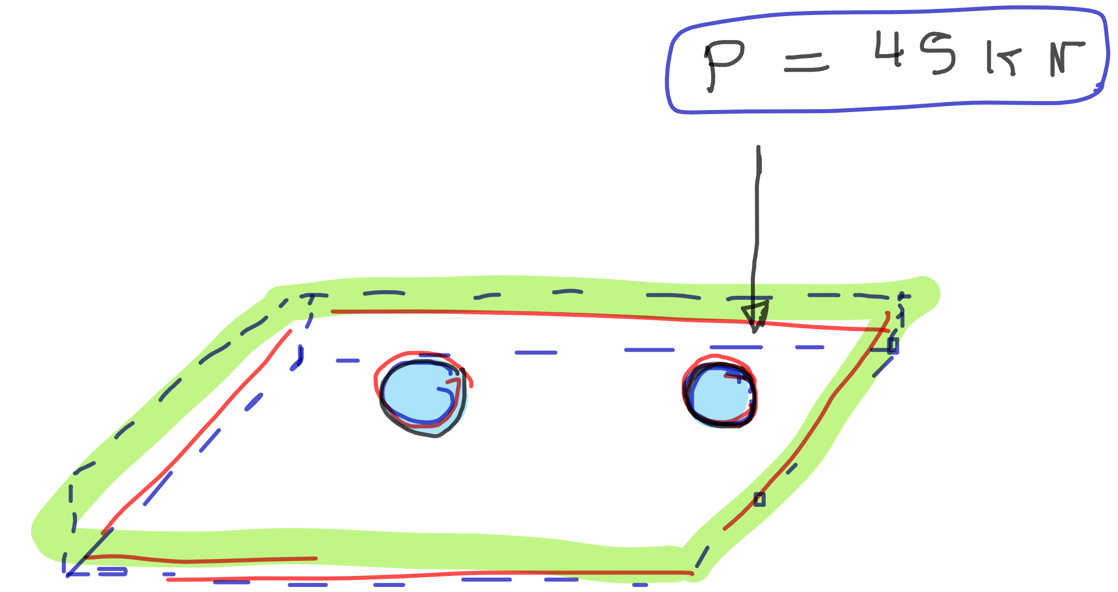
$t = 6\ mm$

$P = 45\ kN$

$\tau_{all} = 55\ MPa$



P // A



Solution

$$\tau_{all} = \frac{P}{A_{shear}} = \frac{P}{\pi D t}$$

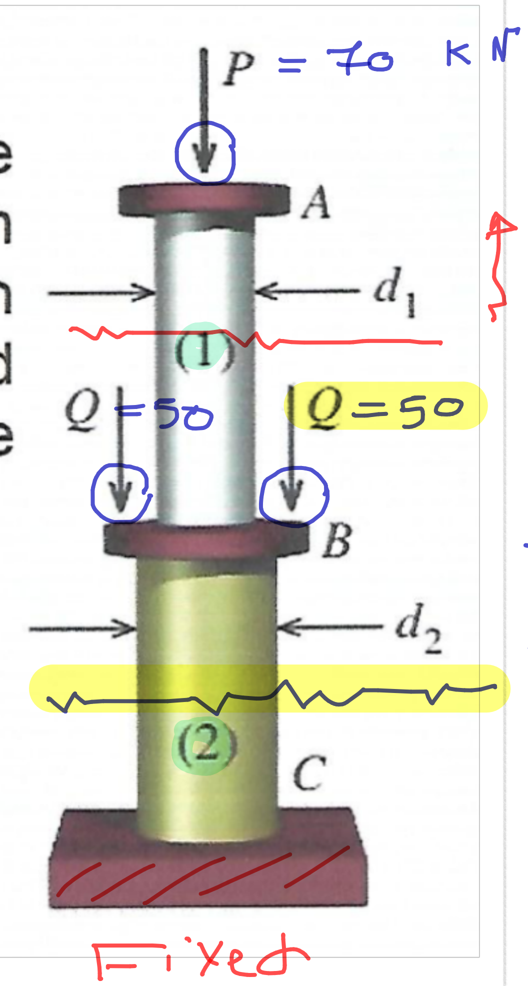
$$D = \frac{P}{\pi t \tau_{all}} = \frac{45 * 10^{3}}{\pi * 6 * 10^{-3} * 55 * 10^{6}}$$

$$= 43.4 * 10^{-3}\ (m) = 43.4\ mm$$

Quiz (1)

P1.3

Two solid cylindrical rods (1) and (2) are joined together at flange B and loaded with loads of P = 70 kN and Q = 50 kN as shown in Figure. If the normal stress in each rod must be limited to **210 MPa**, determine the minimum diameter required for each rod.



$\sigma_{all} = 210$ MPa

$\sigma_{all} = \dfrac{P}{A} = \dfrac{4P}{\pi D^2}$

$D = \sqrt{\dfrac{4P}{\pi \sigma_{all}}}$

Rod AB

$P_{AB} = -70$ kN

$\sigma_{all} = -210$ MPa

$d_1 = \sqrt{\dfrac{4 * -70 * 10^3}{\pi * -210 * 10^6}} = 20.6$ mm

Rod BC

$P_{BC} = -50 - 50 - 70 = -170$ kN

$\sigma_{all} = -210$ MPa

$d_2 = \sqrt{\dfrac{4 * -170 * 10^3}{\pi * -210 * 10^6}}$

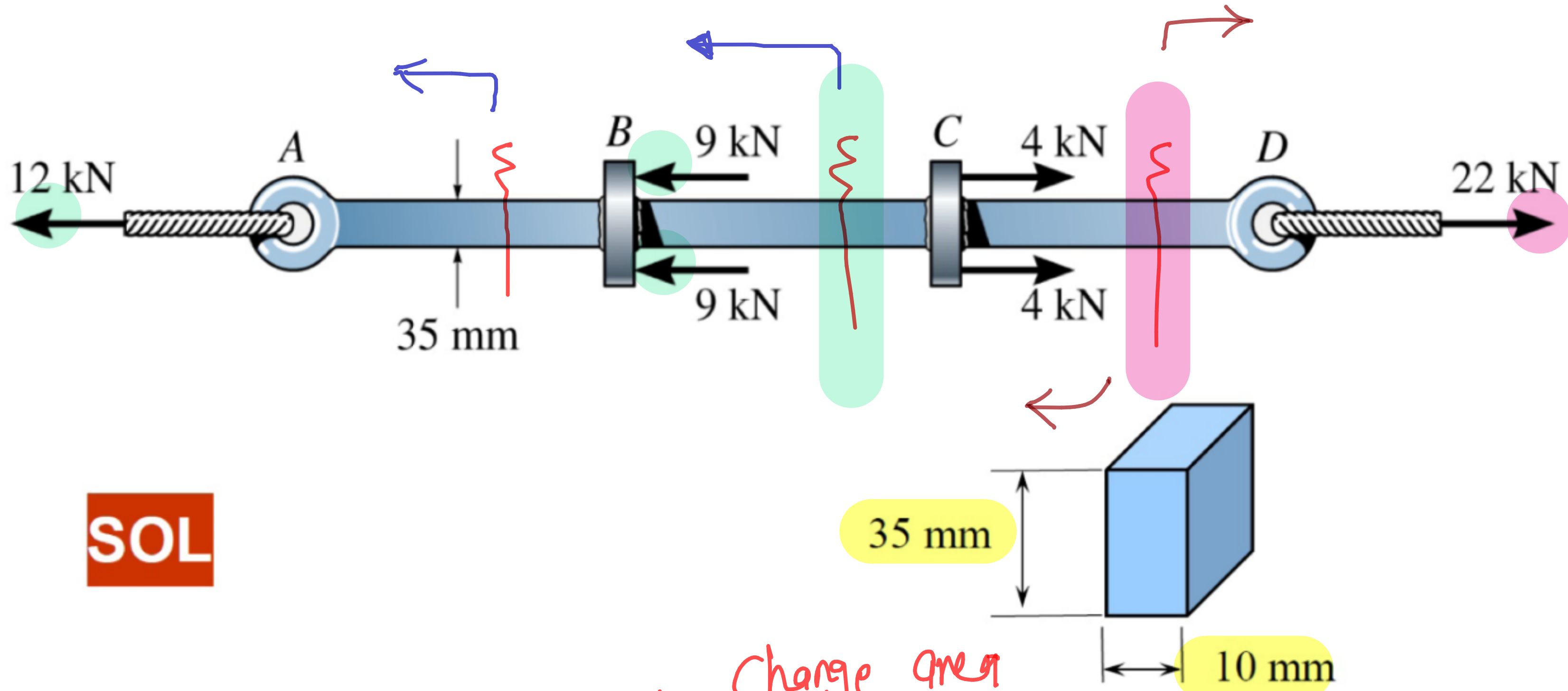
$= 31.1$ mm

Quiz ①

EXAMPLE 1.6

Bar width = 35 mm, thickness = 10 mm

Determine max. average normal stress in bar when subjected to loading shown.



SOL

Section → Change area / Change Force

$P_{AB} = 12\ kN$ (tension)

$P_{BC} = 12 + 9 + 9 = 30\ kN$ (tension)

$P_{CD} = 22\ kN$ (tension)

$$\sigma_{max} = \frac{P_{max}}{A} = \frac{P_{BC}}{A}$$

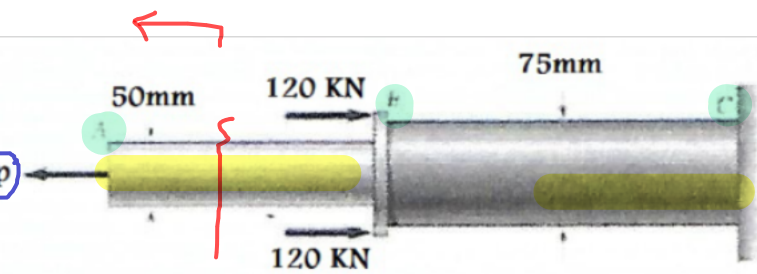
$$= \frac{30 * 10^3}{0.035 * 0.01}$$

$$= 85.7\ MPa$$

tensile stress

QUIZ 1

1. Two solid cylindrical rods AB and BC are welded together at B and loaded as shown in Fig 1. Determine the magnitude of the force P for which, the tensile stress in rod AB is twice the magnitude of the compressive stress in rod BC.



Rod AB :-

$F_{AB} = P$

$A_{AB} = \frac{\pi}{4}\left(\frac{50}{1000}\right)^2 = 1.96 * 10^{-3} \ M^2$

$\sigma_{AB} = \frac{P}{1.96 * 10^{-3}} = 509.3 \ P$

rod BC :-

$F_{BC} = P - 120 - 120 = P - 240$

$A_{BC} = \frac{\pi}{4}\left(\frac{75}{1000}\right)^2 = 4.42 * 10^{-3} \ M^2$

$\sigma_{BC} = \frac{P - 240}{4.42 * 10^{-3}} = 226.4 \ P - 54324.9$

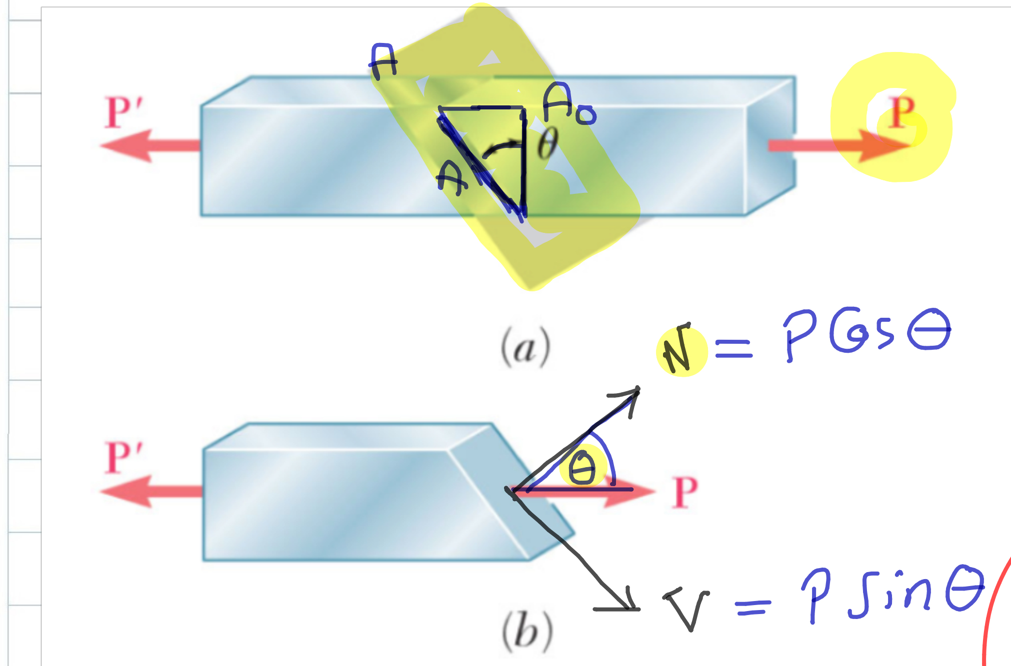
$\sigma_{AB} = -2 \ \sigma_{BC}$

$509.3 \ P = -2 \left(226.4 \ P - 54324.9\right)$

$\Downarrow$

$P = 112.9 \ KN$

Stress on an Oblique Plane



$A_o \Rightarrow$ Normal area

$A \Rightarrow$ oblique area

$\theta \Rightarrow$ angle between A_o and A

$$\cos\theta = \frac{A_o}{A}$$

$$\Downarrow$$

$$A = \frac{A_o}{\cos\theta}$$

$$\sigma_{Normal} = \frac{N}{A} = \frac{P\cos\theta}{\frac{A_o}{\cos\theta}} = \frac{P}{A_o}\cos^2\theta$$

$$\tau_{Shear} = \frac{V}{A} = \frac{P\sin\theta}{\frac{A_o}{\cos\theta}} = \frac{P}{A_o}\sin\theta\cos\theta$$

Factor of Safety