جامعة قطر
QATAR UNIVERSITY

Class

COLLEGE OF ENGINEERING
DEPARTMENT OF CIVIL ENGINEERING AND ARCHITECTURE

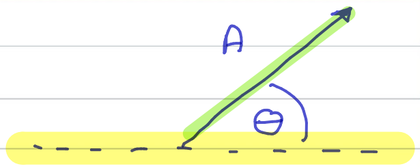
Lecture on Wednesday, 13 Jan. 2021

Vectors and vector operations

CVEN 213 – Statics
Bachelor of Science in Civil Engineering

Vagelis Plevris
13 January 2021

Vectors } Magnitude (A), direction (θ)

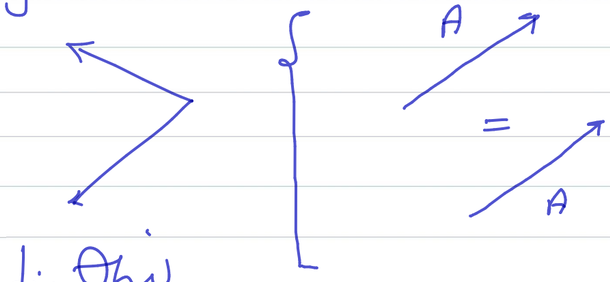


Ex ⇒ Force & velocity

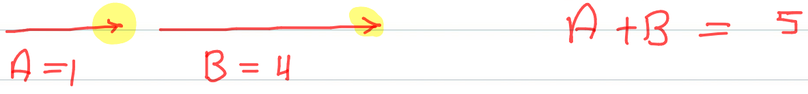
Scalars } only Quantity (+, −); No-direction

Ex. Mass & volume

Same Magnitude



Same direction



$\vec{A} + \vec{B} = 5$

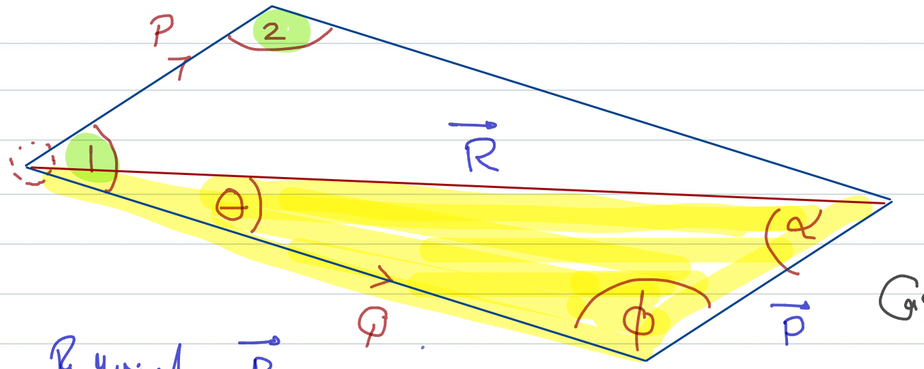
Parallelogram law

Trigonometric Method

only two Forces

$\vec{R} = \vec{P} + \vec{Q}$

$\hat{1} + \hat{2} = 180$



Case ①

Given $\vec{P}$, $\vec{Q}$ Required $\vec{R}$

Solve

① internal angle between $\vec{P}$, $\vec{Q}$ (ϕ)

② Case-law ⇒ Mag

$$R = \sqrt{P^2 + Q^2 - 2PQ\cos\phi}$$

③ Direction ⇒ Sin law

$$\frac{P}{\sin\theta} = \frac{Q}{\sin\alpha} = \frac{R}{\sin\phi}$$

Case ②

Given R Required $\vec{P}$, $\vec{Q}$

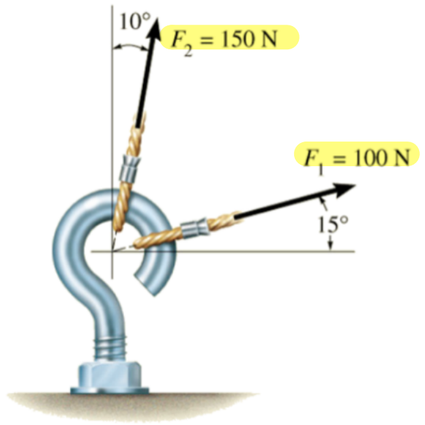
① Calculate all internal angles ($\theta < \alpha < \phi$)

② Sin-law

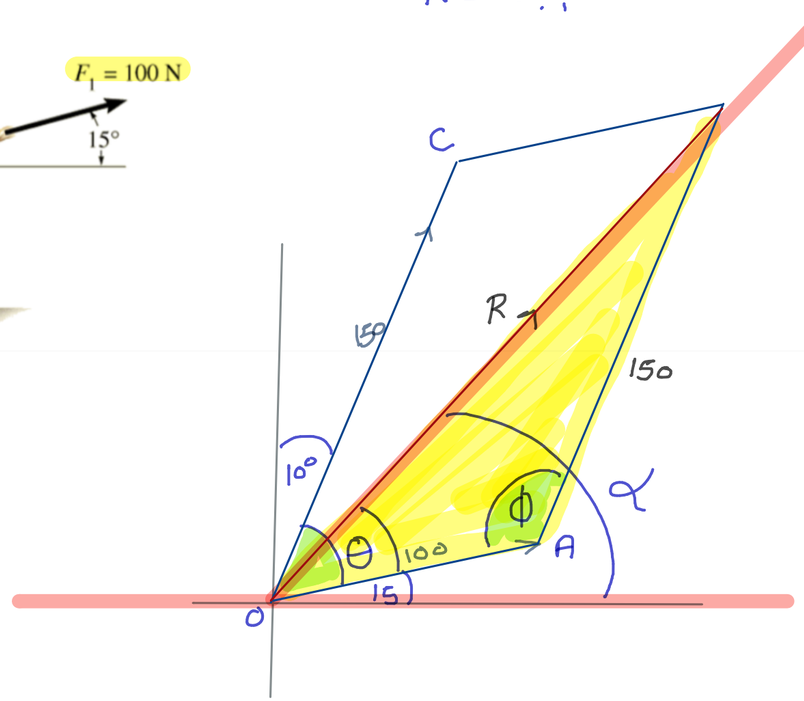
$$\frac{P}{\sin\theta} = \frac{Q}{\sin\alpha} = \frac{R}{\sin\phi}$$

EXAMPLE 2.1

The screw eye in Fig. 2–10*a* is subjected to two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$. Determine the magnitude and direction of the resultant force.



R = ???



* angle CoA = 90 - 15 - 10 = 65

$$\phi = 180 - 65 = 115$$

* $R = \sqrt{100^2 + 150^2 - 2(100)(150)\cos 115}$

$$R = 213 \quad N$$

* Direction $\Rightarrow$ Sin law

$$\alpha = \theta + 15$$

$$\frac{150}{\sin\theta} = \frac{213}{\sin 115}$$

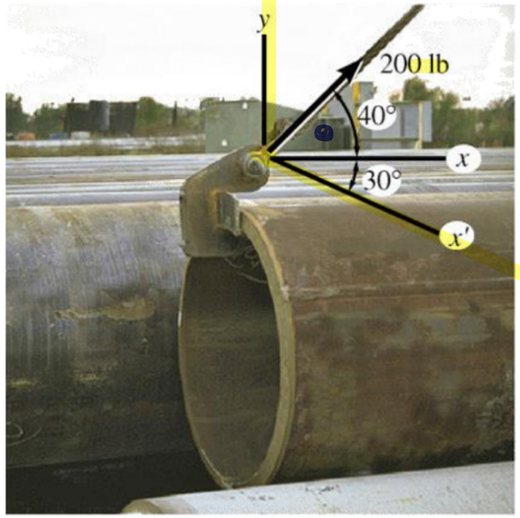
$$\theta = \sin^{-1}\left(\frac{150 \sin 115}{213}\right)$$

$$= 39.7^\circ$$

$$\alpha = 39.7 + 15 = 54.7^\circ$$

EXAMPLE 2.2

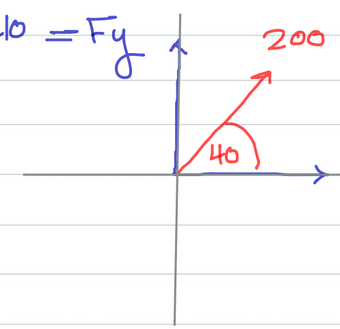
Resolve the 200-lb force acting on the pipe, Fig. 2–11*a*, into components in the (a) x and y directions, and (b) x' and y directions.



(a)

(a)

$200 \sin 40 = F_y$

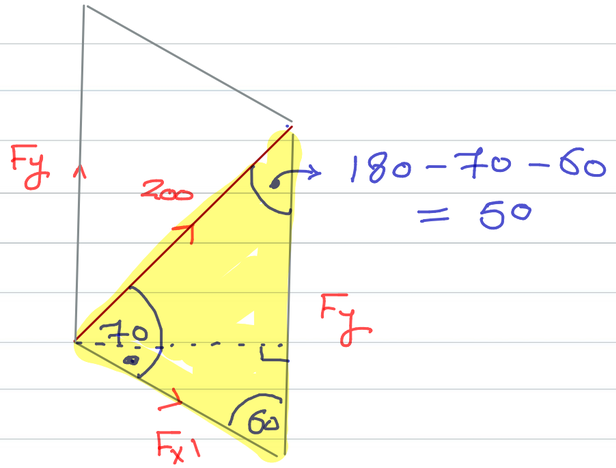


$F_x = 200 \cos 40$

$F_x = 200 \cos 40 = 153 \text{ lb}$

$F_y = 200 \sin 40 = 129 \text{ lb}$

(b)



Sin law

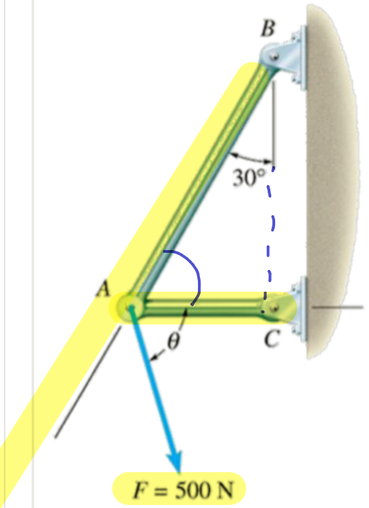
$$\frac{F_x'}{\sin 50} = \frac{F_y}{\sin 70} = \frac{200}{\sin 60}$$

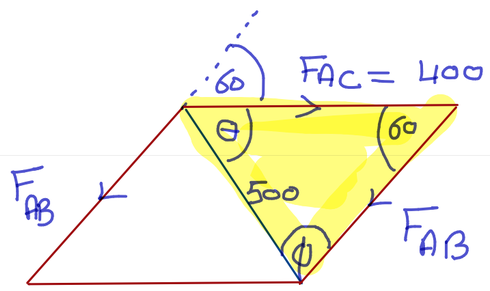
$$F_x' = \frac{200 \sin 50}{\sin 60} = 177 \text{ lb}$$

$$F_y = \frac{200 \sin 70}{\sin 60} = 217 \text{ lb}$$

EXAMPLE 2.3

The force **F** acting on the frame shown in Fig. 2–12*a* has a magnitude of 500 N and is to be resolved into two components acting along members *AB* and *AC*. Determine the angle θ, measured *below* the horizontal, so that the component $\mathbf{F}_{AC}$ is directed from *A* toward *C* and has a magnitude of 400 N.





$$\frac{400}{\sin\phi} = \frac{500}{\sin 60}$$

$$\sin\phi = \frac{400 \sin 60}{500}$$

$$\phi = 43.9^\circ$$

$$\theta = 180 - 60 - 43.9$$

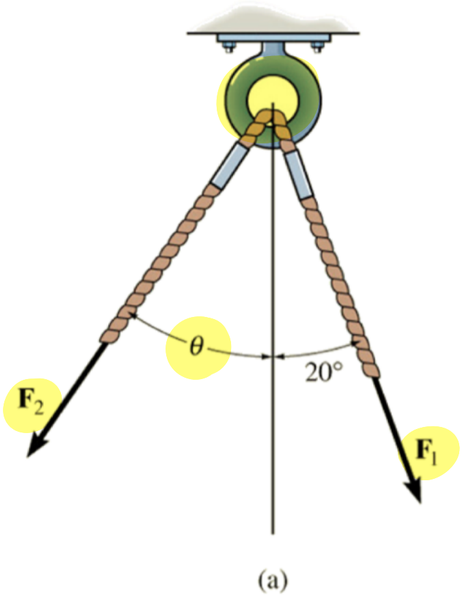
$$= 76.1^\circ \quad \checkmark$$

$$F_{AB} = \sqrt{400^2 + 500^2 - 2(400)(500)\cos 76.1}$$

$$= 561 \text{ N} \quad \checkmark$$

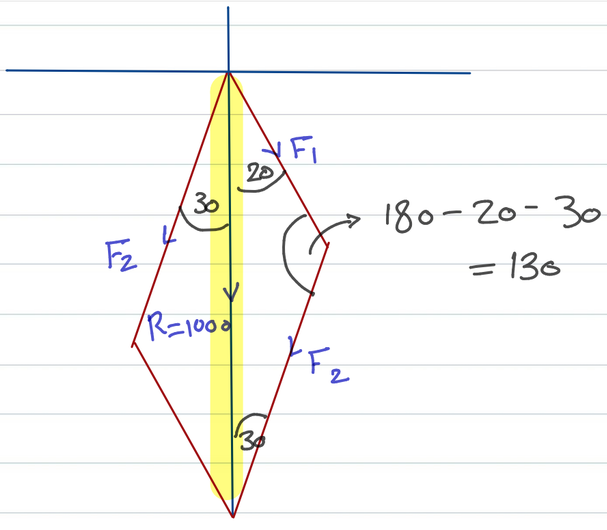
EXAMPLE 2.4

The ring shown in Fig. 2–13*a* is subjected to two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$. If it is required that the resultant force have a magnitude of 1 kN and be directed vertically downward, determine (a) the magnitudes of $\mathbf{F}_1$ and $\mathbf{F}_2$ provided $\theta = 30°$, and (b) the magnitudes of $\mathbf{F}_1$ and $\mathbf{F}_2$ if F_2 is to be a minimum.



(a)

R = 1000 N



(a)

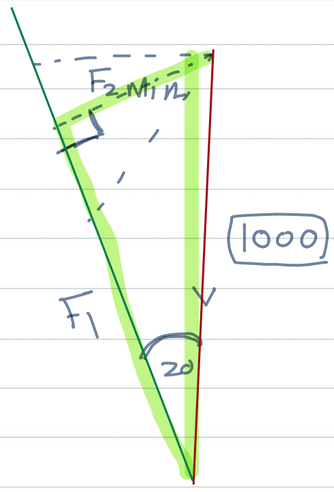
Sine law

$$\frac{F_1}{\sin 30} = \frac{F_2}{\sin 20} = \frac{1000}{\sin 130}$$

$$F_1 = \frac{1000 \sin 30}{\sin 130} = 653 \text{ N}$$

$$F_2 = \frac{1000 \sin 20}{\sin 30} = 446 \text{ N}$$

(b)



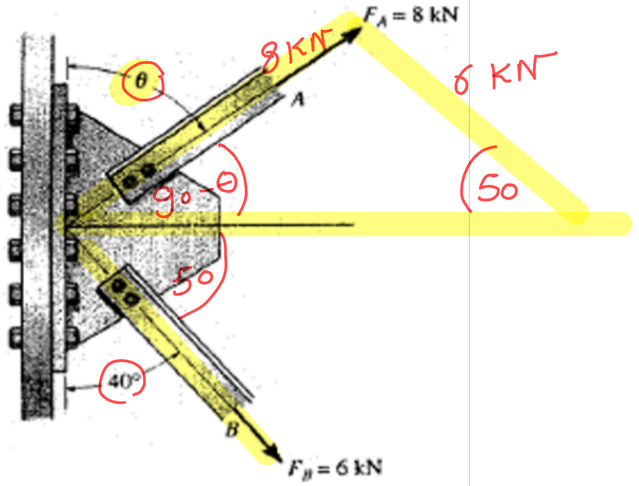
$F_1 = 1000 \cos 20$

$= 940$ N

$F_{2_{Min}} = 1000 \sin 20$

$= 342$

***2-8.** Determine the angle θ for connecting member A to the plate so that the resultant force of F_A and F_B is directed horizontally to the right. Also, what is the magnitude of the resultant force.



8 KN

6 KN

$90 - \theta$

50

50

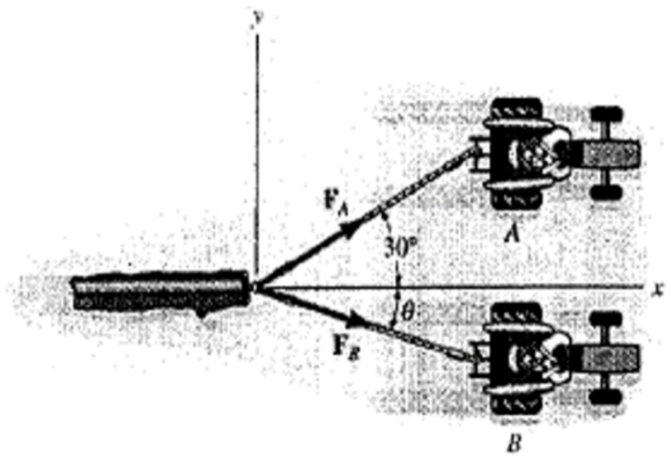
40°

$$\frac{6}{\sin(90-\theta)} = \frac{8}{\sin 50}$$

$$90 - \theta = 35$$

$$\theta = 55^\circ$$

2-25. The log is being towed by two tractors A and B. Determine the magnitude of the two towing forces $\mathbf{F}_A$ and $\mathbf{F}_B$ if it is required that the resultant force have a magnitude $F_R = 10$ kN and be directed along the x axis. Set $\theta = 15°$.



H.W