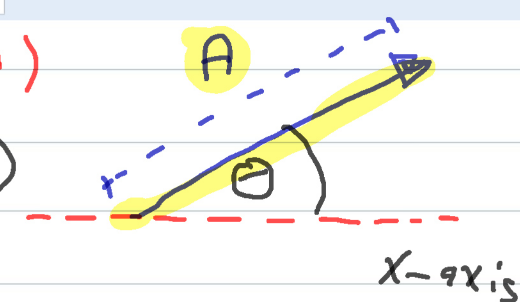
Force VECTORS 2

* Vectors } Magnitude (A), direction (θ)

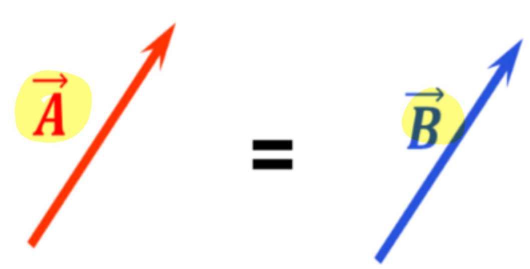


Ex. Force, Velocity

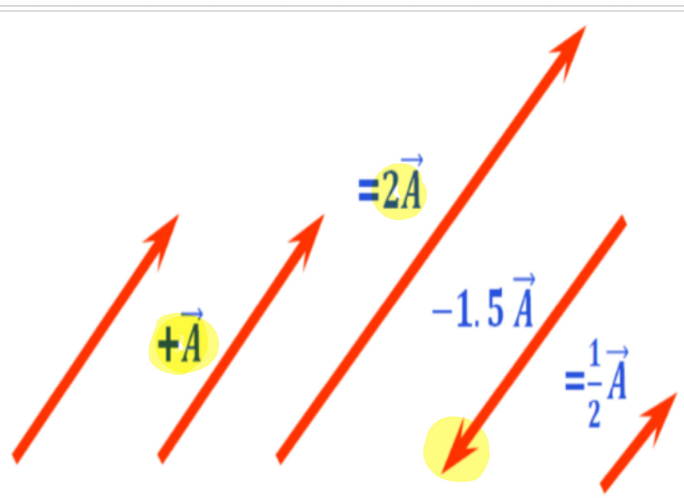
* Scalar } only Quantity (+ , −), No-direction

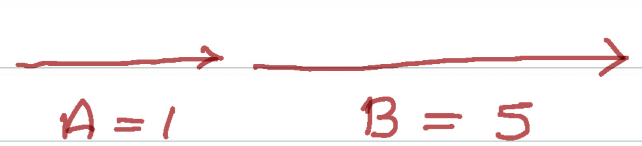
Ex. Mass, Volume

- Vectors are **equal** when they have the same magnitude and same direction



- Vectors can be simply **added** or **subtracted**, if they have the same direction



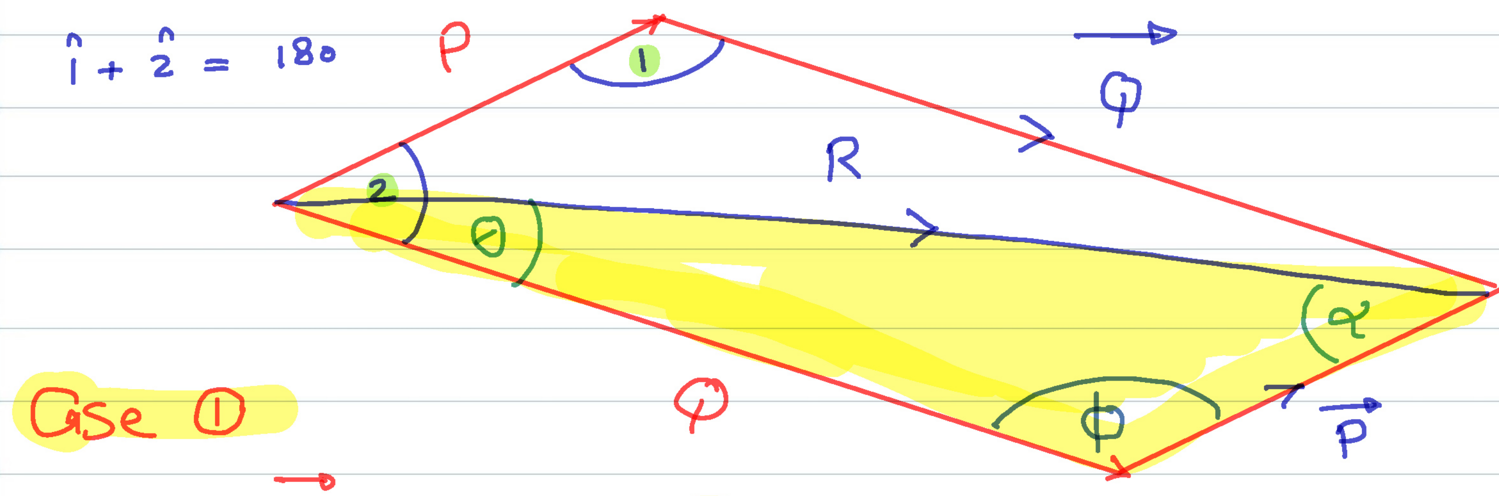


$\vec{A} + \vec{B} = 6$

Parallelogram Law
Trigonometric method

} Resultant of two Forces

$\vec{R} = \vec{P} + \vec{Q}$



Case ①

Given $\vec{P}$, $\vec{Q}$ Required $\vec{R}$ → Mag, dire

① Conclude internal angle between $\vec{P}$ & Q (ϕ)

② Magnitude → Cos law

$$R = \sqrt{P^2 + Q^2 - 2PQ\cos\phi}$$

③ direction → sin law

$$\frac{P}{\sin\theta} = \frac{Q}{\sin\alpha} = \frac{R}{\sin\phi}$$

Case ②

Given $\vec{R}$ Required $\vec{P}$, $\vec{Q}$

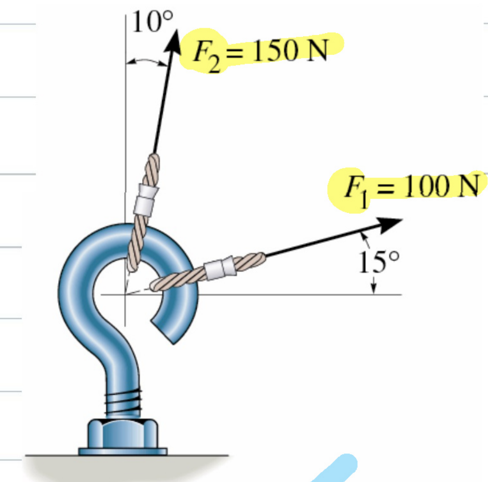
① Conclude all internal angle $\begin{pmatrix}\theta \\ \alpha \\ \phi\end{pmatrix}$

② Sine law

$$\frac{P}{\sin\theta} = \frac{Q}{\sin\alpha} = \frac{R}{\sin\phi}$$

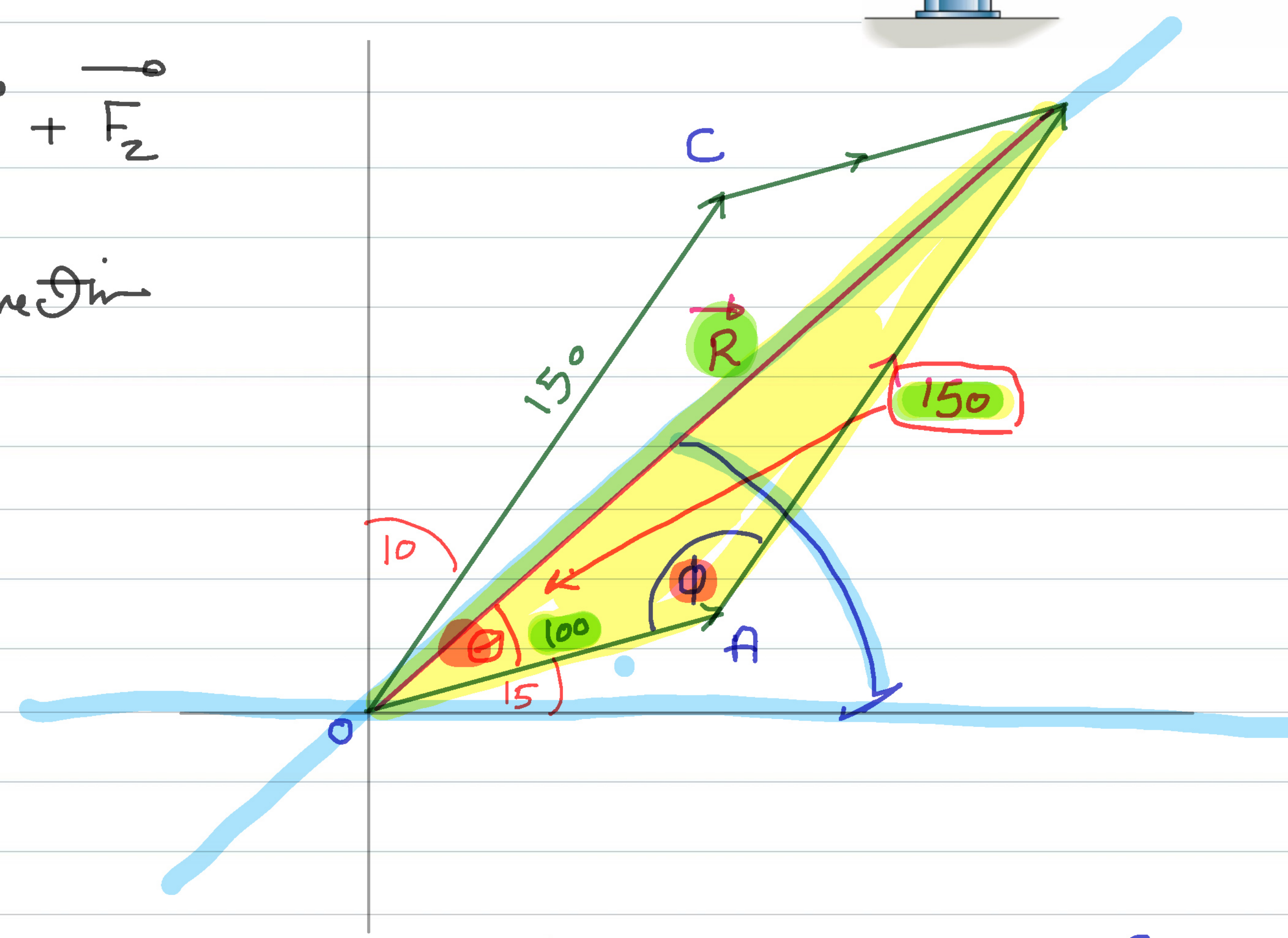
Example 1:-

The screw eye in the figure at the left is subjected to two forces $\vec{F}_1$ and $\vec{F}_2$.
Determine the magnitude and direction of the resultant force.



$\vec{R} = \vec{F_1} + \vec{F_2}$

Mag direction



① angle CoA = 90 − 10 − 15 = 65°

$\phi = 180 - 65 = 115$

② $R = \sqrt{150^2 + 100^2 - 2(150)(100)\cos 115}$

$= 213$ N

③ * to get direction → sin law

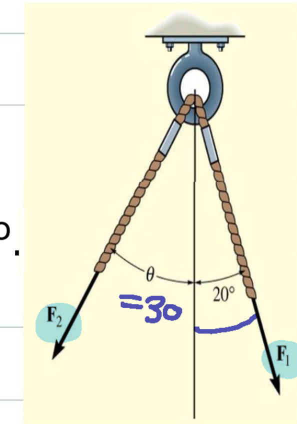
$$\frac{150}{\sin\theta} = \frac{213}{\sin 115}$$

$$\theta = \sin^{-1}\frac{150 \sin 115}{213} = 39.7°$$

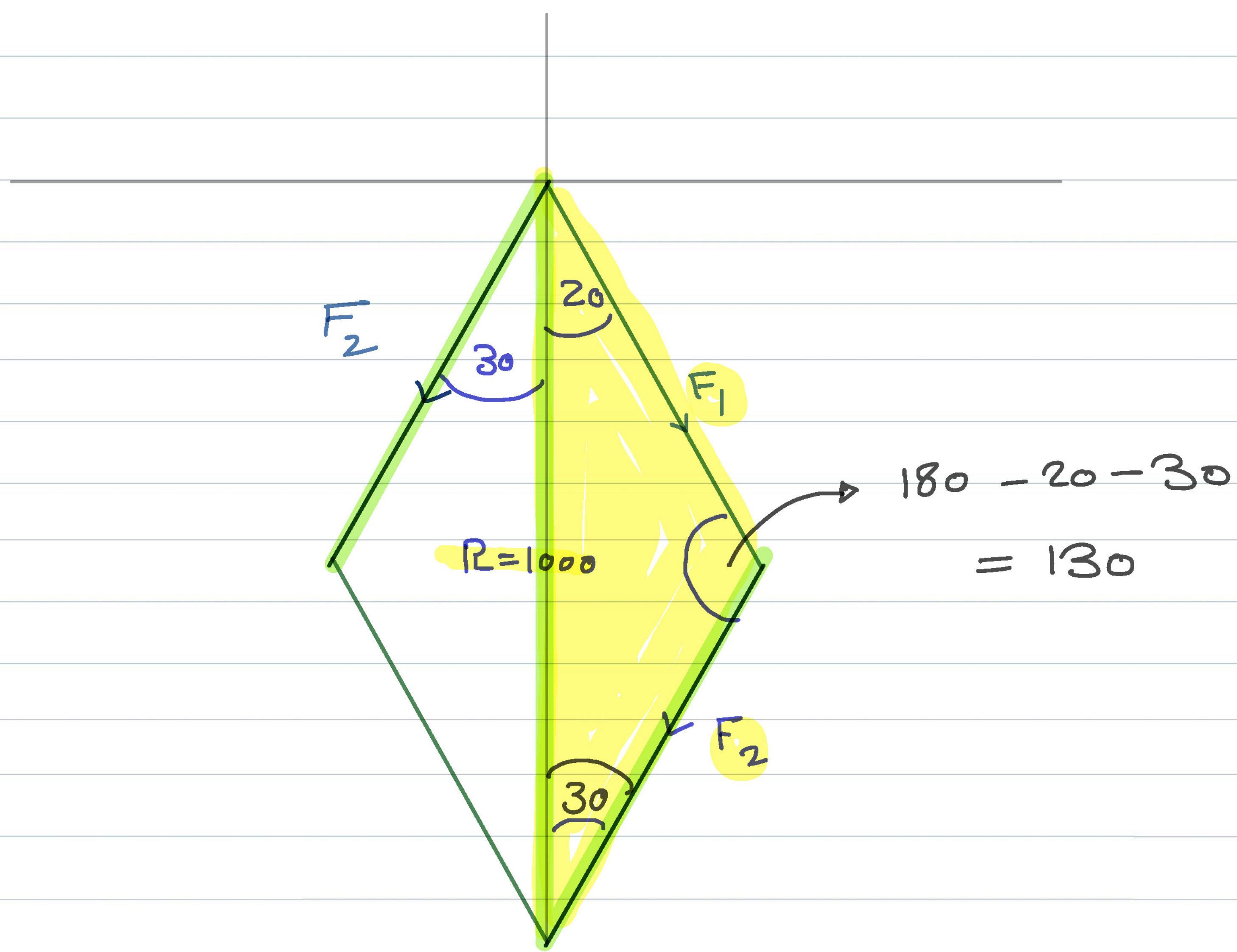
So angle = 39.7 + 15 = 54.7

Example 2 :-

The ring below is subjected to F_1 and F_2. If we want a resultant force of 1kN and directed vertically downward, determine the magnitude of F_1 and F_2 if $\theta = 30^o$.



$R = 1000\ N$

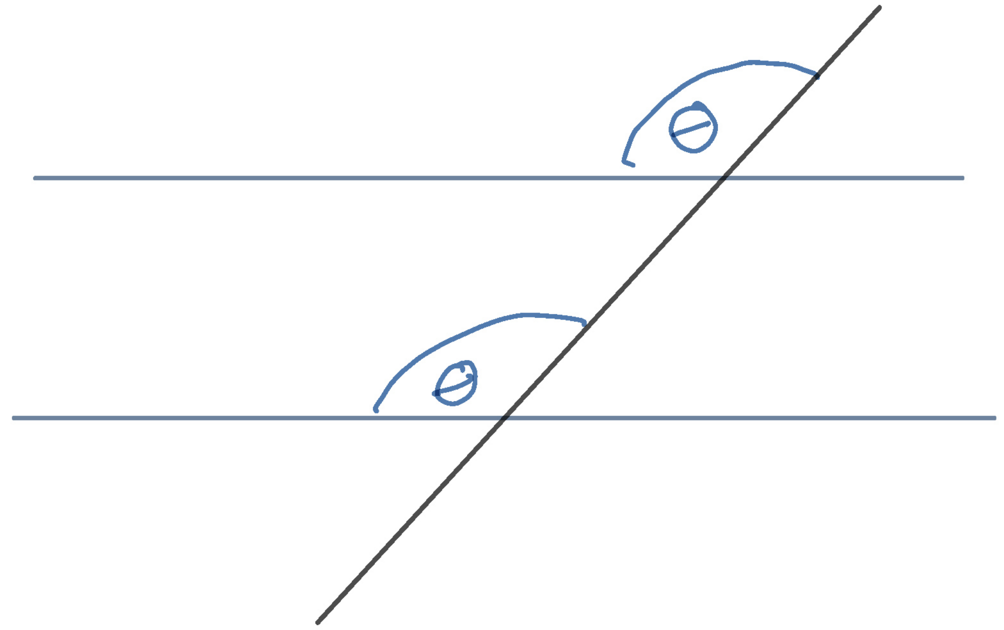


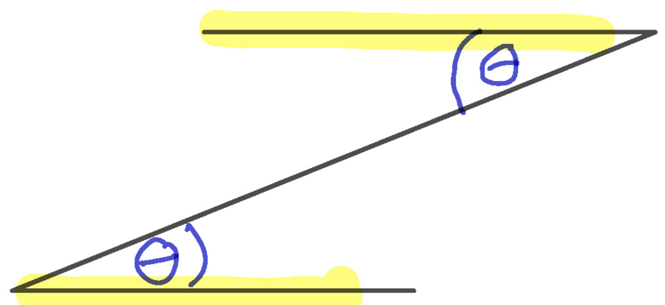
Sine law

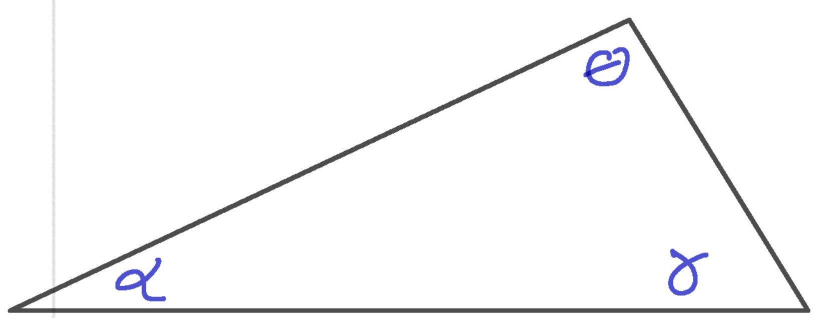
$$\frac{F_1}{\sin 30} = \frac{F_2}{\sin 20} = \frac{1000}{\sin 130}$$

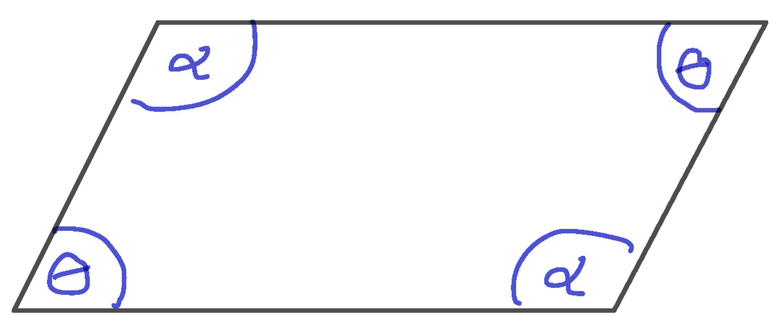
$$F_1 = \frac{1000 \sin 30}{\sin 130} = 653\ N$$

$$F_2 = \frac{1000 \sin 20}{\sin 130} = 446\ N$$









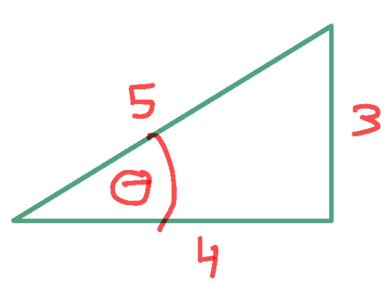
$\theta + \alpha + \gamma = 180$

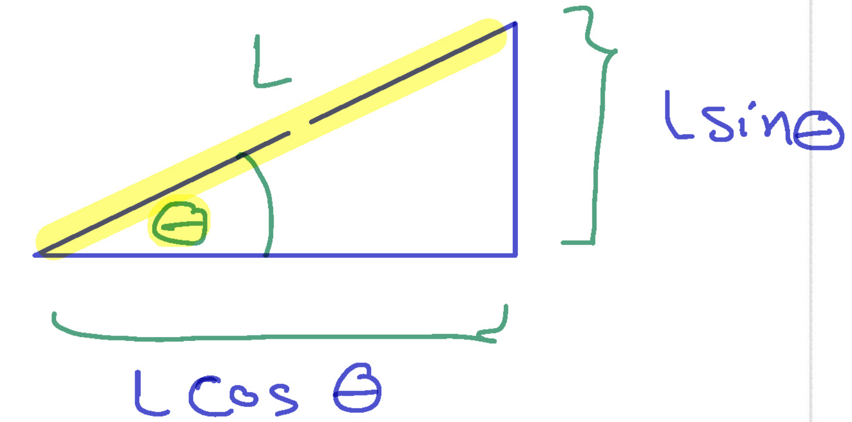
$\theta + \alpha = 180$

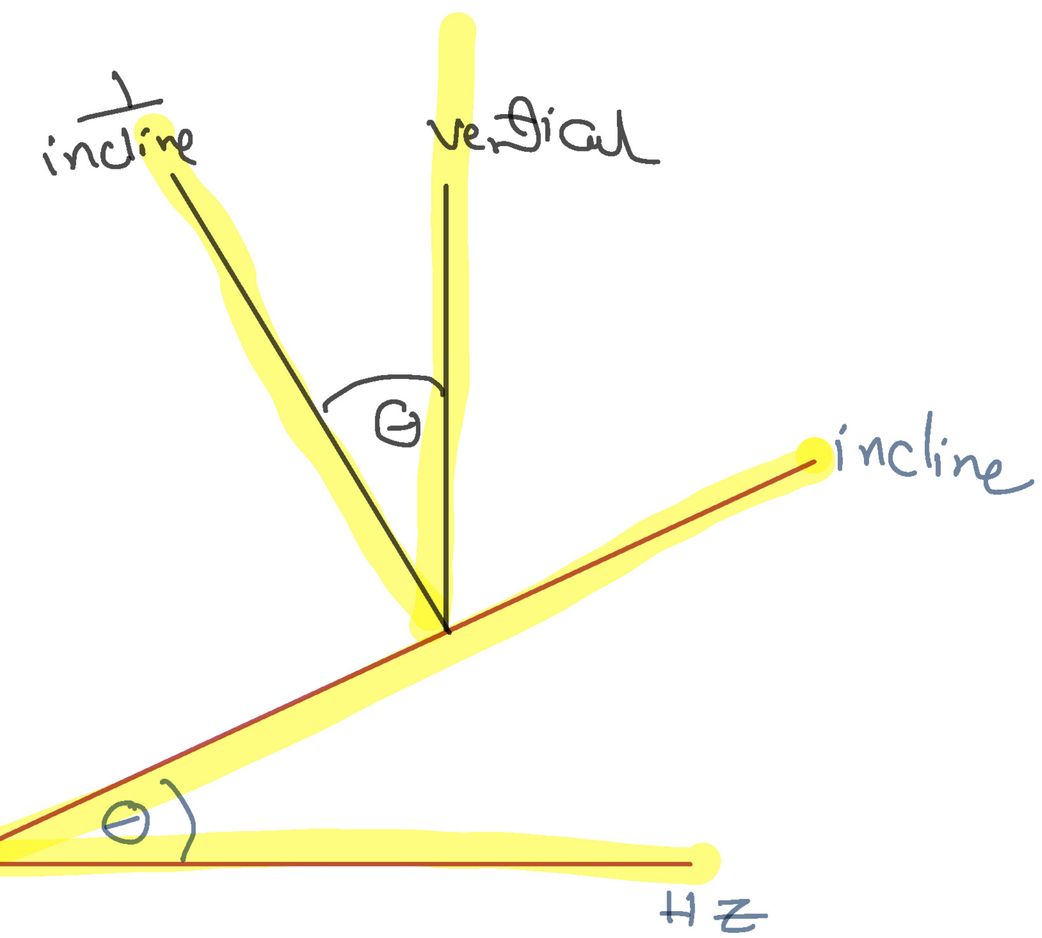
$\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$

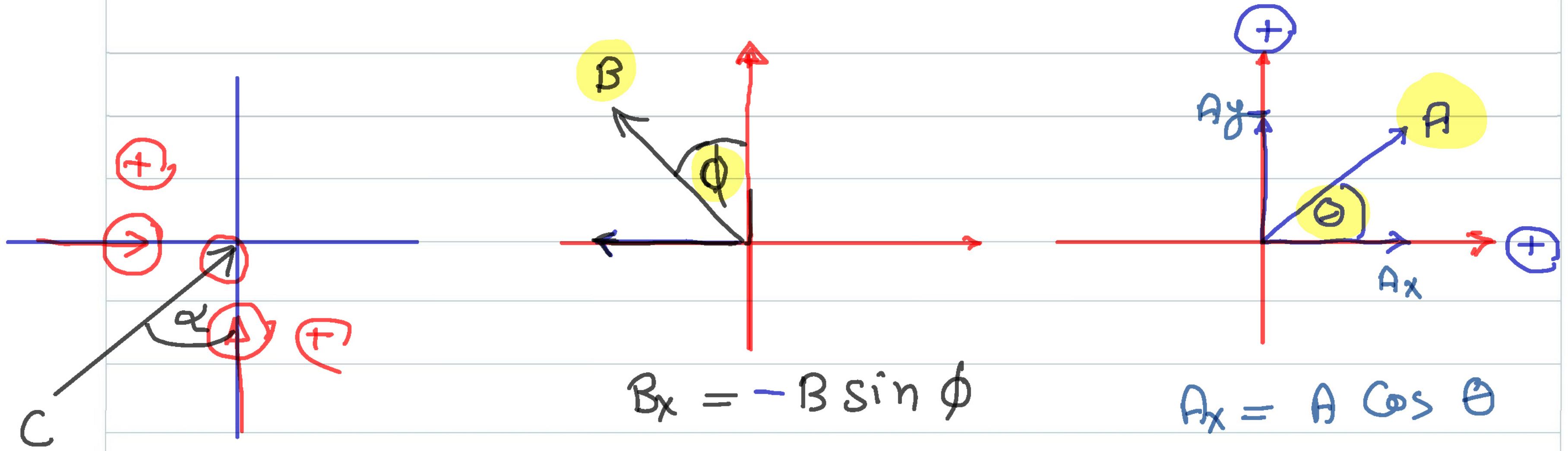
$\tan\theta = \frac{3}{4}$







Rectangular/Cartesian Components Method

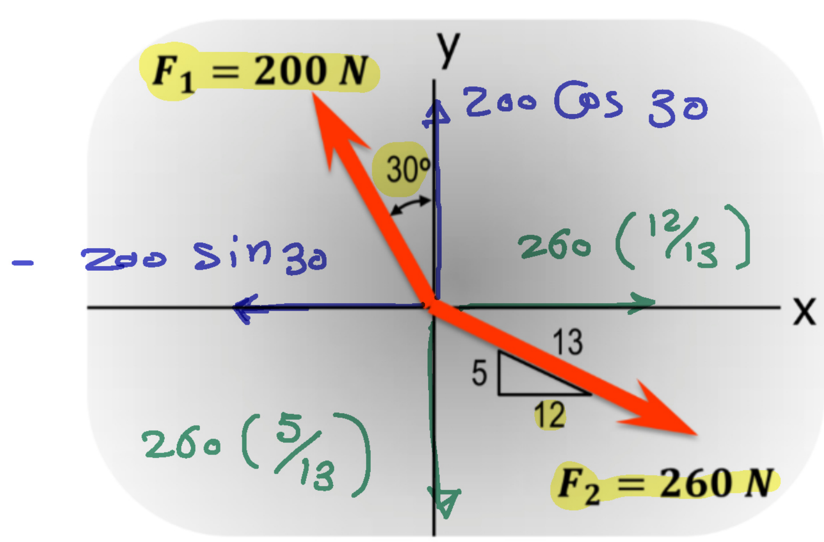


$B_x = -B\sin\phi$ $\quad$ $A_x = A\cos\theta$

$C_x = +C\sin\alpha$ $\quad$ $B_y = +B\cos\phi$ $\quad$ $A_y = A\sin\theta$

$C_y = +C\cos\alpha$ $\quad$ $B = (B_x)\hat{i} + (B_y)\hat{j}$ $\quad$ $A = \sqrt{A_x^2 + A_y^2}$

Determine the x and y Cartesian components of the $\mathbf{F_1}$ and $\mathbf{F_2}$ forces acting on the boom. Put each force in the Cartesian vector form.



$F_{1x} = -200 \sin 30$
$= -100$

$F_{1y} = 200 \cos 30$
$= 173$

$F_{2x} = 260\ (12/13)$
$= 240$

$F_{2y} = -260\ (5/13)$
$= -100$

$F_1 = (-100)\hat{i} + (173)\hat{j}$

$F_2 = (240)\hat{i} + (-100)\hat{j}$

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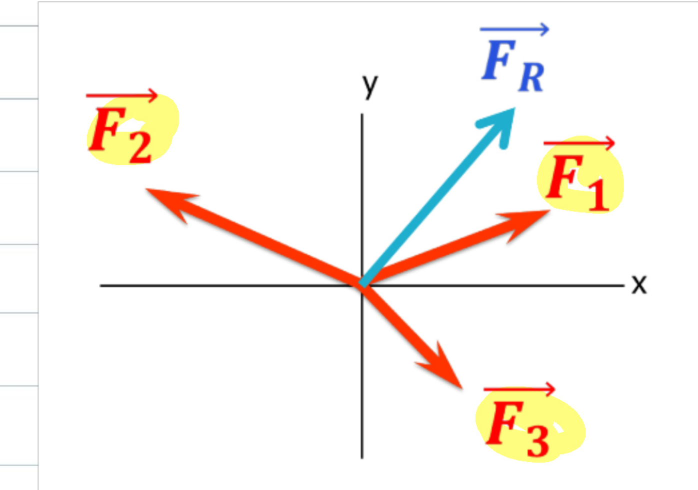
Coplanar Force Resultants

More than 2-Forces

① Resolve

F_{1x}	F_{2x}	F_{3x}
F_{1y}	F_{2y}	F_{3y}



② $\vec{R} = \vec{F_1} + \vec{F_2} + \vec{F_3}$

$F_{Rx} = \boxed{\Sigma F_x} = F_{1x} + F_{2x} + F_{3x}$

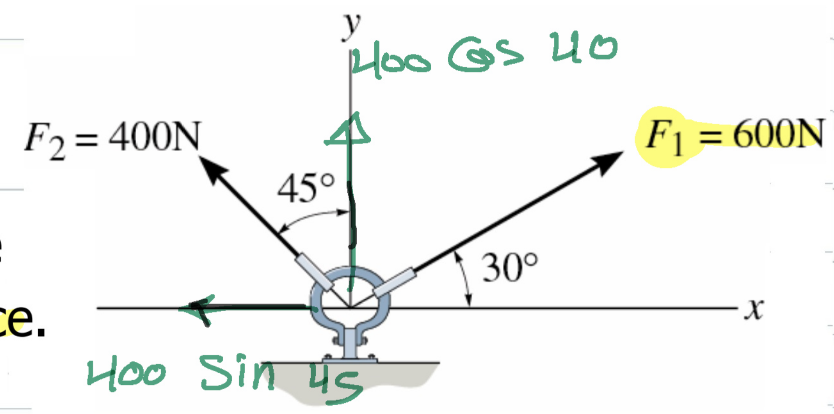
$F_{Ry} = \Sigma F_y = F_{1y} + F_{2y} + F_{3y}$

③ $F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$

④ $\theta = \tan^{-1} \dfrac{F_{Ry}}{F_{Rx}}$

Example 3 :-

The link in the figure is subjected to two forces, $\mathbf{F_1}$ and $\mathbf{F_2}$. Determine the resultant magnitude and orientation of the resultant force.



* Resolve :-

$F_{1x} = 600 \cos 30 = 519.6$

$F_{1y} = 600 \sin 30 = 300$

$F_{2x} = -400 \sin 45 = -282.8$

$F_{2y} = 400 \cos 45 = 282.8$

* $F_{Rx} = 519.6 - 282.8$

$= 236.8$

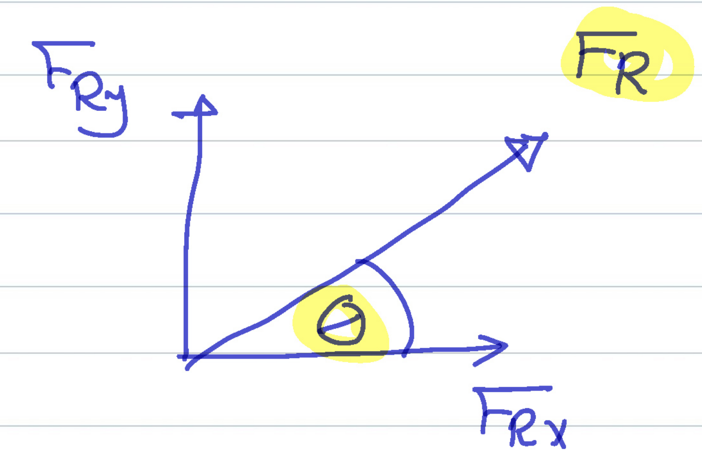
$F_{Ry} = 300 + 282.8 = 582.8$

* $F_R = \sqrt{(236.8)^2 + (582.8)^2}$

$= 629.1$ (N)

* $\theta = \tan^{-1} \dfrac{582.8}{236.8}$

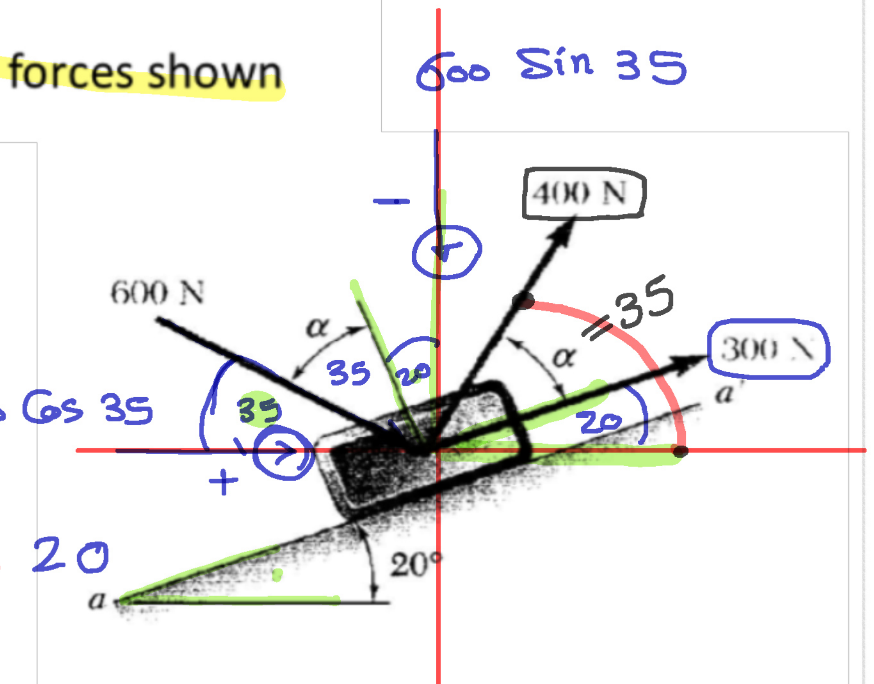
$= 67.9°$



Problem # 3

Knowing that $\alpha = 35°$,

Determine: The resultant of the three forces shown



* Resolve :—

$F_1 = 300$

with angle with HZ = 20

$F_{1x} = 300 \cos 20 = 281.9 \text{ N}$

$F_{1y} = 300 \sin 20 = 102.9 \text{ N}$

$F_2 = 400 \text{ N}$

with angle with HZ $= 20 + 35 = 55$

$F_{2x} = 400 \cos 55 = 229.4 \text{ N}$

$F_{2y} = 400 \sin 55 = 327.7 \text{ N}$

$F_3 = 600 \text{ N}$

with angle with HZ $= 35°$

$F_{3x} = 600 \cos 35 = 491.5 \text{ N}$

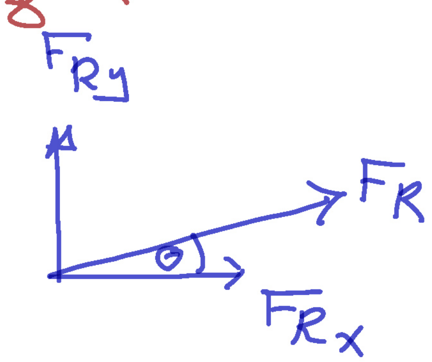
$F_{3y} = -600 \sin 35 = -344.1 \text{ N}$

* $F_{Rx} = \Sigma F_x = 281.9 + 229.4 + 491.5 = 1002.8 \text{ N}$

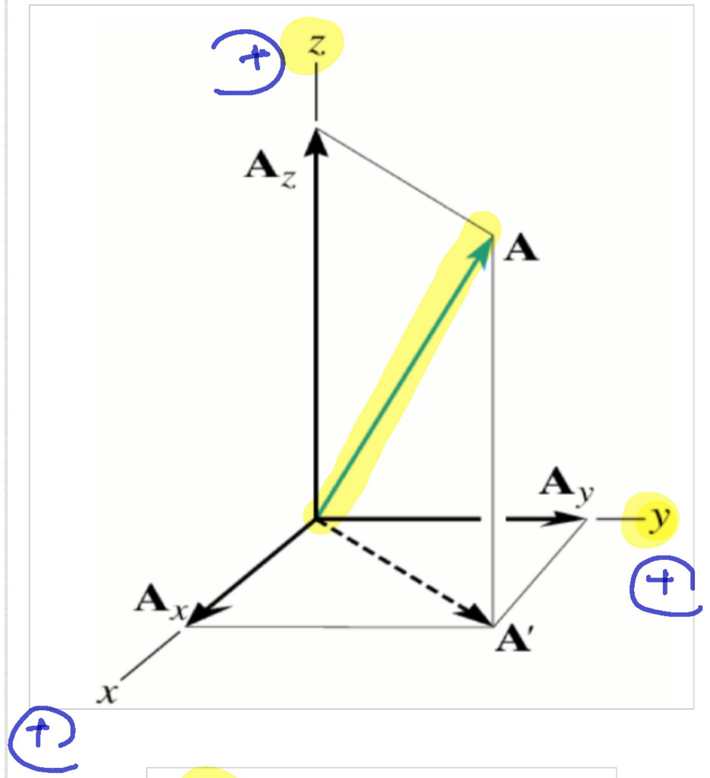
$F_{Ry} = \Sigma F_y = 102.9 + 327.7 - 344.1 = 86.2 \text{ N}$

* $F_R = \sqrt{(1002.8)^2 + (86.2)^2} = 1006.5 \text{ N}$

* $\theta = \tan^{-1} \dfrac{86.2}{1002.8} = 4.91°$



2.7. Cartesian Vectors



Unit Vectors in Coordinate Directions:

$\vec{i}$, $\hat{i}$: Unit vector in the x-direction

$\vec{j}$, $\hat{j}$: Unit vector in the y-direction

$\vec{k}$, $\hat{k}$: Unit vector in the z-direction

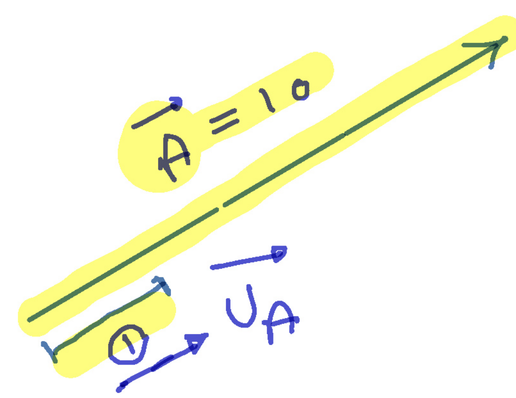
$$\vec{A} = (A_x)\hat{i} + (A_y)\hat{j} + (A_z)\hat{k}$$

Unit Vectors

$$|\vec{U}_A| = 1$$

$$\vec{U}_A = \frac{\vec{A}}{|\vec{A}|} = \frac{\vec{A}}{A}$$

$$\vec{A} = |\vec{A}|\,\vec{U}_A$$

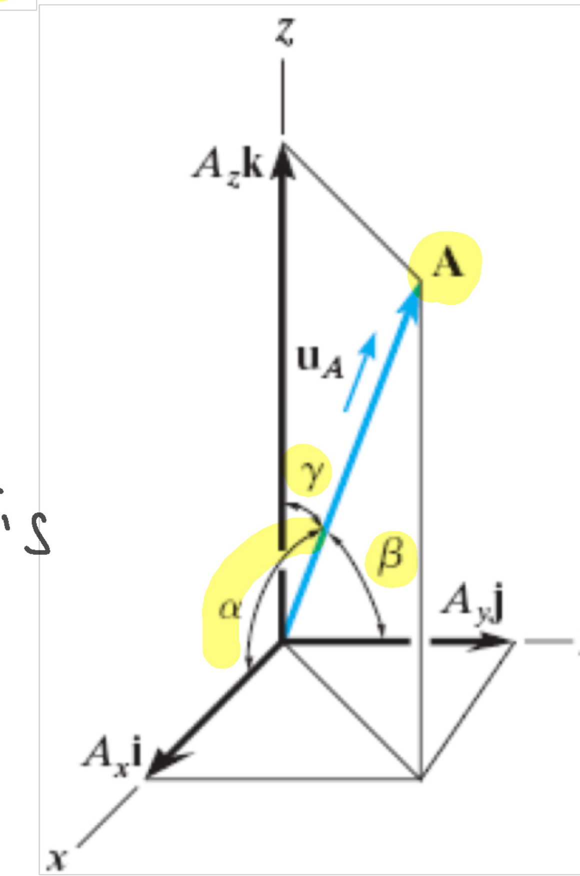


Magnitude

$$A = |A| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$\vec{U}_A = \frac{A_x}{A}\hat{i} + \frac{A_y}{A}\hat{j} + \frac{A_z}{A}\hat{k}$$

Direction of a Cartesian Vector



Direction angles :-

α angle with ⊕ x-axis

β angle with ⊕ y-axis

γ angle with ⊕ z-axis

$\alpha, \beta \geq 0$ & $\gamma \leq 180^\circ$

Direction Cosines of A is

$$\cos\alpha = \frac{A_x}{A}$$

$$\cos\beta = \frac{A_y}{A}$$

$$\cos\gamma = \frac{A_z}{A}$$

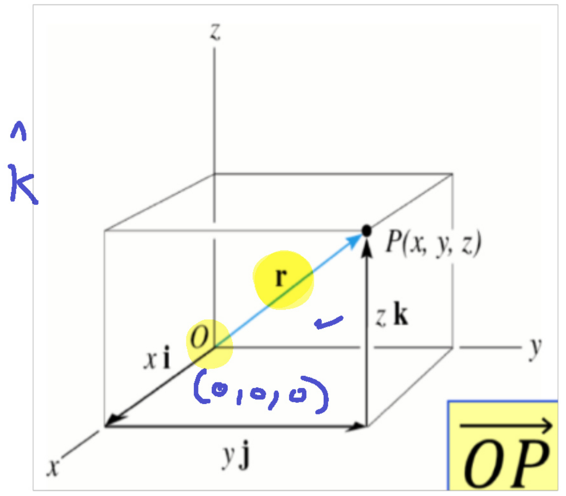
$$U_A = \cos\alpha\,\hat{i} + \cos\beta\,\hat{j} + \cos\gamma\,\hat{k}$$

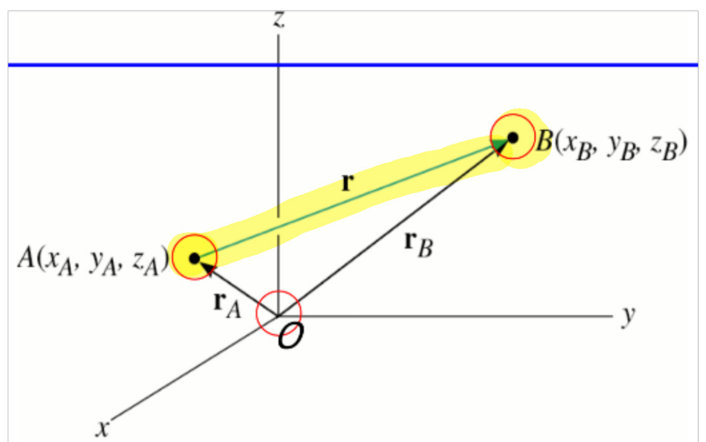
$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

2.9. Coordinates of Relative Position Vectors

$\vec{r} = \vec{OP}$

$= (x)\hat{i} + (y)\hat{j} + (z)\hat{k}$





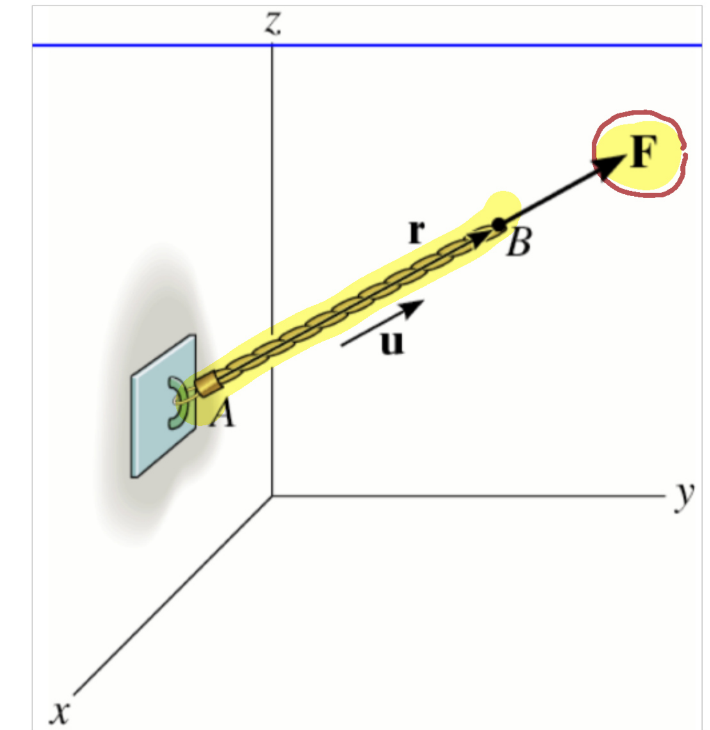
$\vec{AB} = \vec{r_B} - \vec{r_A}$

$= (x_B - x_A)\hat{i} + (y_B - y_A)\hat{j} + (z_B - z_A)\hat{k}$

2. 10. Force Along a Line

$\vec{F} = F\,\vec{U_{AB}}$

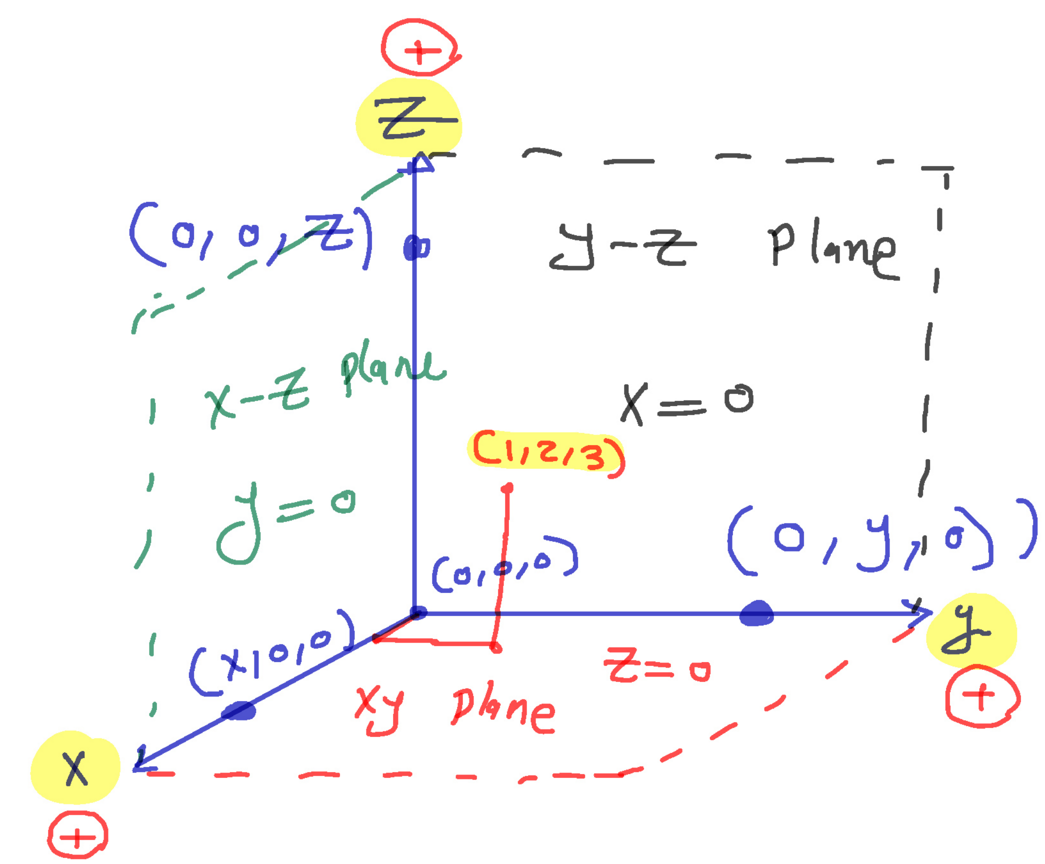
$= F \dfrac{\vec{AB}}{|\vec{AB}|}$

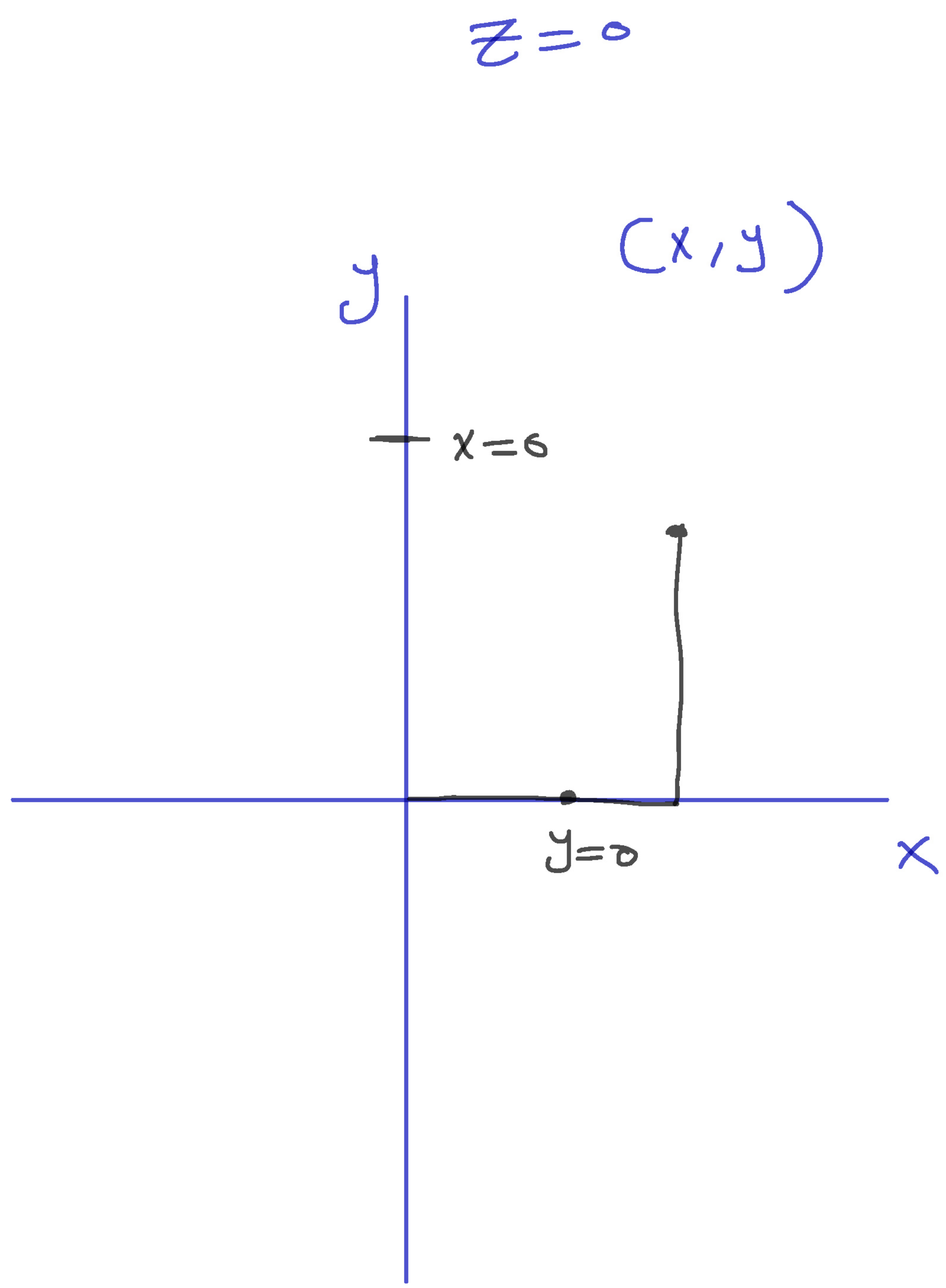


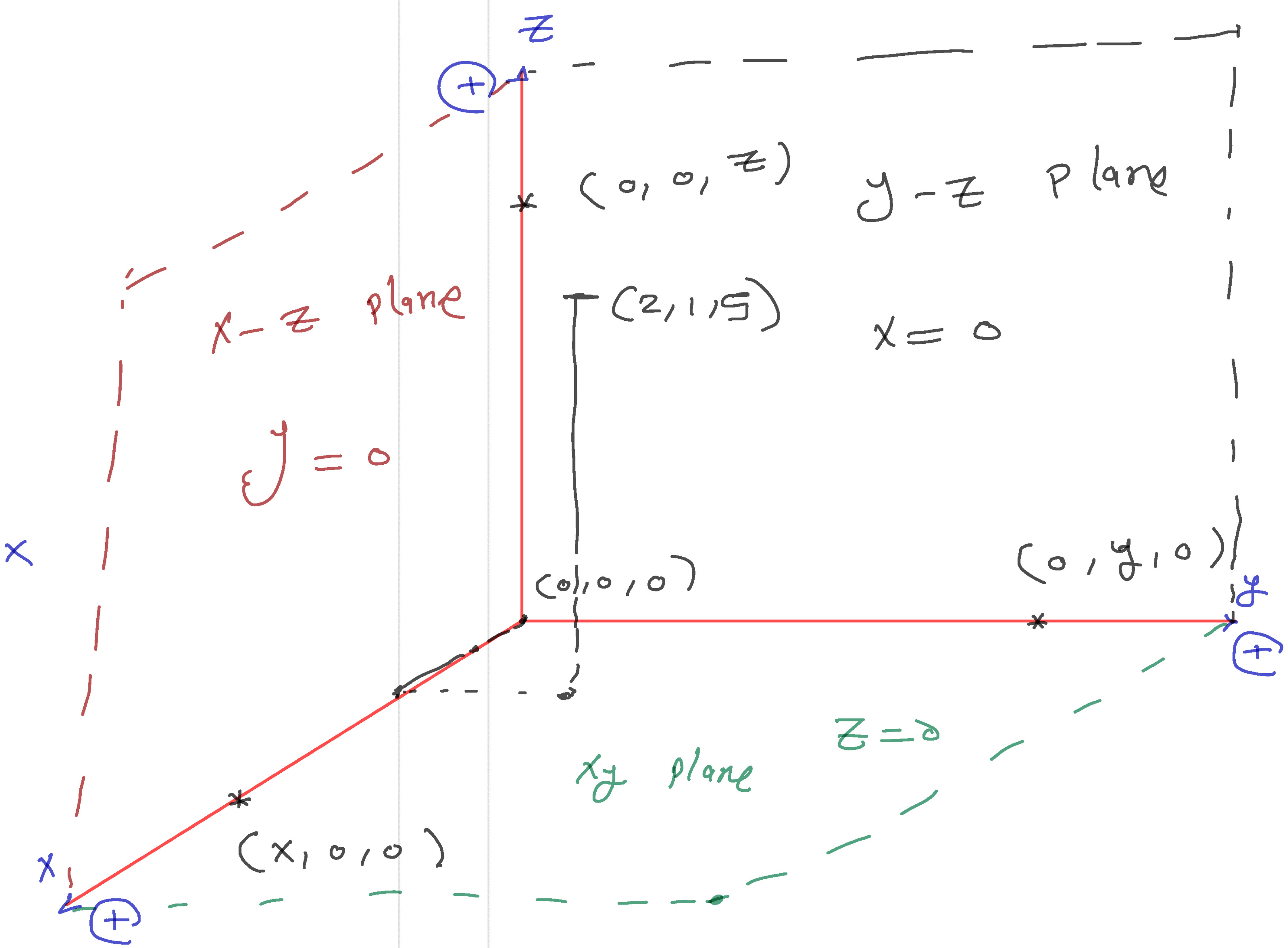
Direction of Force = Direction of Cable

Unit vector of Force = unit vector of Cable

(1, 2, 3)

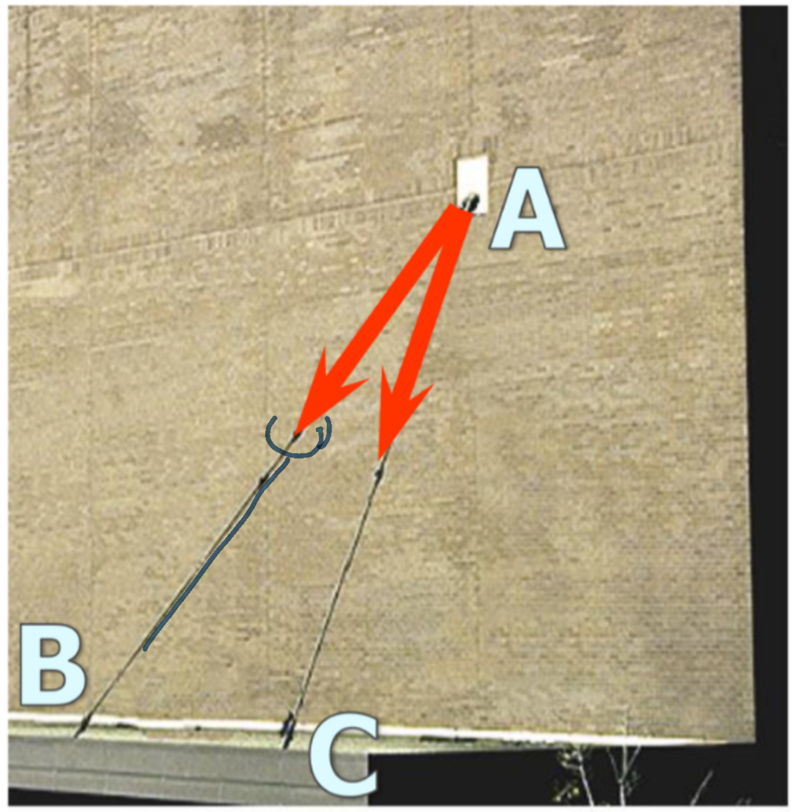



Z = 0
(x, y)
y
x = 0
y = 0
x


Z
+
(0, 0, Z)
y - Z plane
(2, 1, 5)
x = 0
x - Z plane
y = 0
(0, 0, 0)
(0, y, 0)
y
+
Z = 0
xy plane
(x, 0, 0)
x
+

(2, 1, 5)

Example



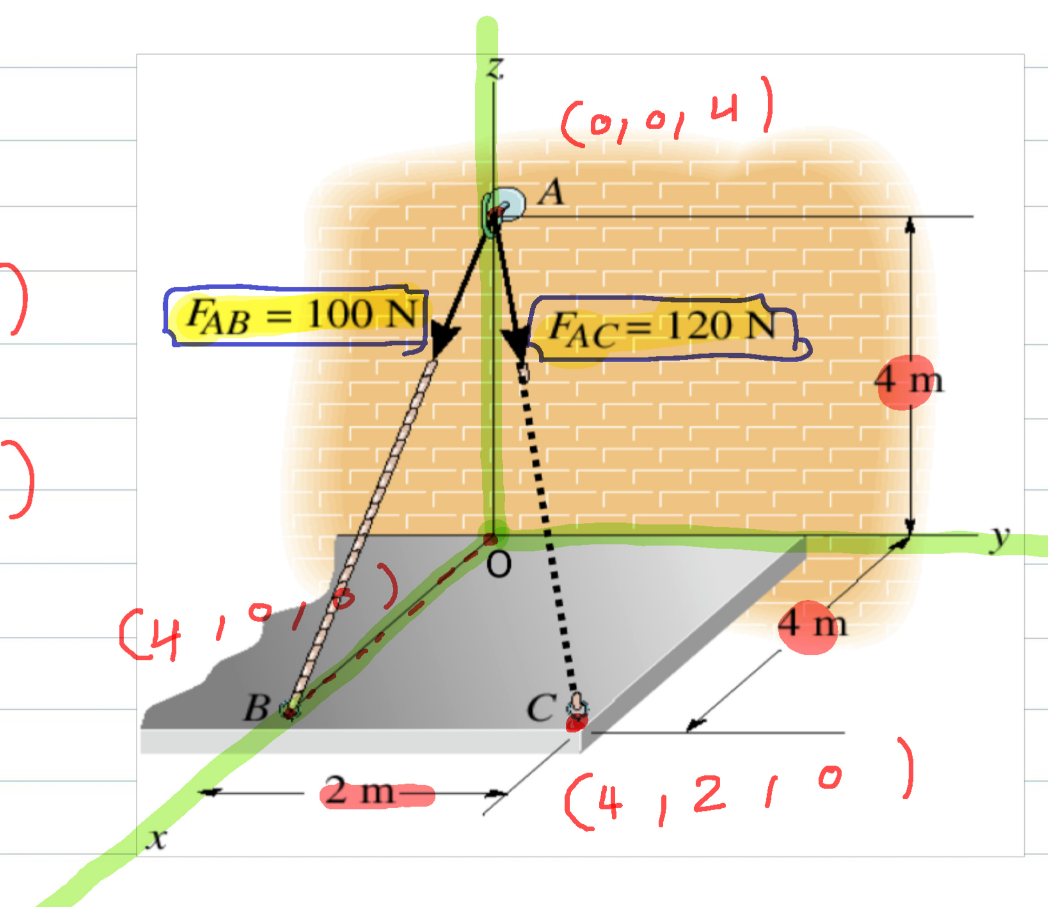
The roof is supported as shown. If the cables exert forces of **F_{AB} = 100 N** and **F_{AC} =120 N** on the wall hook at A, determine the magnitude of the resultant force acting at A.

Step (A) Identify the absolute coordinates of all points (x, y, z)

$A = (0,0,4)$

$B = (4,0,0)$

$C = (4,2,0)$



Step (B) Identify the absolute position vectors

$$\vec{r}_A = \vec{OA} = (0)\hat{i} + (0)\hat{j} + 4\hat{k}$$

$$\vec{r}_B = \vec{OB} = 4\hat{i} + (0)\hat{j} + 0(\hat{k})$$

$$\vec{r}_C = \vec{OC} = 4\hat{i} + 2\hat{j} + 0\hat{k}$$

Step (C) Identify the position vectors of the mechanical elements

$$\vec{AB} = \vec{r}_B - \vec{r}_A = (4)\hat{i} + (0)\hat{j} - (4)\hat{k}$$

$$|\vec{AB}| = \sqrt{(4)^2 + (-4)^2} = 5.66 \text{ m}$$

$$\vec{AC} = \vec{r}_C - \vec{r}_A = (4)\hat{i} + (2)\hat{j} - (4)\hat{k}$$

$$|\vec{AC}| = \sqrt{(4)^2 + (2)^2 + (-4)^2} = 6 \text{ m}$$

Step (D) Find the unit position vectors

$$\vec{u}_{AB} = \frac{\vec{AB}}{|\vec{AB}|} = \frac{(4)\hat{i} - (4)\hat{k}}{5.66}$$

$$= \left(\frac{4}{5.66}\right)\hat{i} - \left(\frac{4}{5.66}\right)\hat{k}$$

$$\vec{U}_{AC} = \frac{\vec{AC}}{|\vec{AC}|} = \frac{4\hat{i} + 2\hat{j} - 4\hat{k}}{6}$$

$$= 4/6\,\hat{i} + 2/6\,\hat{j} - 4/6\,\hat{k}$$

Step (E) Identify the force vectors

$$\vec{F} = F\,\vec{U}$$

$$\vec{F}_{AB} = 100\,\vec{U}_{AB} = 100\left(4/5.66\,\hat{i} - 4/5.66\,\hat{k}\right)$$

$$= 70.7\,\hat{i} - 70.7\,\hat{k}$$

$$\vec{F}_{AC} = 120\,\vec{U}_{AC} = 120\left(4/6\,\hat{i} + 2/6\,\hat{j} - 4/6\,\hat{k}\right)$$

$$= 80\,\hat{i} + 40\,\hat{j} - 80\,\hat{k}$$

Step (F) Find the resultant force

$$\vec{F}_R = \vec{F}_{AB} + \vec{F}_{AC}$$

$$= (70.7 + 80)\,\hat{i} + (40)\,\hat{j} + (-70.7 - 80)\,\hat{k}$$

$$= 150.7\,\hat{i} + 40\,\hat{j} - 150.7\,\hat{k}$$

Step (G) Identify the magnitude and direction of the resultant

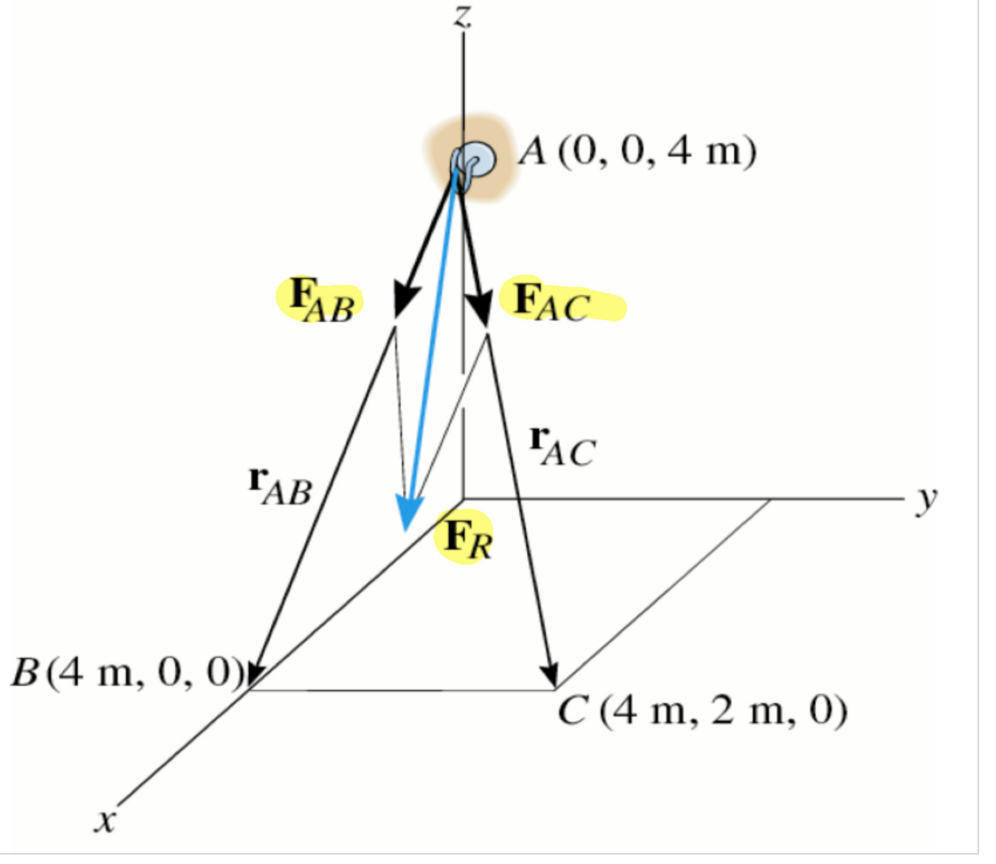
$$F_R = \sqrt{(150.7)^2 + (40)^2 + (-150.7)^2}$$

$$= 217\ N$$

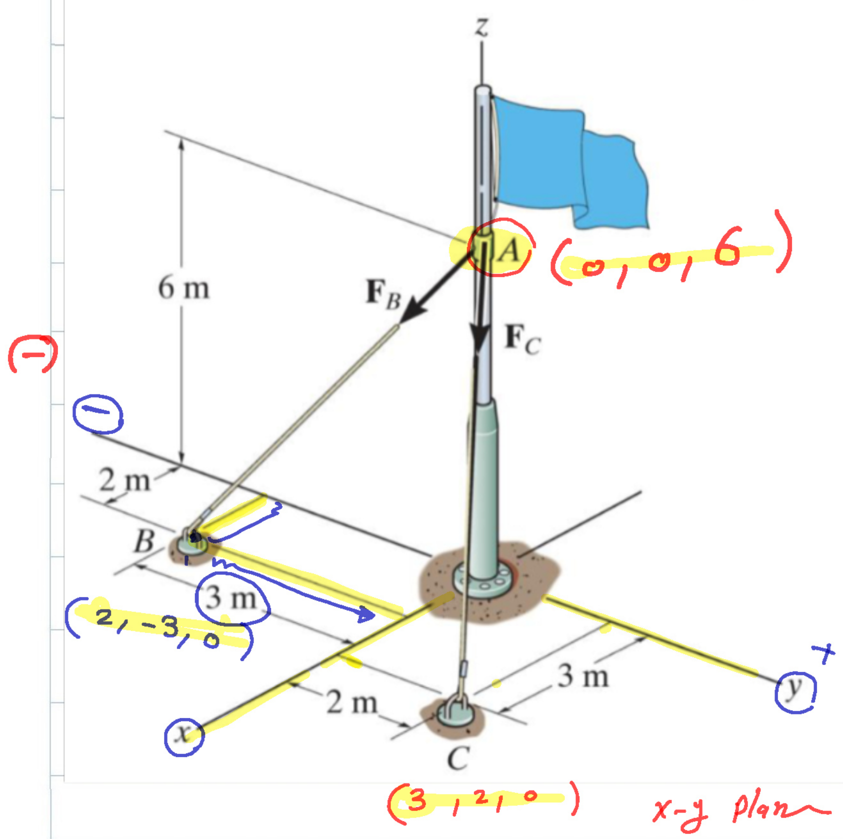
$$\cos\alpha = \frac{A_x}{A} = \frac{150.7}{217} \Rightarrow \alpha = \checkmark$$

$$\cos\beta = \frac{A_y}{A} = \frac{40}{217} \Rightarrow \beta = \checkmark$$

$$\cos\gamma = \frac{A_z}{A} = \frac{-150.7}{217} \Rightarrow \gamma = \checkmark$$



Try Yourself



Given: Two forces are acting on a flag pole as shown in the figure. F_B = 700 N and F_C = 560 N

Find: The magnitude and the coordinate direction angles of the resultant force.

Step (A) Identify the absolute coordinates of all points (x, y, z)

Step (B) Identify the absolute position vectors

Step (C) Identify the position vectors of the mechanical elements

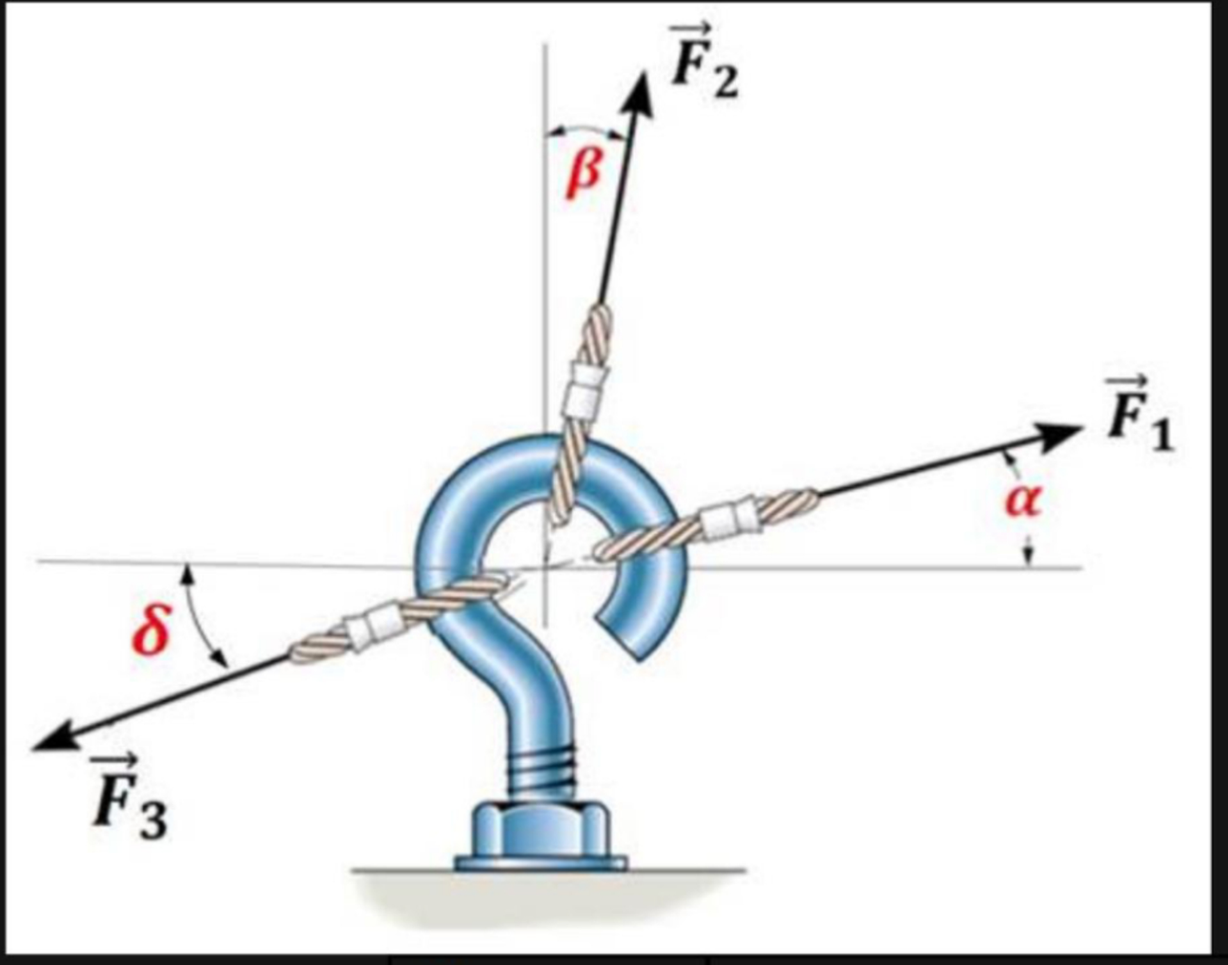
Step (D) Find the unit position vectors

Step (E) **Identify the force vectors**

Step (F) **Find the resultant force**

Step (G) **Identify the magnitude and direction of the resultant**

Determine the **magnitude** (**R**) and **direction** (**θ**) of the resultant force $\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$, by resolving the force vectors into **Cartesian components** (using the projection method).

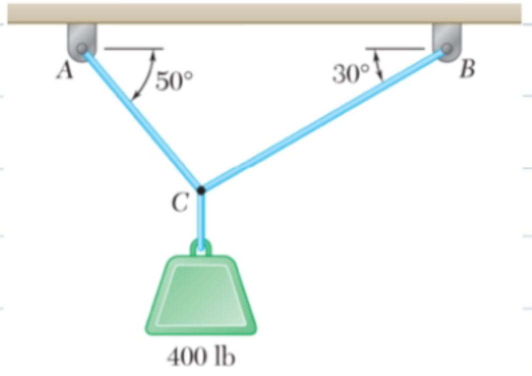


F_1	110 N
F_2	140 N
F_3	60 N
α	34 degrees
β	25 degrees
δ	20 degrees

PROBLEM 2.43

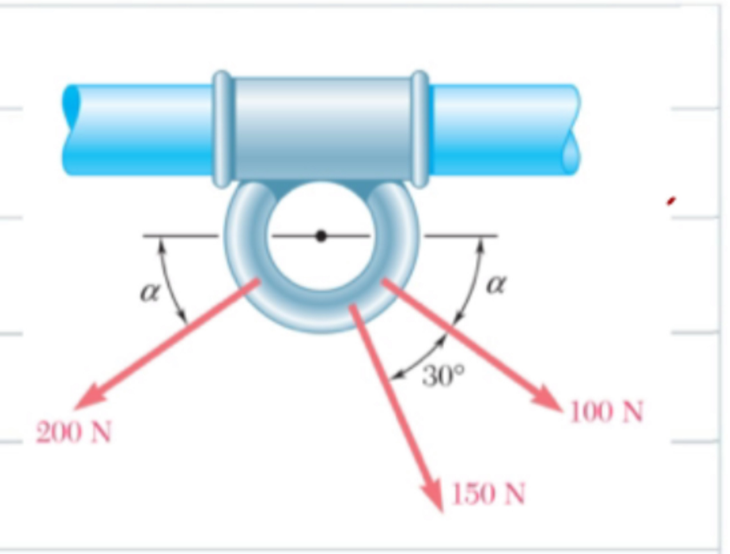
Two cables are tied together at C and are loaded as shown. Determine the tension (*a*) in cable AC, (*b*) in cable BC.

SOLUTION

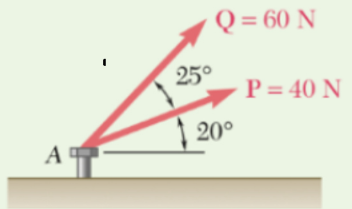


PROBLEM 2.35

Knowing that $\alpha = 35°$, determine the resultant of the three forces shown.



Sample Problem 2.1



Two forces **P** and **Q** act on a bolt A. Determine their resultant.

STRATEGY: Two lines determine a plane, so this is a problem of two coplanar forces. You can solve the problem graphically or by trigonometry.

MODELING: For a graphical solution, you can use the parallelogram rule or the triangle rule for addition of vectors. For a trigonometric solution, you can use the law of cosines and law of sines or use a right-triangle approach.

ANALYSIS:

Graphical Solution. Draw to scale a parallelogram with sides equal to **P** and **Q** (Fig. 1). Measure the magnitude and direction of the resultant. They are

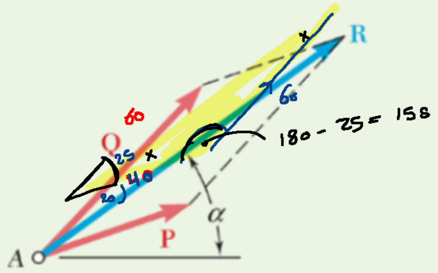


Fig. 1 Parallelogram law applied to add forces **P** and **Q**.

$$R = 98 \text{ N} \qquad \alpha = 35° \qquad \mathbf{R} = 98 \text{ N} \measuredangle 35°$$

You can also use the triangle rule. Draw forces **P** and **Q** in tip-to-tail fashion (Fig. 2). Again measure the magnitude and direction of the resultant. The answers should be the same.

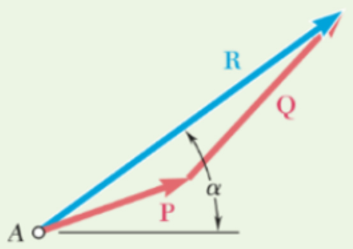


Fig. 2 Triangle rule applied to add forces **P** and **Q**.

$$R = 98 \text{ N} \qquad \alpha = 35° \qquad \mathbf{R} = 98 \text{ N} \measuredangle 35°$$

Trigonometric Solution. Using the triangle rule again, you know two sides and the included angle (Fig. 3). Apply the law of cosines.

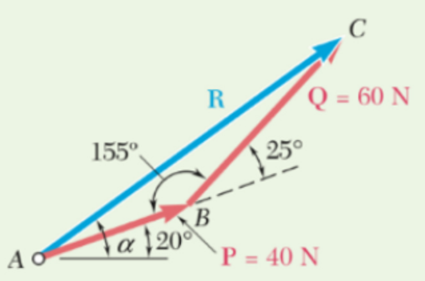


Fig. 3 Geometry of triangle rule applied to add forces **P** and **Q**.

$$R^2 = P^2 + Q^2 - 2PQ \cos B$$
$$R^2 = (40 \text{ N})^2 + (60 \text{ N})^2 - 2(40 \text{ N})(60 \text{ N}) \cos 155°$$
$$R = 97.73 \text{ N}$$

Now apply the law of sines:

$$\frac{\sin A}{Q} = \frac{\sin B}{R} \qquad \frac{\sin A}{60 \text{ N}} = \frac{\sin 155°}{97.73 \text{ N}} \tag{1}$$

Solving Eq. (1) for sin A, you obtain

$$\sin A = \frac{(60 \text{ N}) \sin 155°}{97.73 \text{ N}}$$

Using a calculator, compute this quotient, and then obtain its arc sine:

$$A = 15.04° \qquad \alpha = 20° + A = 35.04°$$

Use three significant figures to record the answer (cf. Sec. 1.6):

$$\mathbf{R} = 97.7 \text{ N} \measuredangle 35.0°$$

Alternative Trigonometric Solution. Construct the right triangle BCD (Fig. 4) and compute

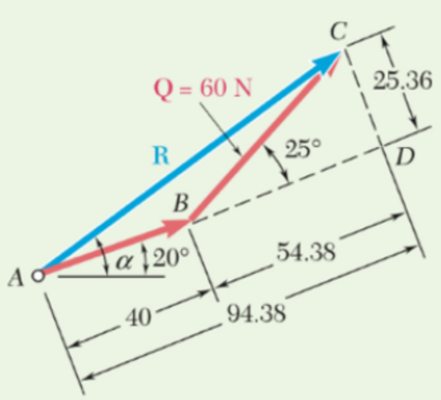


Fig. 4 Alternative geometry of triangle rule applied to add forces **P** and **Q**.

$$CD = (60 \text{ N}) \sin 25° = 25.36 \text{ N}$$
$$BD = (60 \text{ N}) \cos 25° = 54.38 \text{ N}$$