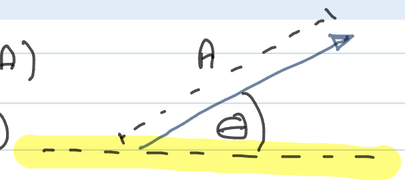
Statics of Particles 2

* Vectors } Magnitude (A)
direction (θ)

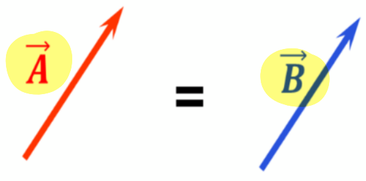


Ex. Force , velocity

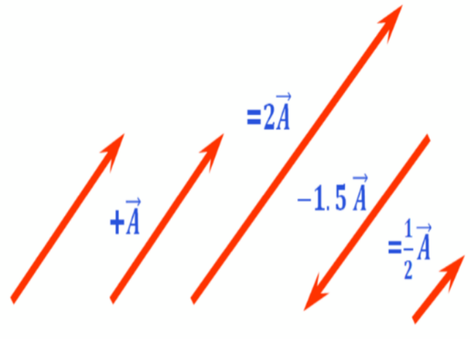
* Scalars } only + or − number
No-direction

Ex. Mass , volume

- Vectors are **equal** when they have the same magnitude and same direction



- Vectors can be simply **added** or **subtracted**, if they have the same direction



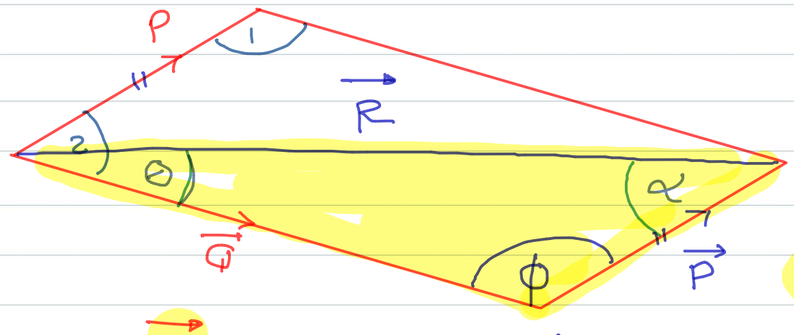
A + B = 5

Parallelogram Law
Trigonometric method

only two forces

$\vec{R} = \vec{P} + \vec{Q}$

$\hat{1} + \hat{2} = 180$



Case (1)

given $\vec{P}$ $\vec{Q}$

Required $\vec{R}$

Mag direction

① Conclude internal angle between $\vec{P}$ & $\vec{Q}$ (ϕ)

② Mag ⇒ Cos-law

$$R = \sqrt{P^2 + Q^2 - 2PQ\cos\phi}$$

③ Direction ⇒ Sin law

$$\frac{R}{\sin\phi} = \frac{P}{\sin\theta} = \frac{Q}{\sin\alpha}$$

Case (2)

given $\vec{R}$ Required $\vec{P}$ $\vec{Q}$

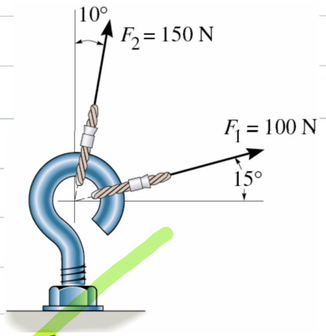
① Conclude all internal angles (ϕ α θ)

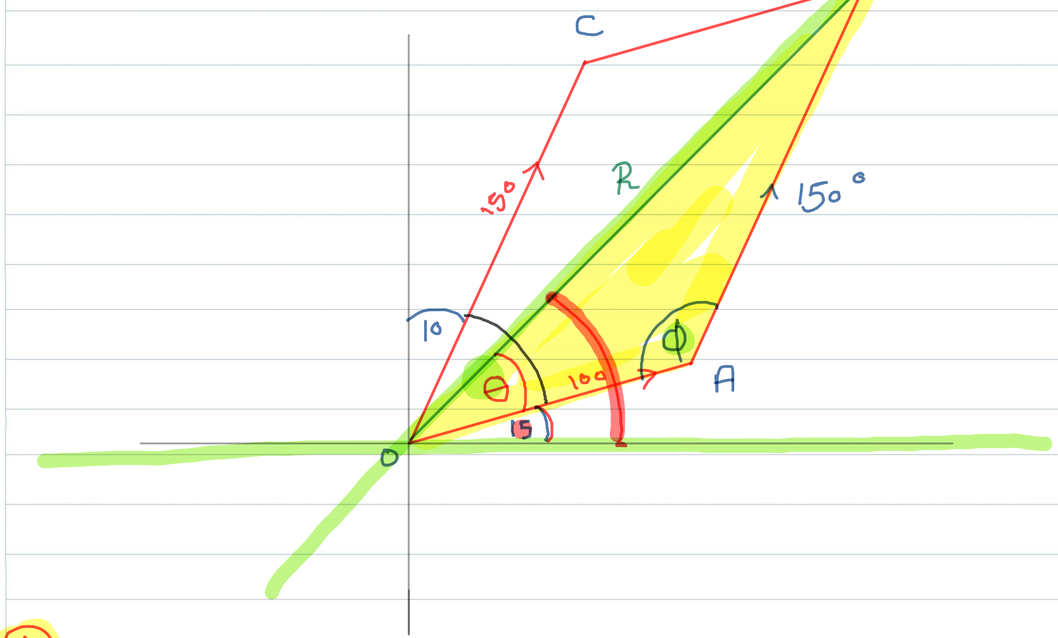
② Sin law

$$\frac{R}{\sin\phi} = \frac{P}{\sin\theta} = \frac{Q}{\sin\alpha}$$

Example 1:-

The screw eye in the figure at the left is subjected to two forces $\vec{F}_1$ and $\vec{F}_2$.
Determine the magnitude and direction of the resultant force.





(1) angle C o A = 90 − 15 − 10 = 65

$\phi = 180 - 65 = 115$

(2) $R = \sqrt{100^2 + 150^2 - 2(100)(150)\cos 115}$

$= 213$ N

(3) to get θ direction of R $\Rightarrow$ sin law

$$\frac{150}{\sin\theta} = \frac{213}{\sin 115}$$

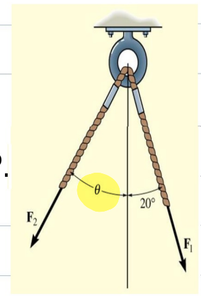
$$\theta = \sin^{-1}\left(\frac{150 \sin 115}{213}\right)$$

$= 39.7°$

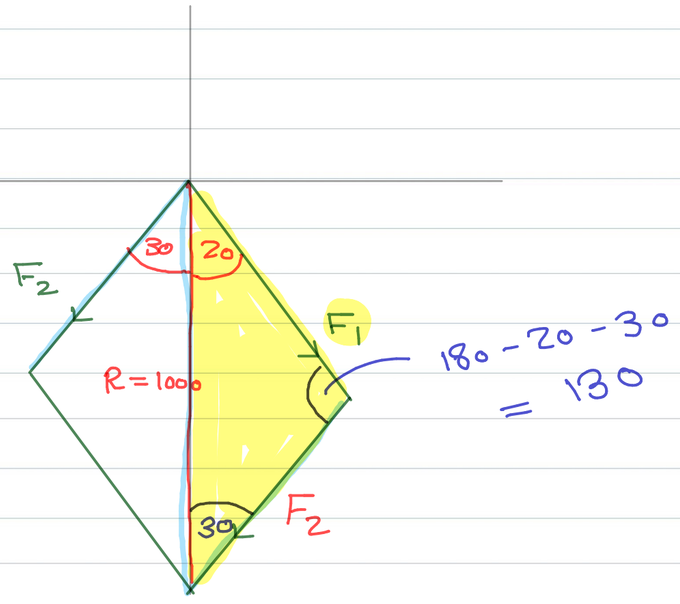
angle $= 39.7 + 15 = 54.7$

Example 2 :-

The ring below is subjected to F_1 and F_2. If we want a resultant force of 1kN and directed vertically downward, determine the magnitude of F_1 and F_2 if $\theta = 30^o$.



$R = 1000\ N$



Sine law

$$\frac{F_1}{\sin 30} = \frac{F_2}{\sin 20} = \frac{1000}{\sin 130}$$

$$F_1 = \frac{1000 \sin 30}{\sin 130} = 653\ N$$

$$F_2 = \frac{1000 \sin 20}{\sin 130} = 446\ N$$

Problem # 1

Two control rods are attached at *A* to lever *AB* .

Using trigonometry and knowing that the force in the left-hand rod is **F1=30N,**

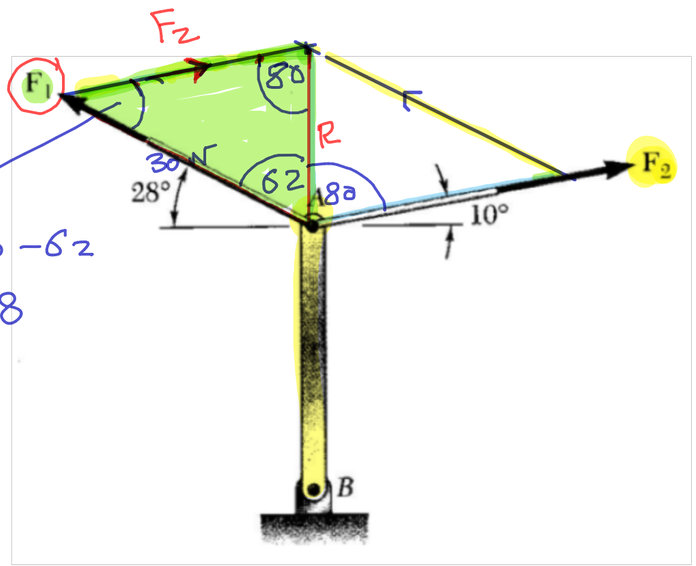
Determine: (a) The required force F2 in the right-hand rod if the resultant R of that forces exerted by the rods on the lever is to be vertical.

(b) The corresponding magnitude of R.

$F_1 = 30\ N$

$F_2 = ??$

$R = ??$



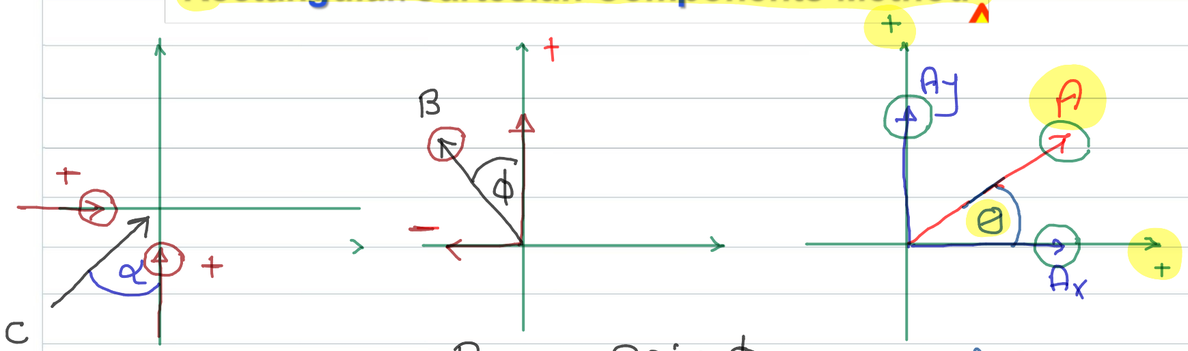
$180 - 80 - 62 = 38$

$$\frac{F_2}{\sin 62} = \frac{R}{\sin 38} = \frac{30}{\sin 80}$$

$$F_2 = \frac{30 \sin 62}{\sin 80} = 27\ N$$

$$R = \frac{30 \sin 38}{\sin 80} = 19\ N$$

Rectangular/Cartesian Components Method

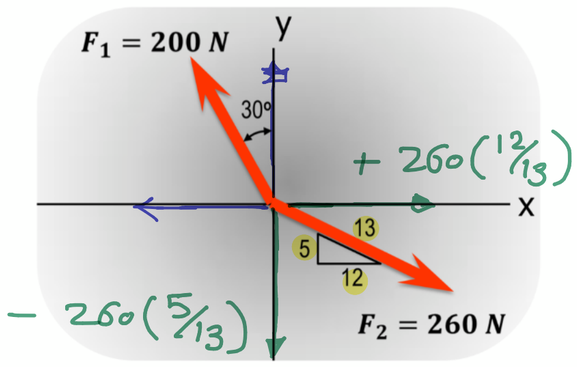


$C_x = +C\sin\alpha$ $\quad B_x = -B\sin\phi$ $\quad A_x = A\cos\theta$

$C_y = +C\cos\alpha$ $\quad B_y = +B\cos\phi$ $\quad A_y = A\sin\theta$

$\vec{B} = (B_x)\hat{i} + (B_y)\hat{j}$ $\quad A = \sqrt{A_x^2 + A_y^2}$

Determine the x and y Cartesian components of the $\mathbf{F}_1$ and $\mathbf{F}_2$ forces acting on the boom. Put each force in the Cartesian vector form.



$F_{1x} = -200\sin 30 = -100$

$F_{1y} = 200\cos 30 = 173$

$F_{2x} = 260\left(\frac{12}{13}\right) = 240$

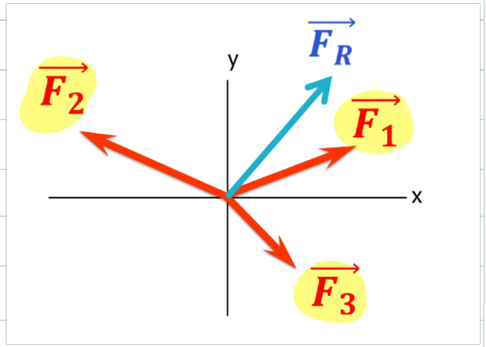
$F_{2y} = -260\left(\frac{5}{13}\right) = -100$

$+260\left(\frac{12}{13}\right)$ $\quad -260\left(\frac{5}{13}\right)$

$\vec{F}_1 = (-100)\hat{i} + (173)\hat{j}$

Coplanar Force Resultants

More than 2-Forces



1. Resolve

F_{1x} $\quad F_{2x}$ $\quad F_{3x}$

F_{1y} $\quad F_{2y}$ $\quad F_{3y}$

2. $R_x = \Sigma F_x$ $\quad \rightarrow +$

$R_y = \Sigma F_y$ $\quad \uparrow +$

3. $R = \sqrt{R_x^2 + R_y^2}$

4. $\theta = \tan^{-1} R_y/R_x$

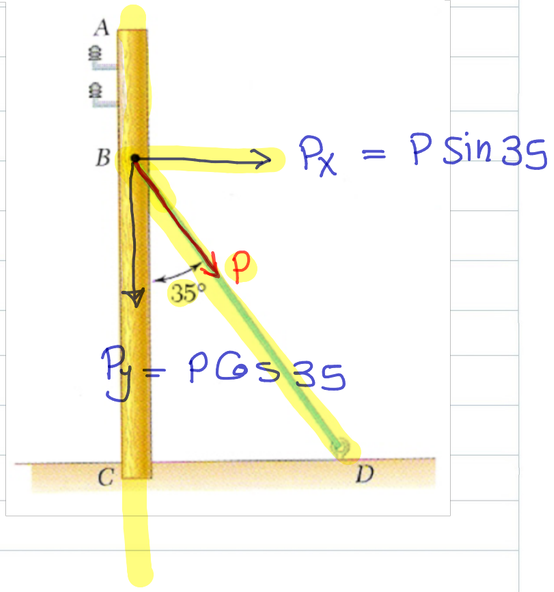
Problem # 2

tension

The Guy wire *BD* exerts on the telephone pole *AC* a force **P** directed along *BD*. Knowing that **P** has a 450-N component along line *AC*,

Determine: (a) The magnitude of the force **P**,

(b) Its component in a direction perpendicular to *AC*.



(a) $P_y = 450\ N$

$P \cos 35 = 450$

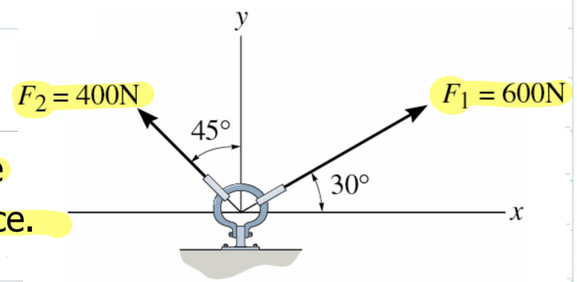
$P = \dfrac{450}{\cos 35} = 549.3$

(b) $P_x = P \sin 35$

$= 549.3 \sin 35 = 315\ N$

Example 3 :-

The link in the figure is subjected to two forces, $\mathbf{F_1}$ and $\mathbf{F_2}$. Determine the resultant magnitude and orientation of the resultant force.



* Resolve :—

$F_{1x} = 600 \cos 30 = 519.6$

$F_{1y} = 600 \sin 30 = 300$

$F_{2x} = -400 \sin 45 = -282.8$

$F_{2y} = 400 \cos 45 = 282.8$

* $R_x = \Sigma F_x \quad \xrightarrow{+}$

$= 519.6 - 282.8 = 236.8$

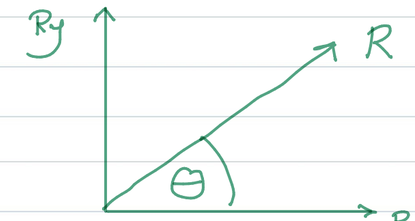
$R_y = \Sigma F_y \quad \uparrow +$

$= 300 + 282.8 = 582.8\ N$

* $R = \sqrt{R_x^2 + R_y^2}$

$= \sqrt{236.8^2 + 582.8^2} = 629.1\ N$

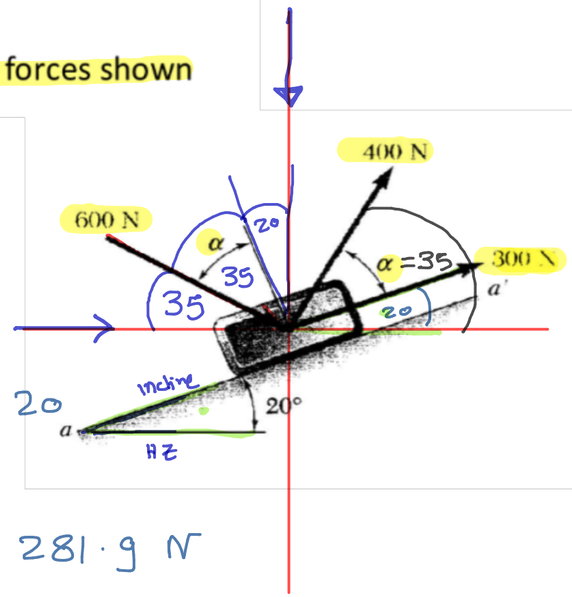
* $\theta = \tan^{-1} \frac{582.8}{236.8} = 67.9°$



Problem # 3

Knowing that $\alpha = 35°$,

Determine: The resultant of the three forces shown



* Resolve :—

$F_1 =$ 300 N

with angle with HZ = 20

$F_{1x} = 300 \cos 20 = 281.9$ N

$F_{1y} = 300 \sin 20 = 102.9$ N

$F_2 = 400$ N

with angle with HZ = 20 + 35 = 55

$F_{2x} = 400 \cos 55 = 229.4$ N

$F_{2y} = 400 \sin 55 = 327.7$ N

$F_3 = 600$ N

with angle with HZ = 35

$F_{3x} = +600 \cos 35 = 491.5$ N

$F_{3y} = -600 \sin 35 = -344.1$ N

* $R_x = \Sigma F_x = 281.9 + 229.4 + 491.5$

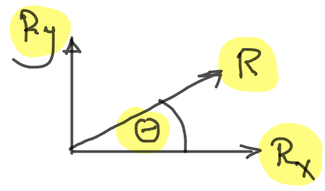
$= 1002.8$

$R_y = \Sigma F_y = 102.9 + 327.7 - 344.1$

$= 86.2$ N

* $R = \sqrt{1002.8^2 + 86.2^2} = 1006.5$ N

* $\theta = \tan^{-1} \frac{86.2}{1002.8} = 4.91°$



Problem # 3

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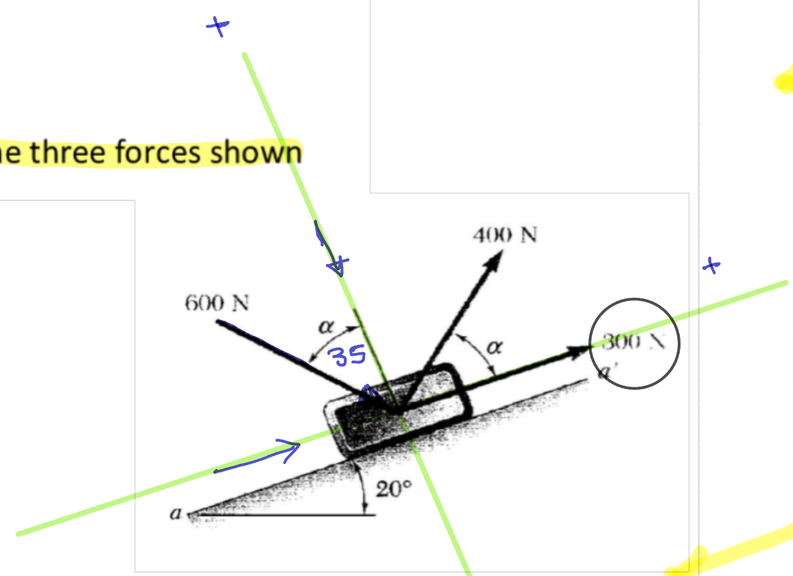


* Resolve :-

$F_1 = 300\ N$

$F_{1x} = 300$

$F_{1y} = 0$

$F_2 = 400\ N$

$F_{2x} = 400\ \cos 35 = 327.7$

$F_{2y} = 400\ \sin 35 = 229.43$

$F_3 = 600\ N$

$F_{3x} = +600\ \sin 35 = 344.15$

$F_{3y} = -600\ \cos 35 = -491.5$

* $R_x = \Sigma F_x = 300 + 327.7 + 344.15$

$= 971.85$

$R_y = \Sigma F_y = 229.43 - 491.5$

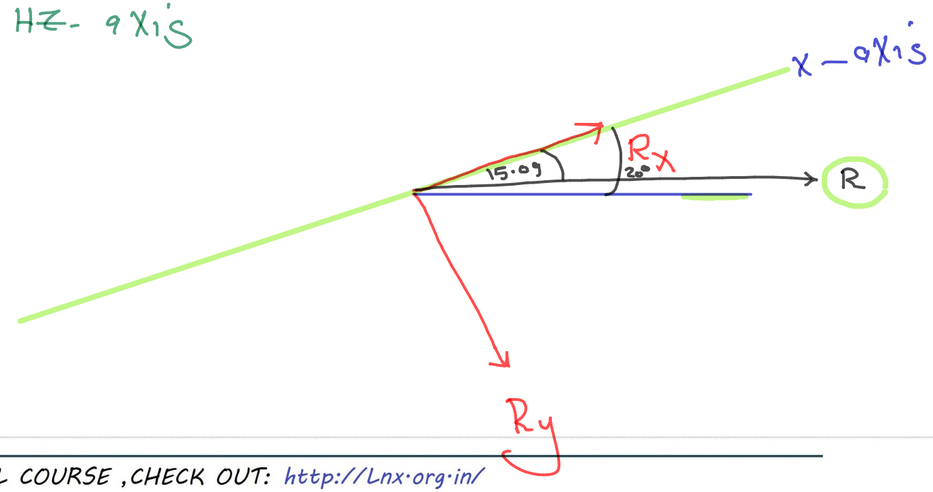
$= -262.07$

* $R = \sqrt{(971.85)^2 + (-262.07)^2} = 1006.5\ N$

* $\Theta = \tan^{-1} \dfrac{-262.07}{971.85} = -15.09°$

$\Theta = 20 - 15.09 = 4.91°$

with Hz. axis

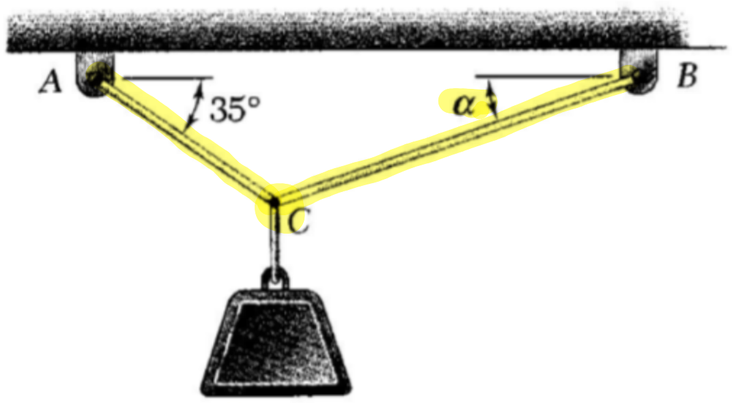


Midterm (1)

Problem # 4

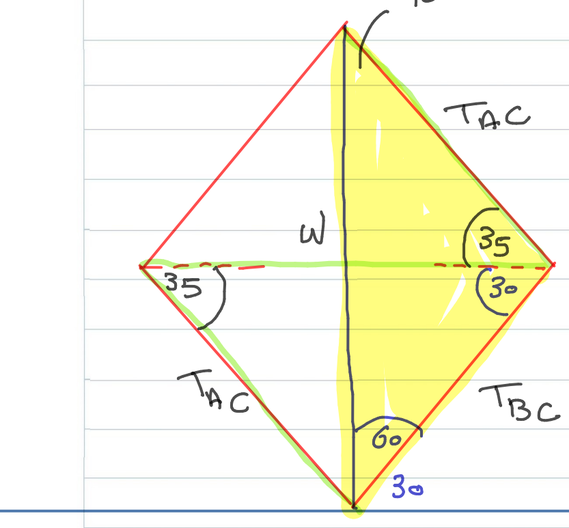
Two cables are tied together at *C* and are loaded as shown. Knowing that **α = 30°**,

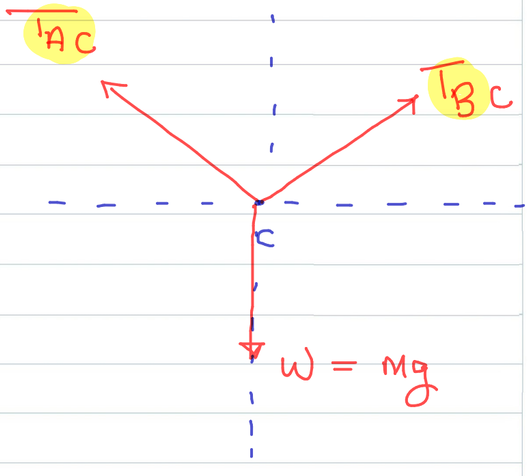
Determine: the tension (a) in cable *AC*, (b) cable *BC*.



$= M$

$180 - 60 - 65 = 55$





$W = Mg$

$= 300 * 9.81 = 2943 \ N$

$$\frac{T_{AC}}{\sin 60} = \frac{T_{BC}}{\sin 55} = \frac{2943}{\sin 65}$$

$$T_{AC} = \frac{2943 \sin 60}{\sin 65} = 2812.19$$

$$T_{BC} = \frac{2943 \sin 60}{\sin 65}$$

$$= 2659.98 \ N$$

2.3 Equilibrium of a Particle

$\Sigma F = 0$

* When Particle @ rest
* Moving with Constant velocity

1. Resolve
2. $\Sigma F_x = 0$ (+→) and $\Sigma F_y = 0$ (+↑) } 2-Eq^{u}/s ⇕ 2-unknowns

Remark

① IF Resultant is Vertical :-

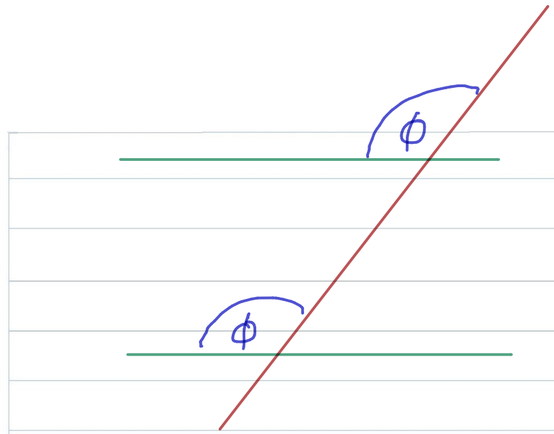
$R = R_y = \Sigma F_y$ (+↑)

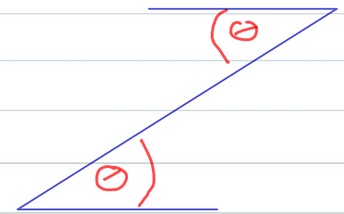
$R_x = \Sigma F_x = 0$ (+→)

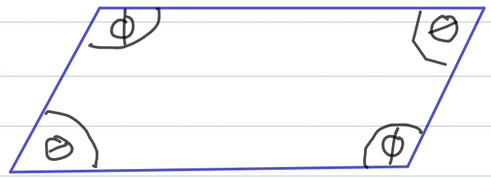
② IF Resultant is Horizontal :-

$R = R_x = \Sigma F_x$ (+→)

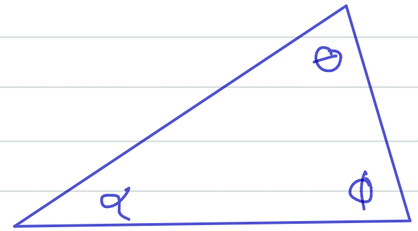
$R_y = \Sigma F_y = 0$ (+↑)



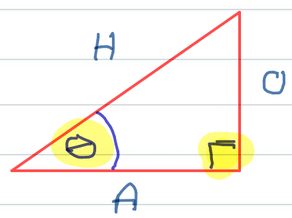




$\theta + \phi = 180$



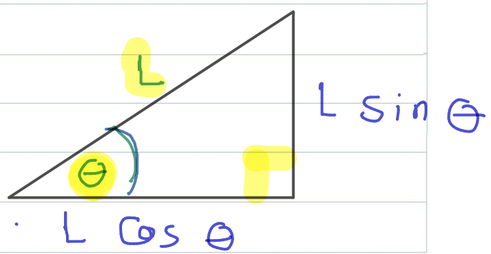
$\theta + \alpha + \phi = 180^{o}$



$\sin\theta = \frac{O}{H}$

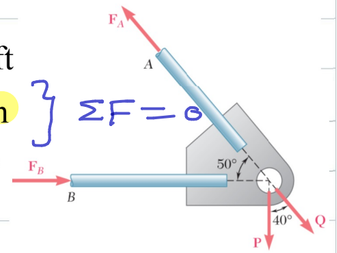
$\cos\theta = \frac{A}{H}$

$\tan\theta = \frac{O}{A}$

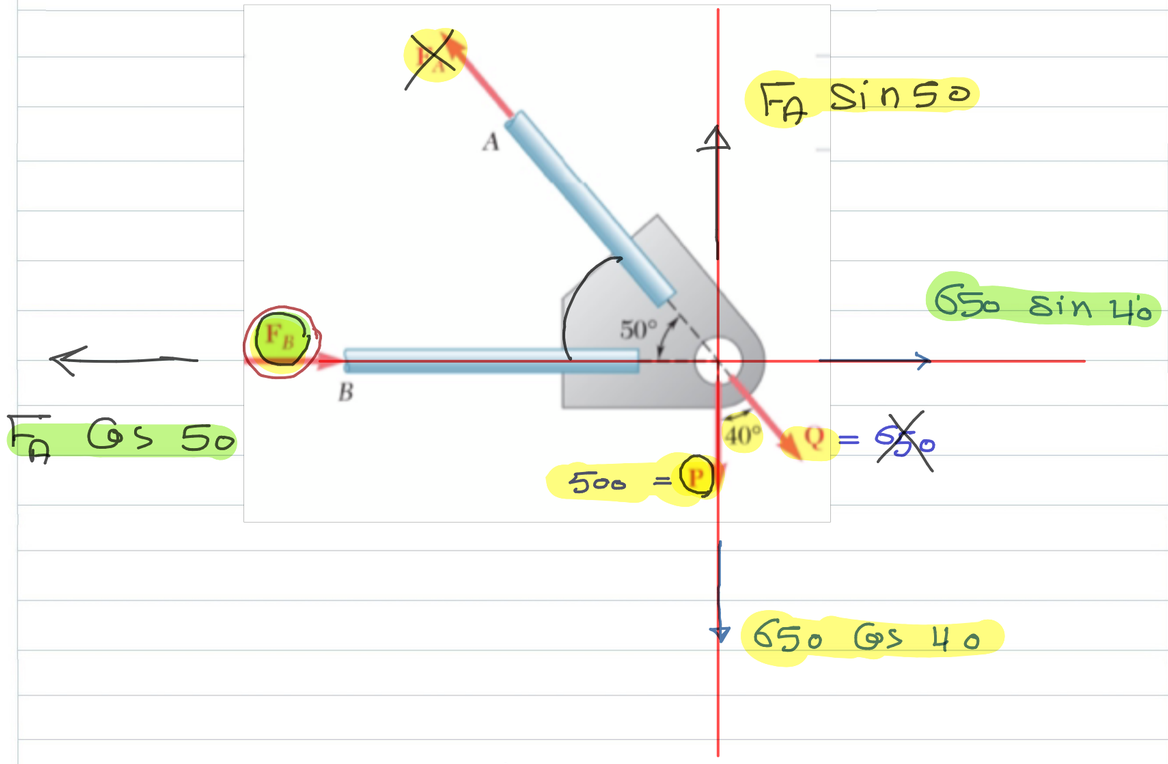


PROBLEM 2.51

Two forces **P** and **Q** are applied as shown to an aircraft connection. Knowing that the connection is in equilibrium and that $P = 500$ N and $Q = 650$ N, determine the magnitudes of the forces exerted on the rods *A* and *B*.



$\Sigma F = 0$



$$\Sigma F_y = 0 \quad \uparrow +$$

$$F_A \sin 50 - 500 - 650 \cos 40 = 0$$

$$F_A \sin 50 = 997.9$$

$$F_A = 1303 \text{ N}$$

$$\Sigma F_x = 0 \quad \rightarrow +$$

$$650 \sin 40 - F_A \cos 50 + F_B = 0$$

1303

$$F_B = 1303 \cos 50 - 650 \sin 40$$

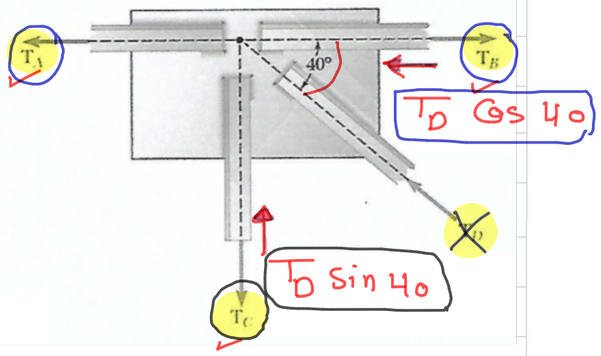
$$= 420 \text{ N}$$

Quiz # 1

Fall 2016

Two forces of magnitude $T_A = 8$ KN and $T_B = 15$ KN are applied as shown to a welded connection. Knowing that the connection is in equilibrium.

Determine: the magnitudes of the forces T_C and T_D.



under Equilibrium :-

* $\Sigma F_x = 0$ $\rightarrow +$

$T_B - T_D \cos 40 - T_A = 0$

$15 - T_D \cos 40 - 8 = 0$

$T_D \cos 40 = 7$

$T_D = 9.14$ KN

* $\Sigma F_y = 0$ $\uparrow +$

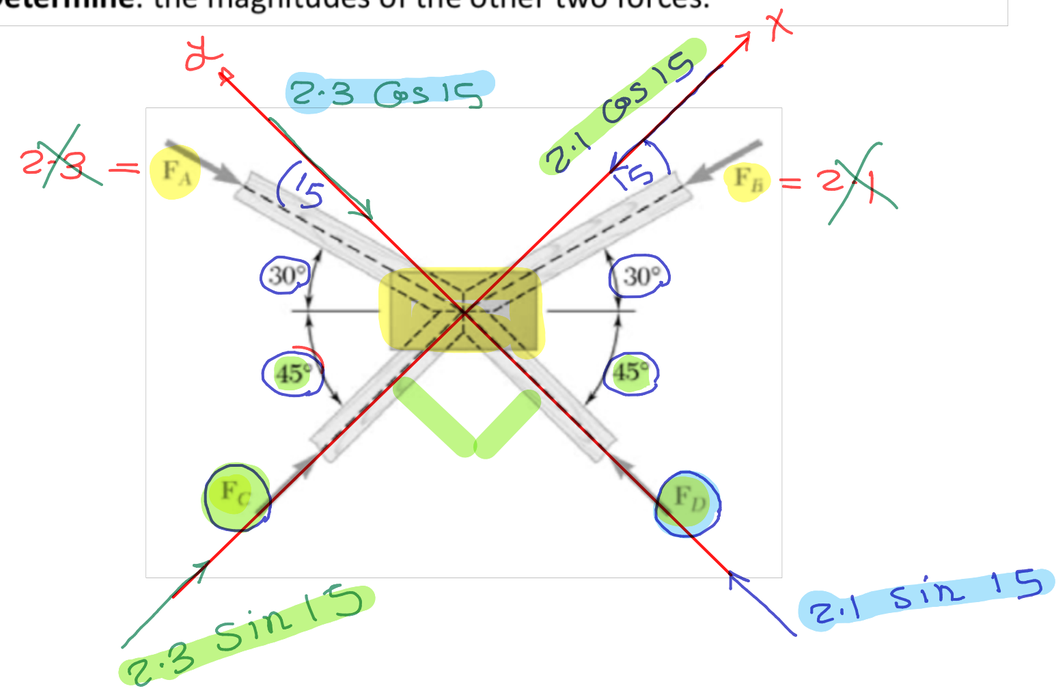
$T_D \sin 40 - T_C = 0$

$T_C = 9.14 \sin 40 = 5.87$ KN

Problem # 5

Four wooden members are joined with metal plate connectors and are in equilibrium under the action of the four forces shown. Knowing that $F_A = 2.3$ kN and $F_B = 2.1$ kN,

Determine: the magnitudes of the other two forces.



$\Sigma F_x = 0$ +

$2.3 \sin 15 + F_C - 2.1 \cos 15 = 0$

$F_C = 1.43$ kN

$\Sigma F_y = 0$ +

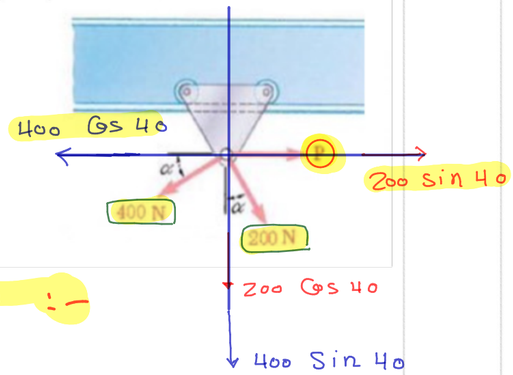
$F_D + 2.1 \sin 15 - 2.3 \cos 15 = 0$

$F_D = 1.68$ kN

Quiz

2.29 A hoist trolley is subjected to the three forces shown. Knowing that $\alpha = 40°$, determine (*a*) the magnitude of the force **P** for which the resultant of the three forces is vertical, (*b*) the corresponding magnitude of the resultant.

solution



IF Resultant is Vertical :-

$$R_x = \Sigma F_x = 0 \qquad \rightarrow +$$

$$200 \sin 40 + P - 400 \cos 40 = 0$$

$$P = 177.9 \text{ kN}$$

$$\Rightarrow R_y = \Sigma F_y \qquad \uparrow +$$

$$= -400 \sin 40 - 200 \cos 40$$

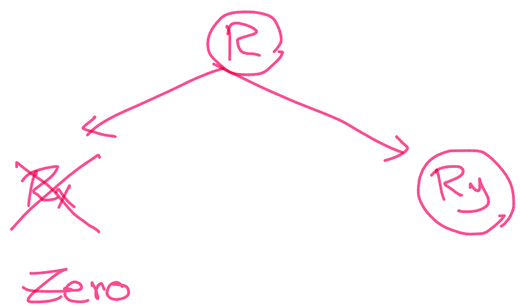
$$= -410.3$$

$$R = |R_y| = 410.3 \text{ N}$$

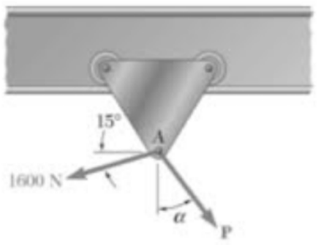
① IF Resultant is Vertical :-

$$R = R_y = \Sigma F_y \qquad \uparrow +$$

$$R_x = \Sigma F_x = 0 \qquad \rightarrow +$$



midterm 1



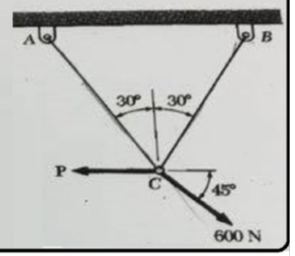
PROBLEM 2.6

A trolley that moves along a horizontal beam is acted upon by two forces as shown. (*a*) Knowing that $\alpha = 25°$, determine by trigonometry the magnitude of the force **P** so that the resultant force exerted on the trolley is vertical. (*b*) What is the corresponding magnitude of the resultant?

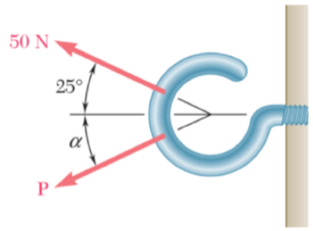
MIDTERM 1
FALL 2016

Problem # 1: (5 points)

Knowing that $P = 400$ N, determine the tension in cables AC and BC.



Mid term ①



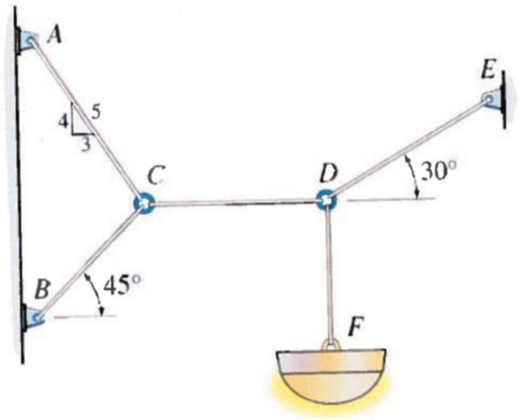
PROBLEM 2.10

Two forces are applied as shown to a hook support. Knowing that the magnitude of **P** is 35 N, determine by trigonometry (*a*) the required angle α if the resultant **R** of the two forces applied to the support is to be horizontal, (*b*) the corresponding magnitude of **R**.

Question # 2: [25 Points]. The bulb support system composed of 5 different wires. Knowing that the bulb mass is 25 kg and the tension in the wire DF (F_{DF}) equals the weight of the bulb, you are required to:

a) Draw the necessary free-body diagrams of the points C and D.

b) Calculate the forces in the cables F_{DE}, F_{CD}, F_{AC} and F_{BC}

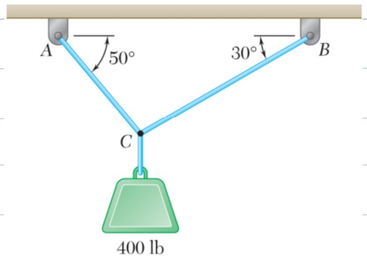


Midterm

PROBLEM 2.43

Two cables are tied together at C and are loaded as shown. Determine the tension (*a*) in cable AC, (*b*) in cable BC.

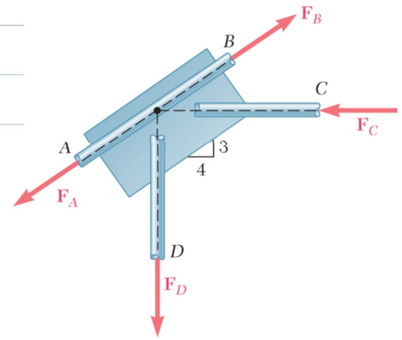
SOLUTION



PROBLEM 2.53

A welded connection is in equilibrium under the action of the four forces shown. Knowing that $F_A = 8$ kN and $F_B = 16$ kN, determine the magnitudes of the other two forces.

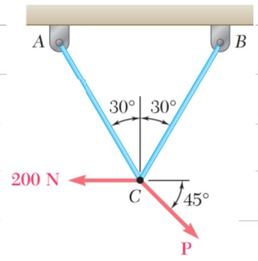
SOLUTION



PROBLEM 2.49

Two cables are tied together at C and are loaded as shown. Knowing that $P = 300$ N, determine the tension in cables AC and BC.

SOLUTION

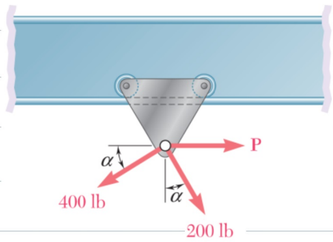


PROBLEM 2.129

A hoist trolley is subjected to the three forces shown. Knowing that $\alpha = 40°$, determine (*a*) the required magnitude of the force **P** if the resultant of the three forces is to be vertical,

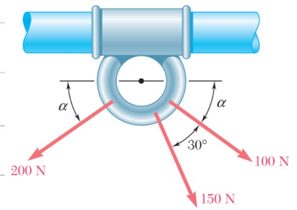
(*b*) the corresponding magnitude of the resultant.

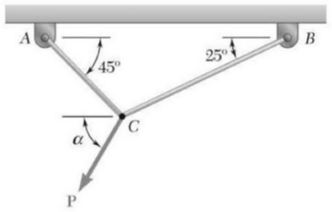
SOLUTION



PROBLEM 2.35

Knowing that $\alpha = 35°$, determine the resultant of the three forces shown.





PROBLEM 2.48

Two cables are tied together at C and are loaded as shown. Knowing that $\mathbf{P} = 500$ N and $\alpha = 60°$, determine the tension in (*a*) in cable AC, (*b*) in cable BC.