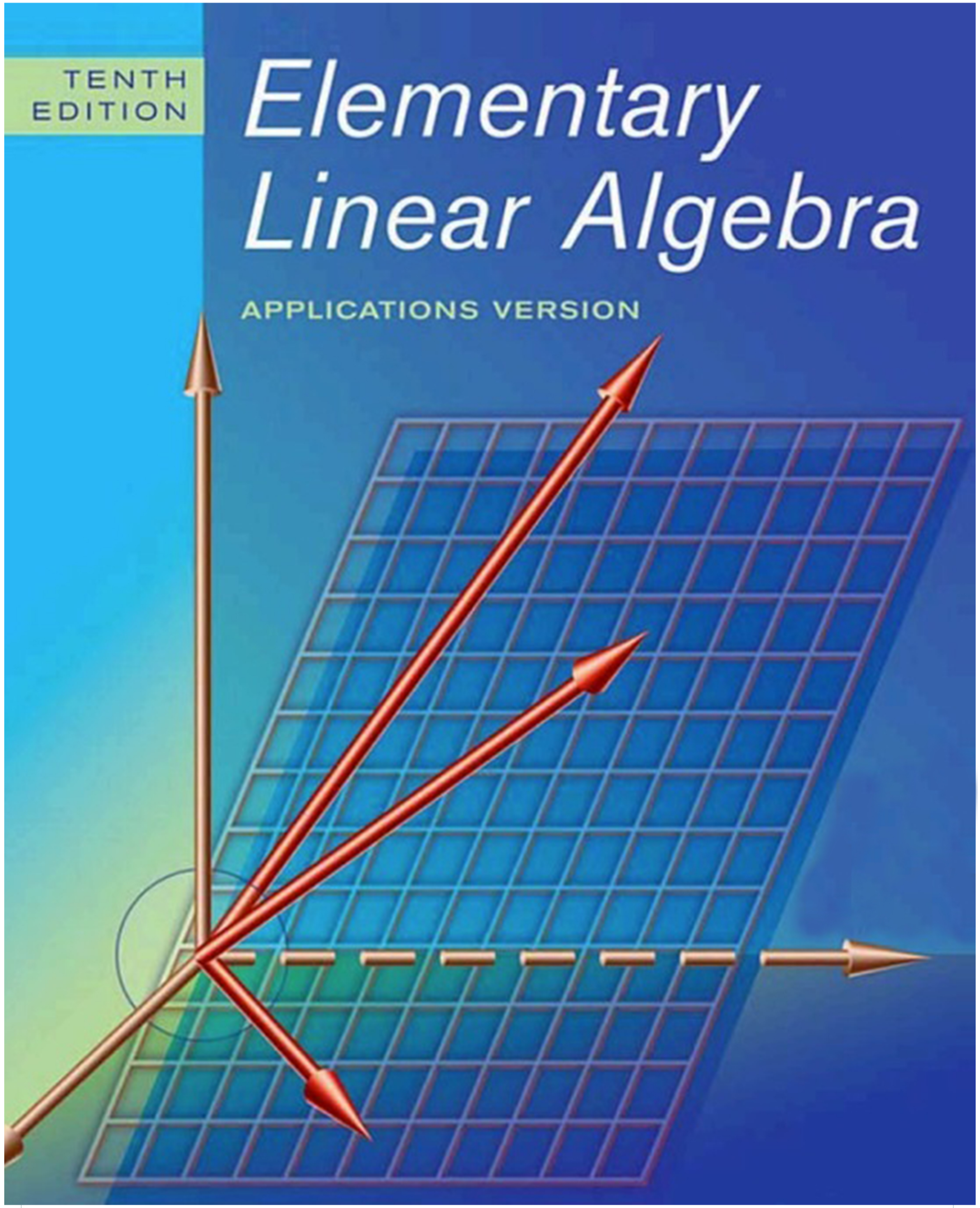

Linear Algebra (MATH 231)

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Chapter 1

Systems of Linear Equations and Matrices

Section 1.1: Introduction to Systems of Linear Equations.

$\boxed{1}$ Equation of Line :-

$$ax + by = c \qquad a \neq 0,\ b \neq 0 \qquad (x-y)$$

$$ax + by + cz = d \qquad a \neq 0,\ b \neq 0,\ c \neq 0 \qquad (x-y-z)$$

$\boxed{2}$ Linear Equations :-

$$a_1 x_1 + a_2 x_2 + a_3 x_3 ---- a_n x_n = b$$

$a_1 = 0 \qquad a_2 = 0 \qquad a_n = 0$

$b \neq 0 \Rightarrow$ non-homogenous

$b = 0 \Rightarrow$ homogenous L. Eq^{ns}

Examples of linear Eq^{ns}

* $3x + 2y = 5$
* $4x + 3y + 7z = 10$
* $x_1 + x_2 - 2^{100}\ x_3 = e^{1000}$

$\boxed{3}$ Examples of non linear equations :-

* $3x^{-1} + 4y = 5$
* $3/x + 4y = 2$
* $\sqrt{x} + y = e^{10}$

$\cancel{xy + z = 5}$

(4) System of Linear equations :-

$$\left.\begin{aligned} x + y &= 2 \\ x - y &= 4 \end{aligned}\right\} \begin{aligned} &2\text{-Eq}^{ns} \\ &2\text{- Unknowns} \end{aligned}$$

$$\left.\begin{aligned} 2x_1 + 4x_2 - x_3 &= 0 \\ x_1 + 10x_2 + 11x_3 &= 0 \end{aligned}\right\} \begin{aligned} &2\text{-Eq}^{ns} \\ &3\text{-Unknowns} \end{aligned}$$

$$\left.\begin{aligned} x + y + z &= 1 \\ x - y + z &= 2 \\ x + y + 3z &= 0 \\ x - y - z &= 2 \end{aligned}\right\} \begin{aligned} &4\text{- Eq}^{ns} \\ &3\text{- Unknowns} \end{aligned}$$

Example.

Verify that $x = 1$ and $y = -2$ is a solution to the linear system

$$\begin{aligned} 5x + y &= 3 \\ 2x - y &= 4 \end{aligned}$$

Sol

$5x + y = 5(1) - 2 = 3$ ✓

$2x - y = 2(1) - (-2) = 2 + 2 = 4$ ✓

Then $\left.\begin{aligned} x &= 1 \\ y &= -2 \end{aligned}\right\}$ is solution to given system

Example.

Verify that $x = 0$ and $y = 3$ is not a solution to the linear system

$$\begin{aligned} 5x + y &= 3 \\ 2x - y &= 4 \end{aligned}$$

$5x + y = 5(0) + 3 = 3$ ✓

$2x - y = 2(0) - 3 = -3 \neq 4$

Then $\left.\begin{aligned} x &= 0 \\ y &= 3 \end{aligned}\right\}$ is not a solution to the system

Example.

Verify that $x_1 = 1$, $x_2 = 2$, and $x_3 = -1$ is a solution to the linear system

$$\begin{aligned} 4x_1 - x_2 + 3x_3 &= -1 \\ 3x_1 + x_2 + 9x_3 &= -4 \end{aligned}$$

Sol

$\Rightarrow 4x_1 - x_2 + 3x_3 = 4(1) - (2) + 3(-1)$

$= 4 - 5 = -1$ ✓

$\Rightarrow 3x_1 + x_2 + 9x_3 = 3(1) + (2) + 9(-1)$

$= -4$ ✓

Then $\left.\begin{aligned} x_1 &= 1 \\ x_2 &= 2 \\ x_3 &= -1 \end{aligned}\right\}$ is a solution to the system

Linear systems in two unknowns :-

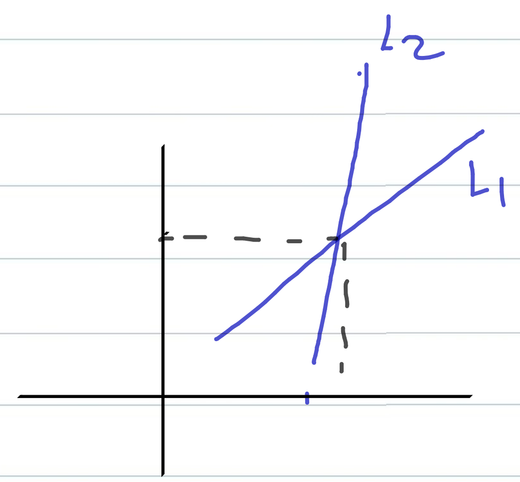
$$a_1 x + b_1 y = c_1 \quad (L_1)$$
$$a_2 x + b_2 y = c_2 \quad (L_2)$$

2-Equ's
2-unknowns

① one solution

intersect @ only one point

unique



Consistent

② ∞-Many solution

Coincide

③ No-solution

Parallel distinct

in Consistent

Elementry row operations :-

$$x - y = 1 \quad (E_1) \Rightarrow R_1$$
$$2x - 3y = 7 \quad (E_2) \Rightarrow R_2$$

① Multiply an Eq^u E_1 or E_2 by c : $c \neq 0$

② $E_1 \Leftrightarrow E_2$ interchange

③ Multiply E_1 by Constant then add to E_2

Example. Solve the linear system $x - y = 1$ E_1
$2x + y = 5$ E_2

$-2E_1 + E_2 \to E_2$ $\begin{cases} x - y = 1 & E_1 \\ 3y = 3 & E_2 \end{cases}$

$\frac{1}{3}E_2 \to E_2$ $\begin{cases} x - y = 1 & E_1 \\ y = 1 & E_2 \end{cases}$

From E_2 $y = 1$

" E_1 $x - 1 = 1 \Rightarrow x = 2$

unique $\left(\begin{matrix} x = 2 \\ y = 1 \end{matrix} \right\}$ is a solution of system

Example. Solve the linear system $x + y = 4$ E_1
$3x + 3y = 6$ E_2

$-3E_1 + E_2 \to E_2$ $\begin{cases} x + y = 4 & E_1 \\ 0 = -6 \end{cases}$ Not possible

The System has no-Solution

Example. Solve the linear system $2x - y = 2$ E_1
$8x - 4y = 8$ E_2

$-4E_1 + E_2 \to E_2$ $\begin{cases} 2x - y = 2 & E_1 \\ 0 = 0 \end{cases}$

∞ - Many solution

$2x - y = 2$

$2x = y + 2$

$x = \frac{1}{2}(y + 2)$

let $y = t$; t is any real number

$x = \frac{1}{2}(t + 2)$

System has ∞ - Many Solution

$x = \frac{1}{2}(t + 2)$

$y = t$

Example. Solve the linear system

$$\begin{aligned} x - y + z &= 0 & E_1\\ -x + 2y + 2z &= 1 & E_2\\ 2x - y - z &= 3 & E_3 \end{aligned}$$

$E_1 + E_2 \rightarrow E_2$

$-2E_1 + E_3 \rightarrow E_3$

$$\left.\begin{aligned} x - y + z &= 0 & E_1\\ y + \boxed{3z} &= 1 & E_2\\ y - 3z &= 3 & E_3 \end{aligned}\right.$$

$-E_2 + E_3 \rightarrow E_3$

$$\left.\begin{aligned} x - y + z &= 0 & E_1\\ y + 3z &= 1 & E_2\\ -6z &= 2 & E_3 \end{aligned}\right.$$

$-\frac{1}{6}E_3 \rightarrow E_3$

$$\left.\begin{aligned} x - y + z &= 0 & E_1\\ y + 3z &= 1 & E_2\\ z &= -\tfrac{1}{3} & E_3 \end{aligned}\right.$$

From $E_3 \Rightarrow z = -\frac{1}{3}$

$E_2 \Rightarrow y + 3\left(-\frac{1}{3}\right) = 1 \Rightarrow y = 2$

$E_1 \Rightarrow x - 2 + \left(-\frac{1}{3}\right) = 0 \Rightarrow x = \frac{7}{3}$

$x = \frac{7}{3}$

$y = 2$

$z = -\frac{1}{3}$

} unique solution

Example. Find the augmented matrix for the linear system

$$\begin{aligned} x - y + z &= 0\\ -x + 2y + 2z &= -1\\ 2x - y - z &= 3 \end{aligned}$$

$$\begin{array}{c} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0\\ -1 & 2 & 2 & -1\\ 2 & -1 & -1 & 3 \end{array}\right] \end{array} \begin{array}{l} R_1\\ R_2\\ R_3 \end{array}$$

Example. Find the augmented matrix for the linear system

$$\left.\begin{aligned} 2x + 5y - z &= 0\\ -2y + 2z &= 0 \end{aligned}\right\}$$

2 - Eqs

3 - unknowns

$$\left[\begin{array}{ccc|c} 2 & 5 & -1 & 0\\ 0 & -2 & 2 & 0 \end{array}\right] \begin{array}{l} R_1\\ R_2 \end{array}$$

homogenous system

Example. For the given augmented matrix write down the corresponding system of equations.

$$\begin{array}{cc} \begin{array}{cc|c} x_1 & x_2 & \end{array} & \\ \left[\begin{array}{cc|c} 2 & -1 & -1\\ -1 & 3 & 1\\ -4 & 5 & 0 \end{array}\right] & \begin{array}{l} R_1\\ R_2\\ R_3 \end{array} \end{array}$$

$$\begin{aligned} 2x_1 - x_2 &= -1\\ -x_1 + 3x_2 &= 1\\ -4x_1 + 5x_2 &= 0 \end{aligned}$$