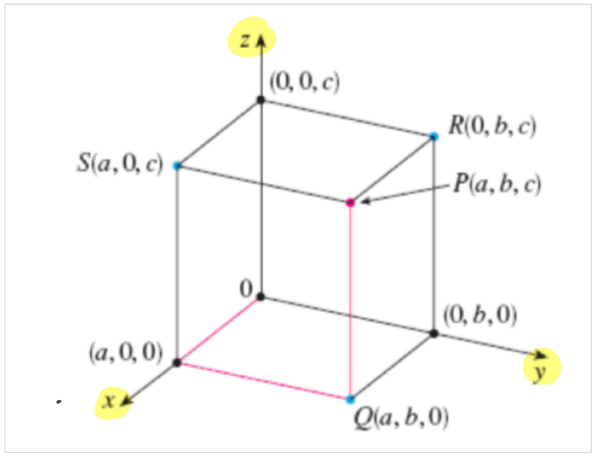
Section 12.1
(Three Dimensional coordinate system)
Basic definitions in 3-dimensional rectangular coordinate system

• The Cartesian product

The Cartesian product $\mathbb{R}\times\mathbb{R}\times\mathbb{R}=\{(x,y,z):x,y,z\in\mathbb{R}\}$

is the set of all ordered triples of real numbers and is denoted by $\mathbb{R}^3$.



• Coordinate Axes

x-axis , y-axis & z-axis

• Coordinate Planes

xy-plane , xz-plane , yz plane

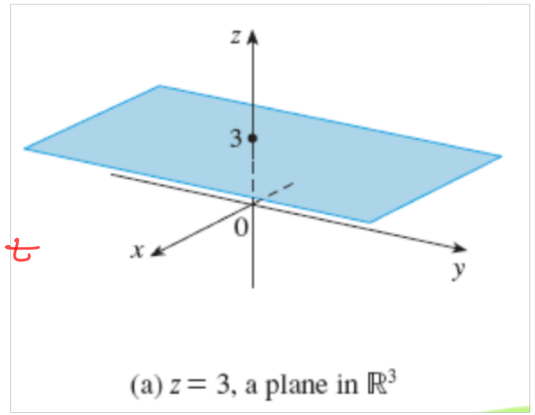
• Octants

8-parts called octants.

EXAMPLE 1 What surfaces in $\mathbb{R}^3$ are represented by the following equations?

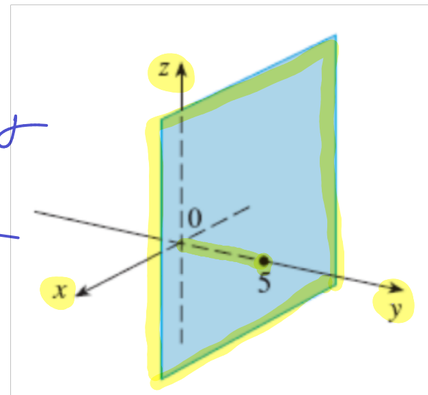
(a) $z = 3$ (b) $y = 5$

(a) This is plane that is parallel to x-y plane and three units above it



(a) $z = 3$, a plane in $\mathbb{R}^3$

(b) This is plane that is parallel to x-z plane and Five units to the right of it



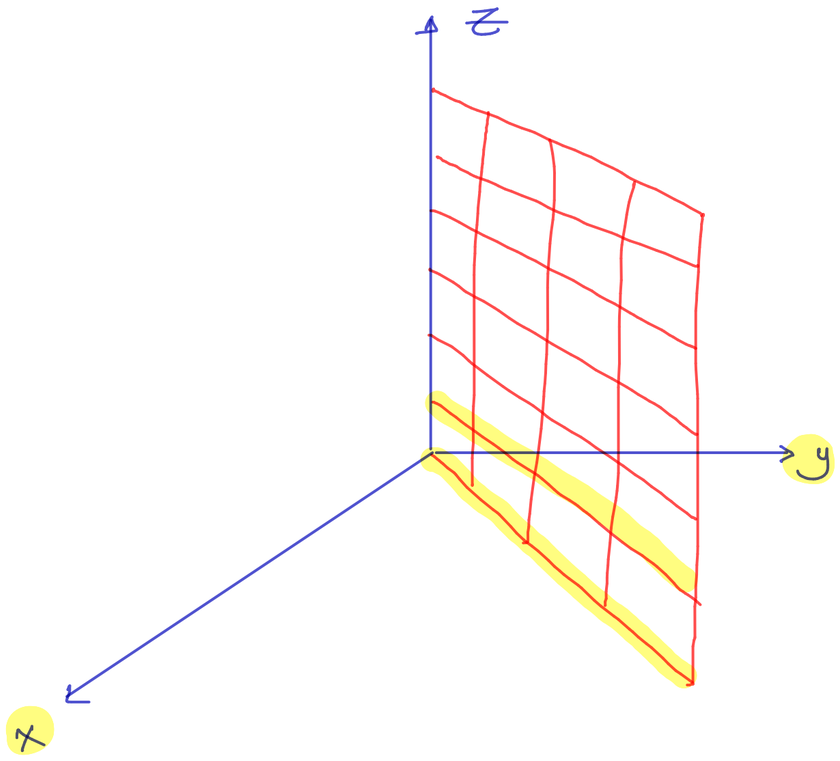
(b) $y = 5$, a plane in $\mathbb{R}^3$

Note:

In general, if k is a constant, then $x = k$ represents a plane parallel to the yz-plane, $y = k$ is a plane parallel to the xz-plane, and $z = k$ is a plane parallel to the xy-plane.

Exercise

Describe and sketch the surface in $\mathbb{R}^3$ represented by the equation $y = x$.



This is the vertical plane that intersects

xy plane in the line $y = x \quad z = 0$

Basic formulas in 3-dimensional coordinate system

Distance Formula

The distance between points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Example Find the distance between $P_1(2, -1, 7)$ and $P_2(1, -3, 5)$.

$$d = \sqrt{(1-2)^2 + (-3+1)^2 + (5-7)^2} = \sqrt{1+4+4}$$

$$= 3$$

Midpoint Formula

The coordinates of midpoint of $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ are

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$$

Example Find the midpoint of $P_1(2, -1, 7)$ and $P_2(1, -3, 5)$.

$$M = \left(\frac{2+1}{2}, \frac{-1-3}{2}, \frac{7+5}{2} \right)$$

$$= \left(3/2, -2, 6 \right)$$

Simple graphs in 3-space

- **Spheres**

Sphere with center (x_0, y_0, z_0) and radius r is given by

$$(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=r^2$$

- center
- radius

Complete Square

Given any equation of the form

$$x^2+y^2+z^2+Gx+Hy+Iz+J=0.$$

We can write it as

$$(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=(K)$$

$K>0$ then sphere

IF $K=0$ then only the Point (x_0, y_0, z_0)

IF $K<0$ then no graph

Example What is the graph of $2x^2+2y^2+2z^2+8x+12y+18=0$?

Divide by (2)

$$x^2+y^2+z^2+4x+6y+9=0$$

$$x^2+4x+y^2+6y+z^2+9=0$$

$$x^2+4x+4-4+y^2+6y+9-9+z^2+9=0$$

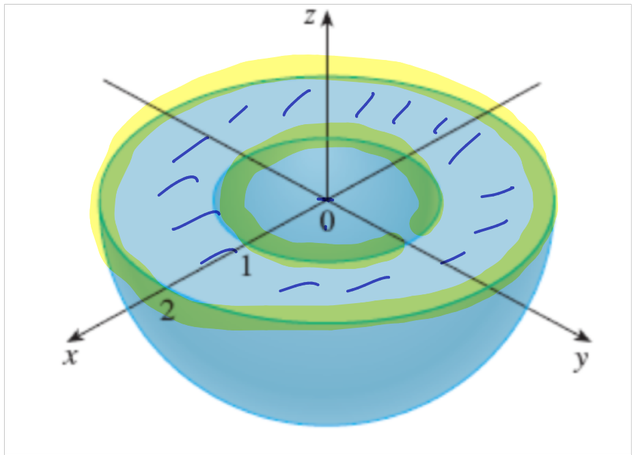
$$(x+2)^2+(y+3)^2+z^2-4-9+9=0$$

$$(x+2)^2+(y+3)^2+z^2=4=2^2$$

$K=4>0$ then sphere with center $(-2, -3, 0)$ and radius $=2$

Example What region is represented by $1 \le x^2 + y^2 + z^2 \le 4,\ z \le 0$?

The set of points between (or on) the sphere $x^2+y^2+z^2=1$ and $x^2+y^2+z^2=4$ and below (or on) the x-y plane



Exercise Find the center and radius of the sphere that has $(1,-2,4)$ and $(3,4,-12)$ as end points of a diameter. Give the equation of the sphere.

Center is $M = \left(\frac{1+3}{2}, \frac{-2+4}{2}, \frac{4-12}{2}\right)$

$= (2, 1, -4)$ ✔

$x_0 \quad y_0 \quad z_0$

Radius is distance between $(1,-2,4)$ and $(2,1,-4)$

$r = \sqrt{(2-1)^2 + (1+2)^2 + (-4-4)^2}$

$= \sqrt{1+9+64} = \sqrt{74}$ ✔

Eq^n of sphere is

$(x-2)^2 + (y-1)^2 + (z+4)^2 = 74$ ✔