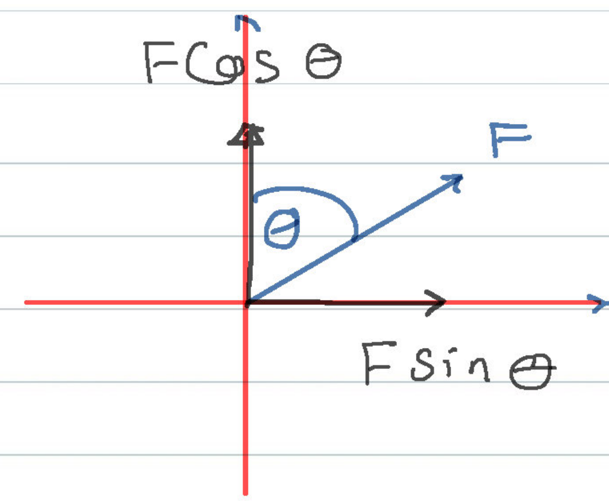
CHAPTER 1

Introduction – Concept of Stress

Rectangular/Cartesian Components Method

$$\vec{F} = (F_x)\,\hat{i} + F_y\,\hat{j}$$

$$F = \sqrt{F_x^2 + F_y^2}$$

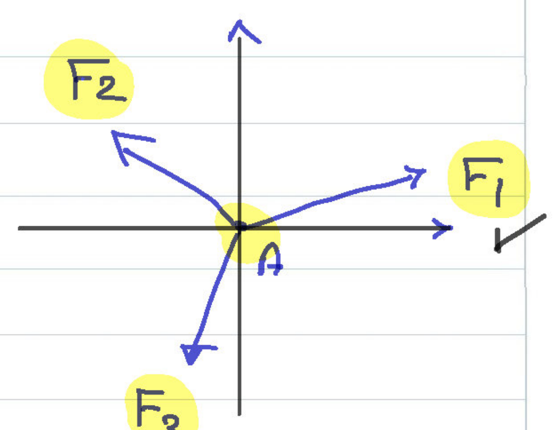


Equilibrium of a Particle

@ rest

1) Resolve
2) $\Sigma F_x = 0$ $\xrightarrow{+}$

$\Sigma F_y = 0$ $\uparrow +$



moment of the force

(vector)

$$M = \Sigma F * d_{\perp}$$

F	d
x	y
y	x

ccw ↺ +

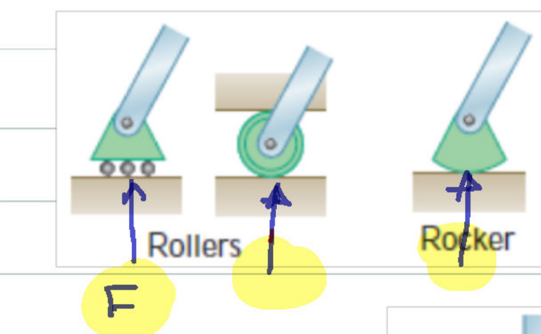
cw ↻ −

Equilibrium of Rigid Bodies

$\Sigma F_x = 0$ $\xrightarrow{+}$ $\Sigma F_y = 0$ $\uparrow +$ $\Sigma M = 0$ ↺+

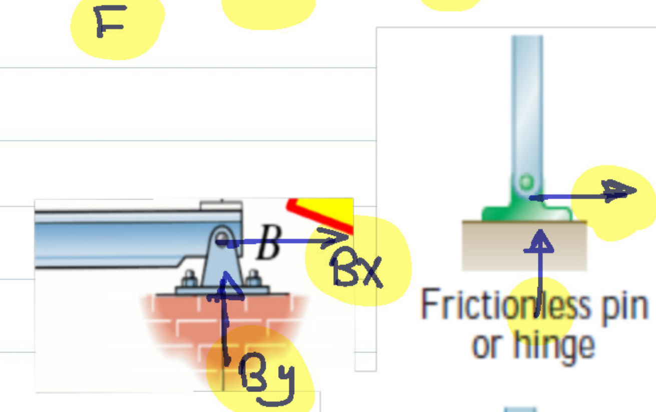
Support Reactions :-

1) Roller



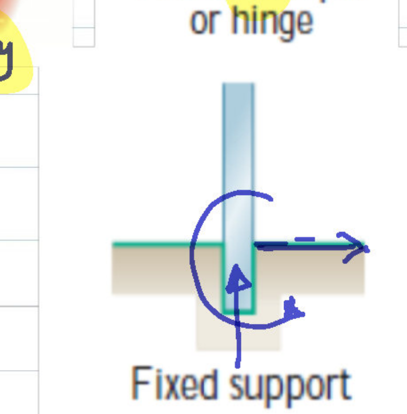
1- Reaction

2) Pin



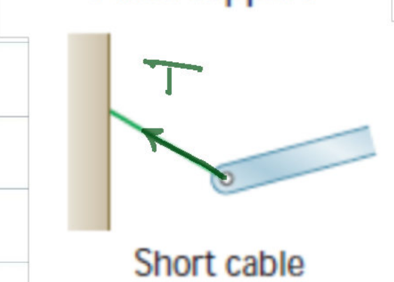
2- Reaction

3) Fixed



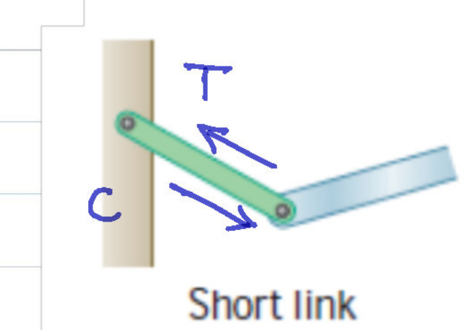
3- Reaction

4) Cable



only tension

5) Link



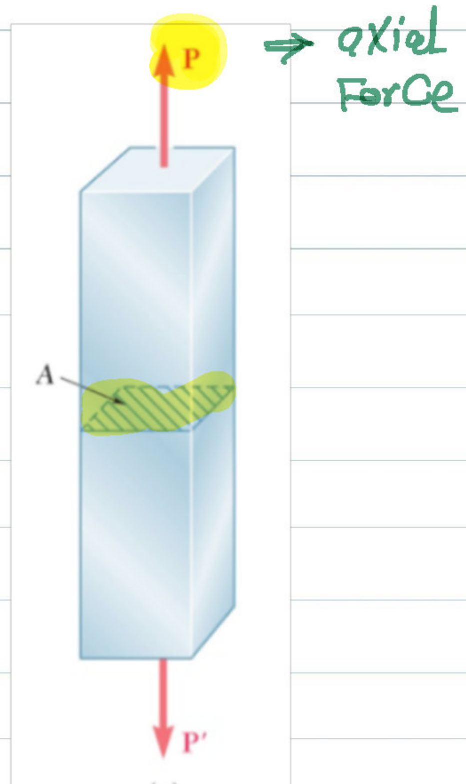
tension or Compression

Concept of Stress

$$\text{Stress} = \frac{\text{Force}}{\text{Area}} \quad \left[\frac{N}{M^2} = Pa \right]$$

Normal stress

$P \perp A$



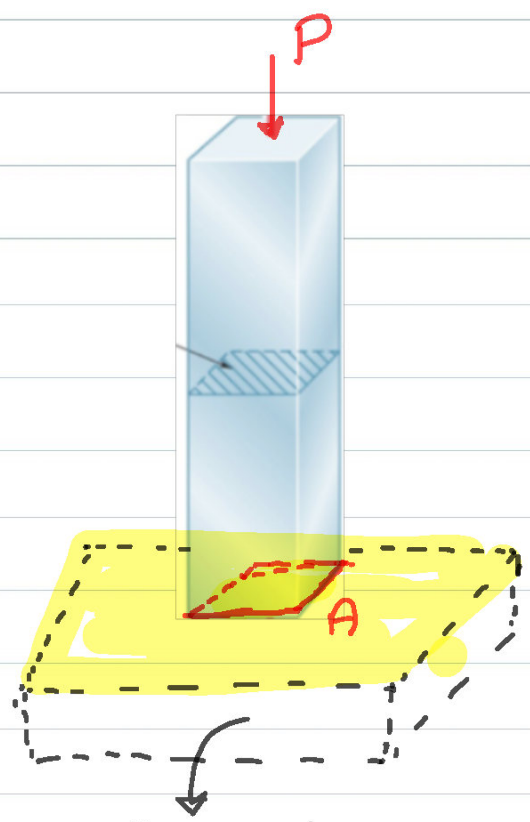
$$\sigma_{av} = \frac{P}{A}$$

⊕ Tension Force

⊖ Compression Force

Bearing stress

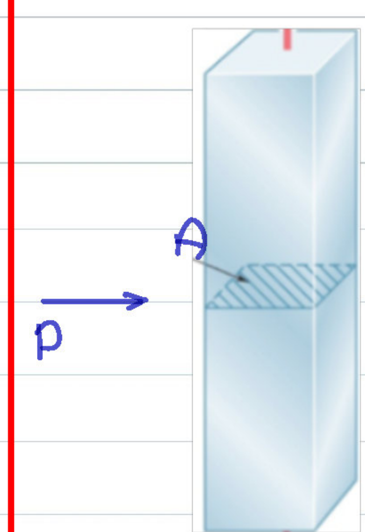
$P \perp A$



$$\sigma_b = \frac{P}{A}$$

Shear stress

$P \parallel A$



$$\tau_{av} = \frac{P}{A}$$

Single Shear

double Shear

Shearing Stress Examples

Single Shear

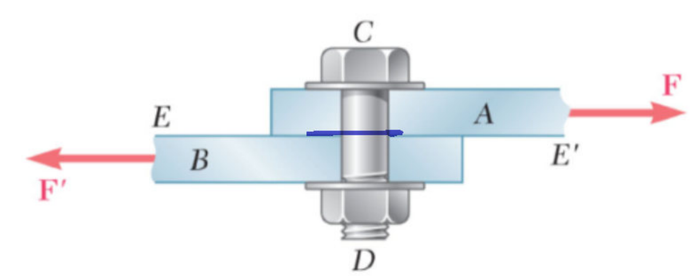
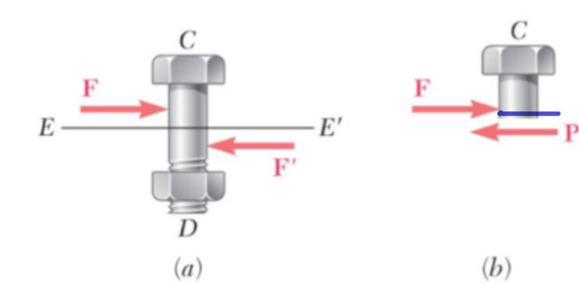


Fig. 1.16 Bolt subject to single shear.



2- Members

1- Cut

$$\tau_{av} = \frac{F}{A_{bolt}}$$

Double Shear

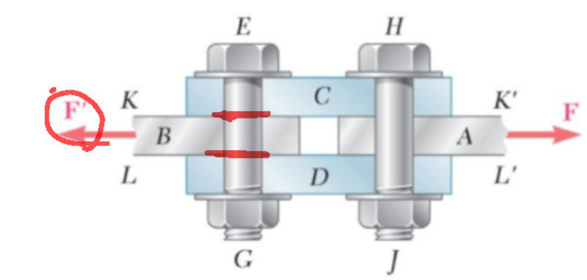
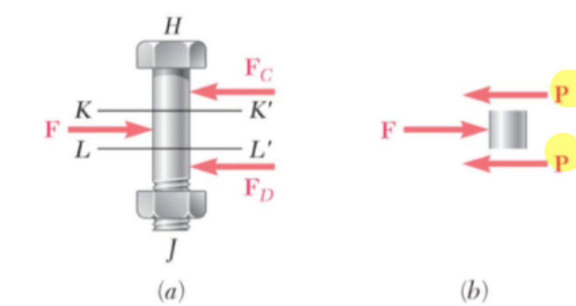


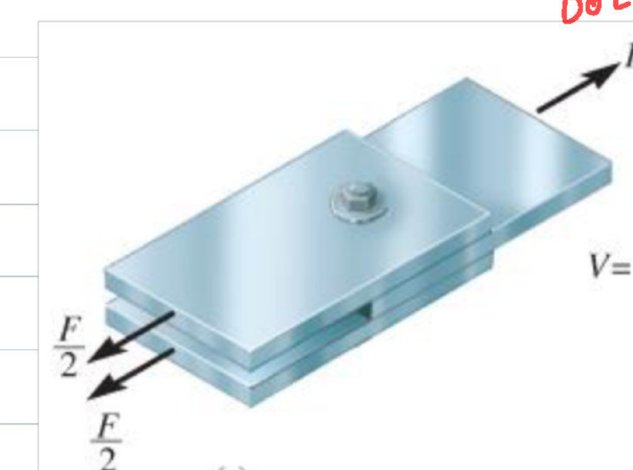
Fig. 1.18 Bolt subject to double shear.



3- Members

2- cuts

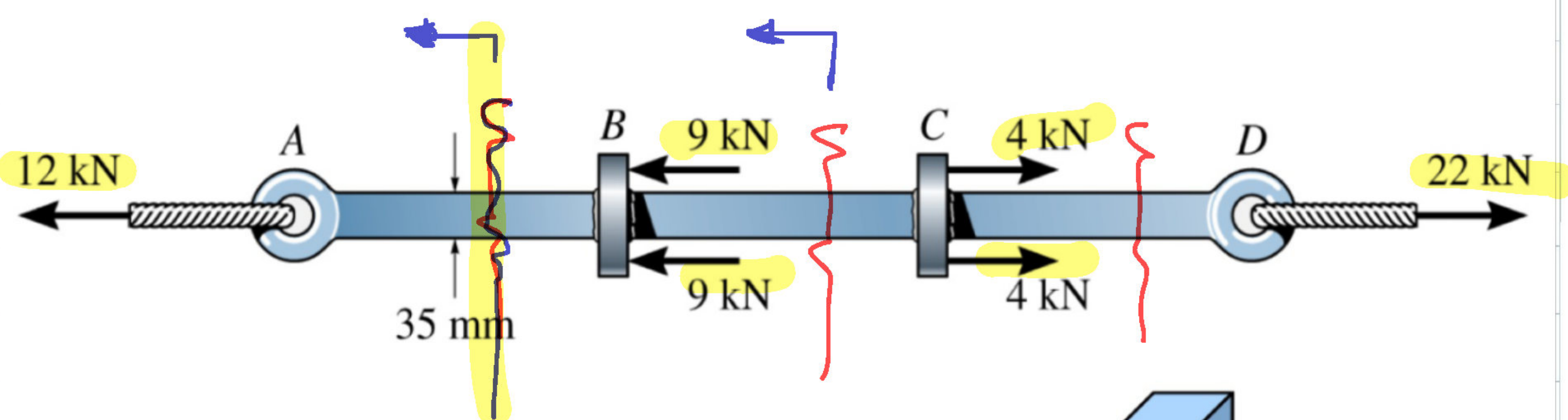
$$\tau_{av} = \frac{F}{2\,A_{bolt}}$$



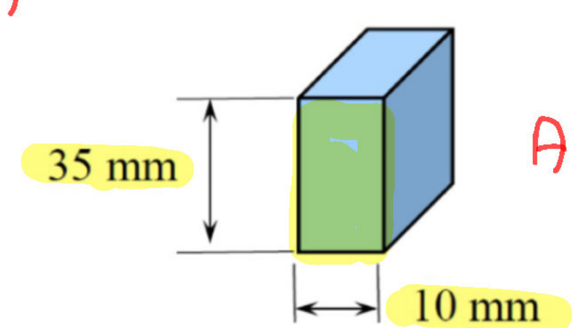
EXAMPLE 1.6

Bar width = 35 mm, thickness = 10 mm

Determine max. average normal stress in bar when subjected to loading shown.



SOL

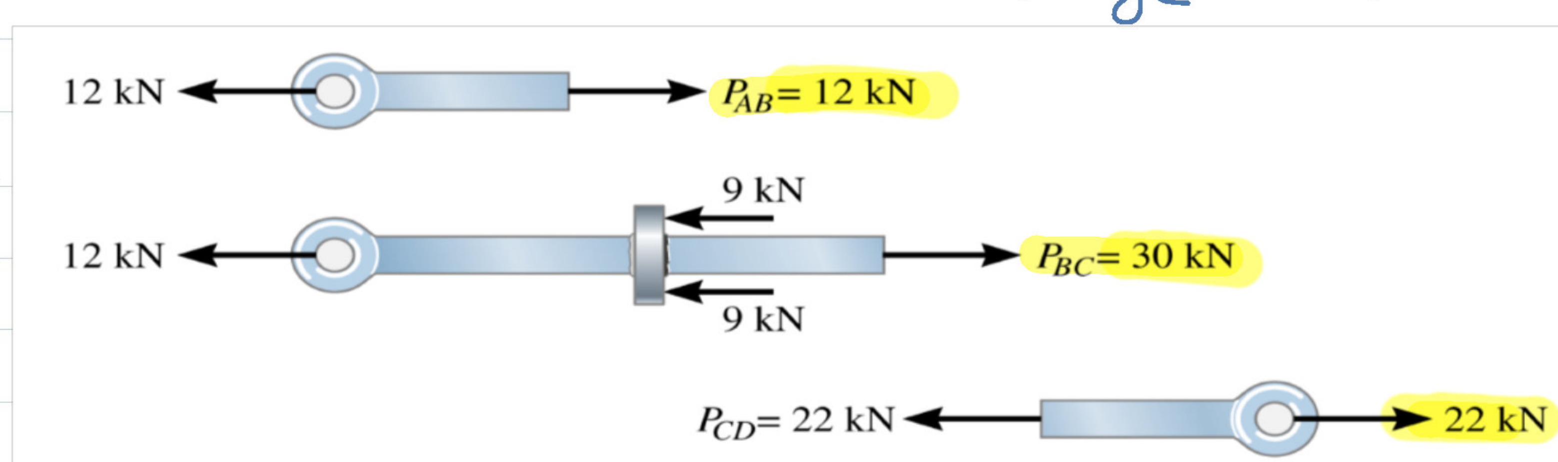


$A = b\,h$

Area is Constant

$$\sigma_{Max} = \frac{P_{Max}}{A}$$

Section → change Force
Section → change area



$P_{AB} = 12$ kN (tension)

$P_{BC} = 12 + 9 + 9 = 30$ kN (tension) ✓

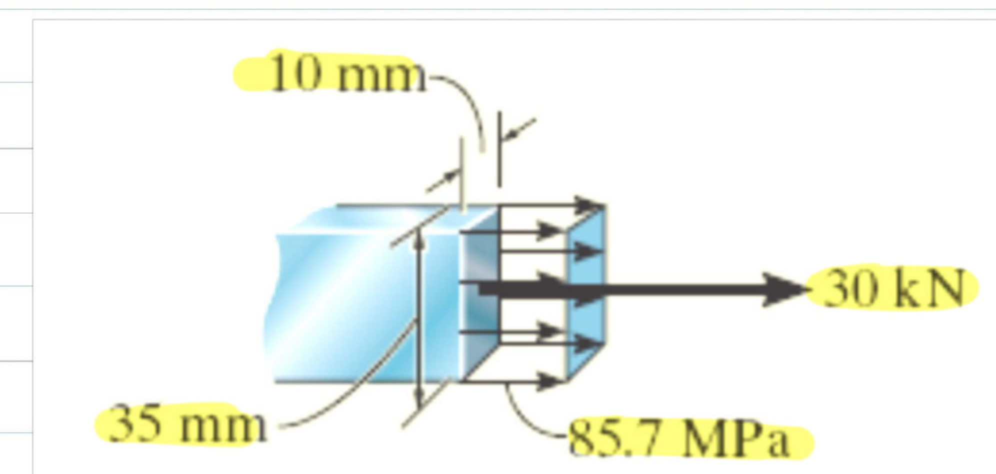
$P_{CD} = 22$ kN (tension)

$$\sigma_{Max} = \frac{P_{Max}}{A} = \frac{P_{BC}}{A}$$

$$= \frac{30 * 10^3}{0.035 * 0.01}$$

$$= 85.7 \text{ MPa}$$

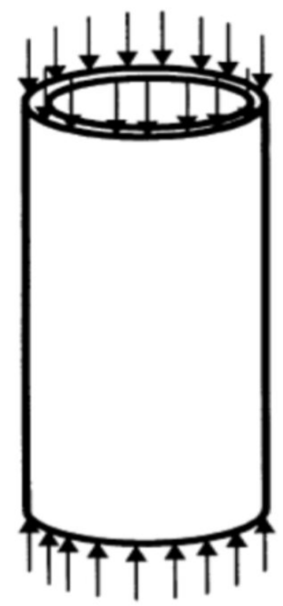
Tensile stress



$$Area = \pi R^2 = \pi D^2/4$$

Challenging Problem P1.1

A stainless steel tube with an outside diameter of 60 mm and a wall thickness of 5 mm is used as a compression member. If the normal stress in the member must be limited to 200 MPa (σ_{max}), determine the maximum load P that the member can support.



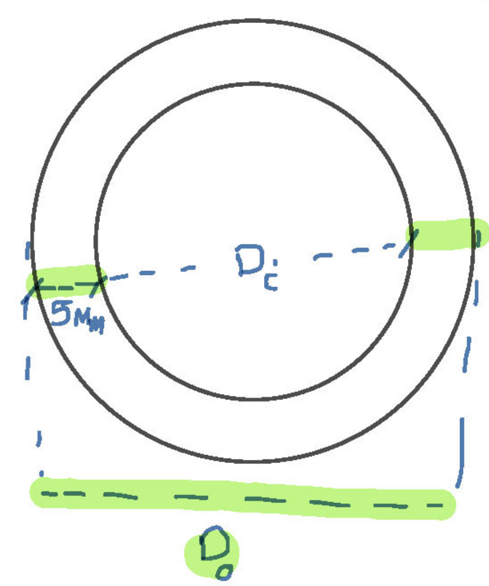
CORRECT ANSWER

$$P_{max} = 172.8\ kN$$

37

$D_o = 60$ MM

$D_i = 60 - 2(5) = 50$ MM



$$A = \frac{\pi}{4}\left(D_o^2 - D_i^2\right)$$

$$= \frac{\pi}{4}\left(\left(\frac{60}{1000}\right)^2 - \left(\frac{50}{1000}\right)^2\right)$$

$$= 8.64 * 10^{-4} \quad M^2$$

$$\sigma_{max} = \frac{P_{max}}{A} \rightarrow P_{max} = \sigma_{max} A$$

$$P_{max} = 200 * 10^6 * 8.64 * 10^{-4}$$

$$= 172.8 \quad kN$$

Challenging Problem P1.2

A 2024-T4 aluminum tube with an outside diameter of 90 mm (D_o) will be used to support a 285 kN load (P). If the normal stress in the member must be limited to 165 MPa (σ_{max}), determine the wall thickness required for the tube.

CORRECT ANSWER

$$t_{min} = 6.59\ mm$$

$$\sigma_{max} = \frac{P}{A} \Rightarrow A = \frac{P}{\sigma_{max}} = \frac{285 * 10^3}{165 * 10^6}$$

$$A = 1.73 * 10^{-3} \quad M^2$$

$$A = \pi\left(R_o^2 - R_i^2\right)$$

$$1.73 * 10^{-3} = \pi\left[\left(\frac{45}{1000}\right)^2 - R_i^2\right]$$

$$R_i = 38.4 \quad MM$$

$$t = R_o - R_i = 45 - 38.4$$

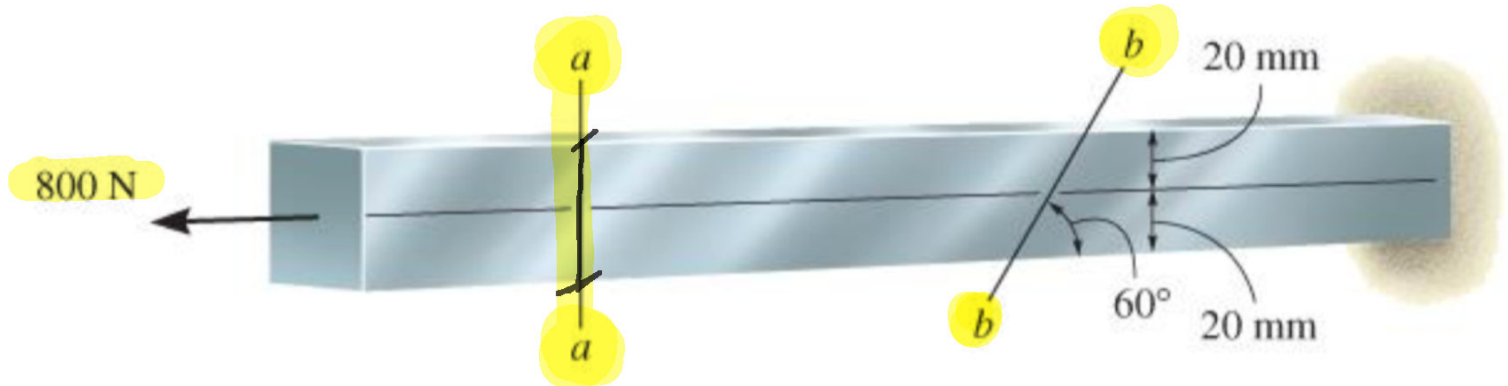
$$= 6.59 \quad MM$$

EXAMPLE 1.10

Determine:

- average **normal** stress
- average **shear** stress

acting along section planes ***a-a***, and section plane ***b-b***.



Depth and thickness = 40 mm × 40 mm

Cut a – a :–

$P = 800\ N$ [axial Force]

$$\sigma_{av} = \frac{P}{A} = \frac{800}{0.04 * 0.04} = 500\ kPa$$

tensile Stress (+)

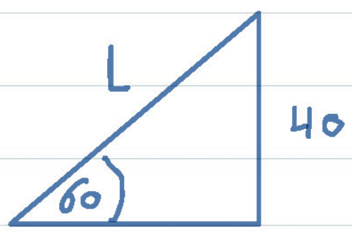
No shear Force @ Section a – a

$$\tau_{avg} = 0$$

Cut b – b :–

$$N = 800\ \cos 30 = 692.8\ N$$

$$V = 800\ \sin 30 = 400\ N$$



$$\sin 60 = \frac{40}{L}$$

$$L = \frac{40}{\sin 60}$$

$$\sigma_{av} = \frac{N}{A} = \frac{692.8}{0.04 * 0.04/\sin 60}$$

$$= 357\ kPa$$

$$\tau_{av} = \frac{V}{A} = \frac{400}{0.04 * 0.04/\sin 60}$$

$$= 217\ kPa$$

Factor of Safety

$$F.S = \frac{\sigma_{ult}}{\sigma_{all}} \qquad \sigma_{ult} \Rightarrow Fail$$

$\sigma_{all} \Rightarrow$ allowable stress $\Rightarrow$ Used for design

$$A = \frac{P}{\sigma_{all}}$$

$$F.S = \frac{F_{ult}}{F_{all}} \Rightarrow F_{Fail}$$

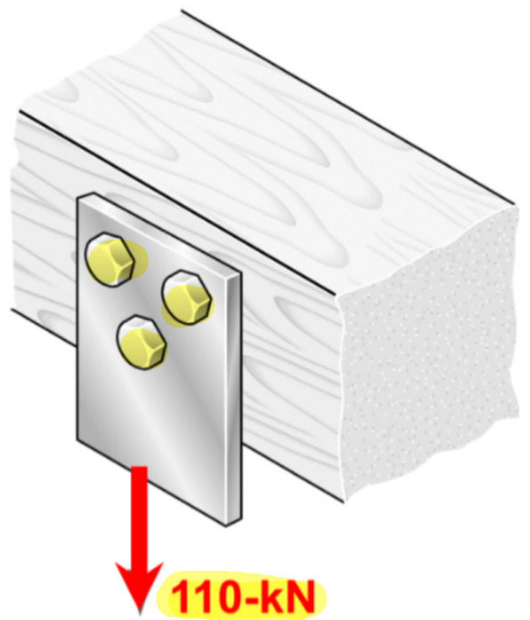
$$F.S = \frac{\tau_{ult}}{\tau_{all}} \Rightarrow \tau_{Fail}$$

$$F.S > 1$$

$$\tau_{ult} = \frac{P_{ult}}{A}$$

PROBLEM 1.45

Three 18-mm-diameter steel bolts are to be used to attach the steel plate shown to a wooden beam. Knowing that the plate will support a 110-kN load and that the ultimate shearing stress for the steel used is 360 MPa, determine the factor of safety for this design.



CORRECT ANSWER **FS = 2.50**

$P = 110\ kN$ $\quad \tau_{ult} = 360\ MPa$

For one bolt

For each bolt

$$A = \frac{\pi d^2}{4} = \frac{\pi}{4}\left(\frac{18}{1000}\right)^2 = 254.47 * 10^{-6}\ m^2$$

$$P_U = A\,\tau_{ult} = 254.47 * 10^{-6} * 360 * 10^6 = 91.61 * 10^3\ N$$

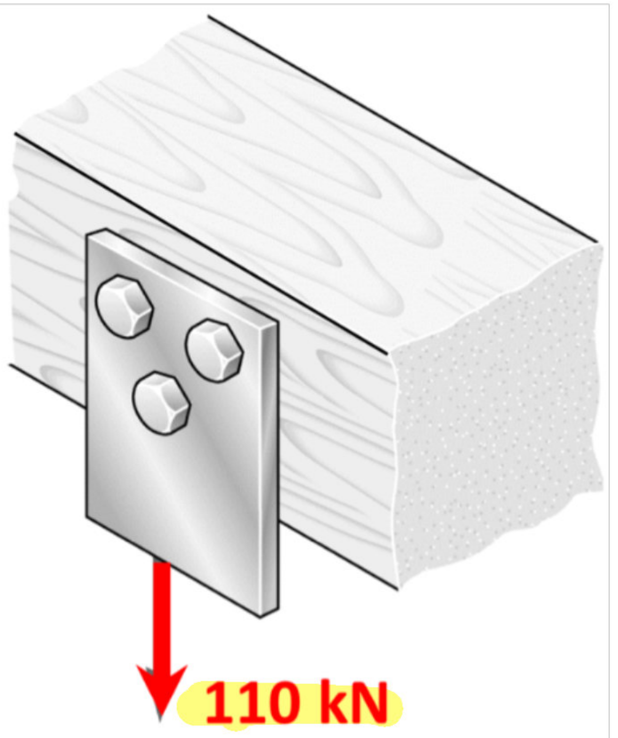
For the three bolts :—

$$P_u = 3 * 91.61 * 10^3 = 274.83 * 10^3 N$$

$$F \cdot S = \frac{P_{ult}}{P} = \frac{274.83 * 10^3}{110 * 10^3} = 2.5$$

PROBLEM 1.46

Three steel bolts are to be used to attach the steel plate shown to a wooden beam. Knowing that the plate will support a 110-kN load, that the ultimate shearing stress for the steel used is 360 MPa, and that a factor of safety of 3.35 is desired, determine the required diameter of the bolts.



CORRECT ANSWER **d = 20.8 mm**

$F \cdot S = 3.35$ $\quad \tau_u = 360\ MPa$

For one bolt

For each bolt :—

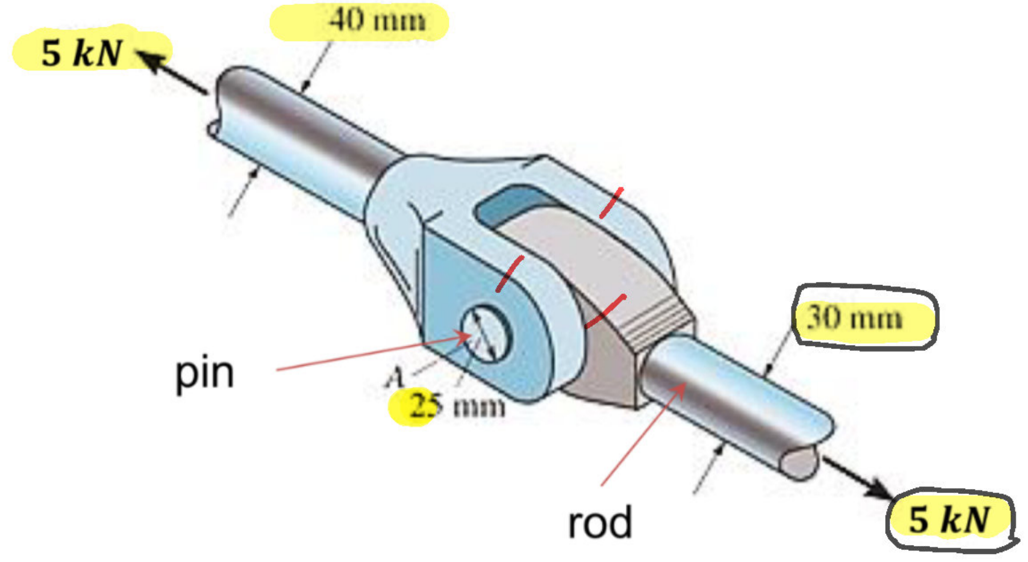
$$P = \frac{110}{3} = 36.667\ kN$$

$$F \cdot S = \frac{P_u}{P} \Rightarrow P_u = F \cdot S * P$$

$$= 3.35 * 36.667 = 122.83\ kN$$

$$\tau_u = \frac{P_{ult}}{A} = \frac{4 P_{ult}}{\pi d^2}$$

$$d = \sqrt{\frac{4\ P_{ult}}{\pi\ \tau_{ult}}} = \sqrt{\frac{4 * 122.83 * 10^3}{\pi * (360 * 10^6)}} = 20.8\ mm$$



Stress on the rod ?
(type & Value)

Stress on the pin ?
(type & Value)

CORRECT ANSWER

Rod	$\sigma = 7.07\ MPa$	Pin	$\sigma = 0$
	$\tau = 0$		$\tau = 5.09\ MPa$

Rod 30 MM :—

$$\sigma_{30\,mm} = P/A = \frac{5 * 10^3}{\pi/4\,(30/1000)^2} = 7.1\ MPa$$

} tensile stress

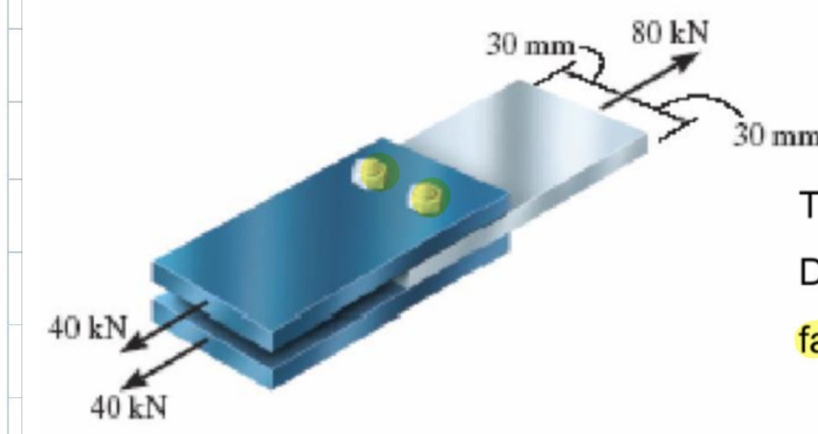
Rod 40 MM :—

$$\sigma_{40\,mm} = \frac{5 * 10^3}{\pi/4\,(40/1000)^2} = 3.98\ MPa$$

} tensile stress

Double Shear @ Pin :—

$$\tau = \frac{P}{2A_{pin}} = \frac{5 * 10^3}{2\left(\pi/4 * (25/1000)^2\right)} = 5.1\ MPa$$



The joint is fastened together using two bolts. Determine the required diameter of the bolts if the failure shear stress for the bolts is $\tau_{Fail} = 350\ MPa$.

Use a factor of safety for shear of F.S. = 2.5. $\Rightarrow F.S = \tau_{ult}/\tau_{av}$

CORRECT ANSWER

d = 13.49 mm

* Shear is double

$$\tau_{av} = \frac{P}{2(2A)} = P/4A$$

$$\tau_{ult}/F.S = P/4A$$

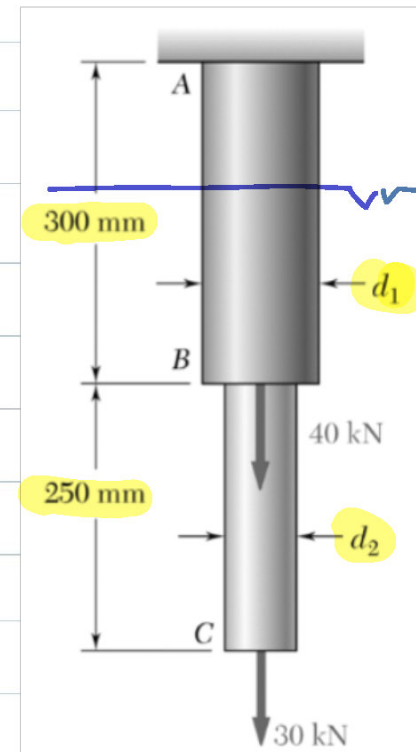
$$\frac{350 * 10^6}{2.5} = \frac{80 * 10^3}{\cancel{4} * \pi d^2/\cancel{4}}$$

$$\Downarrow$$

$$d = 13.5\ mm$$

PROBLEM 1.2

Two solid cylindrical rods AB and BC are welded together at B and loaded as shown. Knowing that $d_1 = 50$ mm and $d_2 = 30$ mm, find the average normal stress at the midsection of (*a*) rod AB, (*b*) rod BC.



Rod AB :-

$$P_{AB} = 30 + 40 = 70 \text{ kN}$$

$$A_{AB} = \pi d_1^2/4$$

$$= \pi/4 \, (50/1000)^2$$

$$= 1.9635 \times 10^{-3} \text{ m}^2$$

$$\sigma_{AB} = P_{AB}/A = \frac{70 \times 10^3}{1.9635 \times 10^{-3}} = 35.7 \times 10^6 \text{ Pa}$$

$$= 35.7 \text{ MPa}$$

Rod BC :-

$$P_{BC} = 30 \text{ kN}$$

$$A_{BC} = \pi d_2^2/4$$

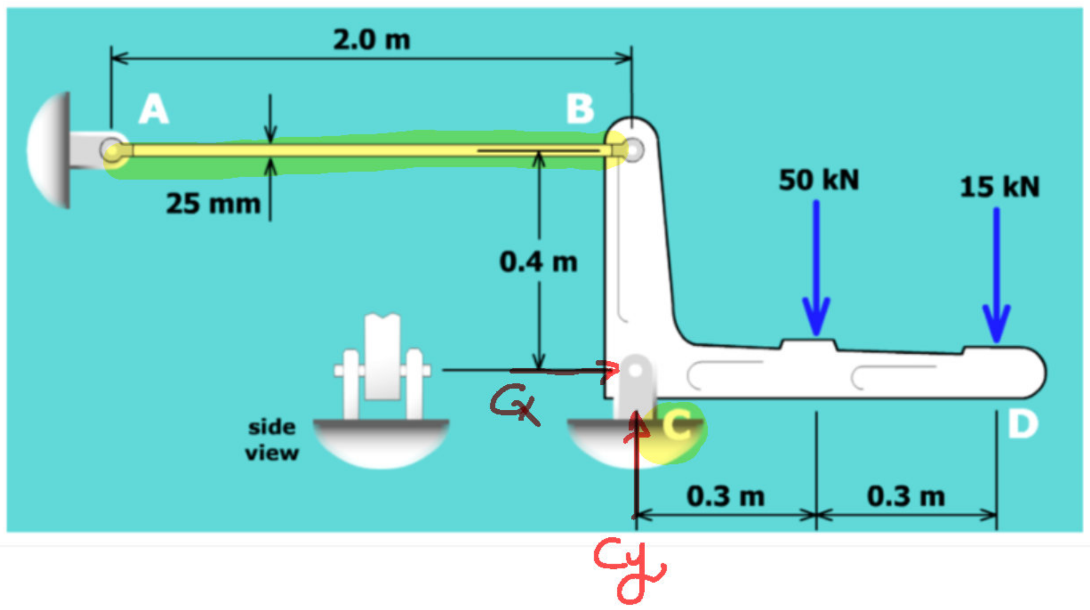
$$A_{BC} = \pi/4 \, (30/1000)^2 = 706.86 \times 10^{-6} \text{ m}^2$$

$$\sigma_{BC} = P/A$$

$$= \frac{30 \times 10^3}{706.86 \times 10^{-6}} = 42.4 \times 10^6 \text{ Pa}$$

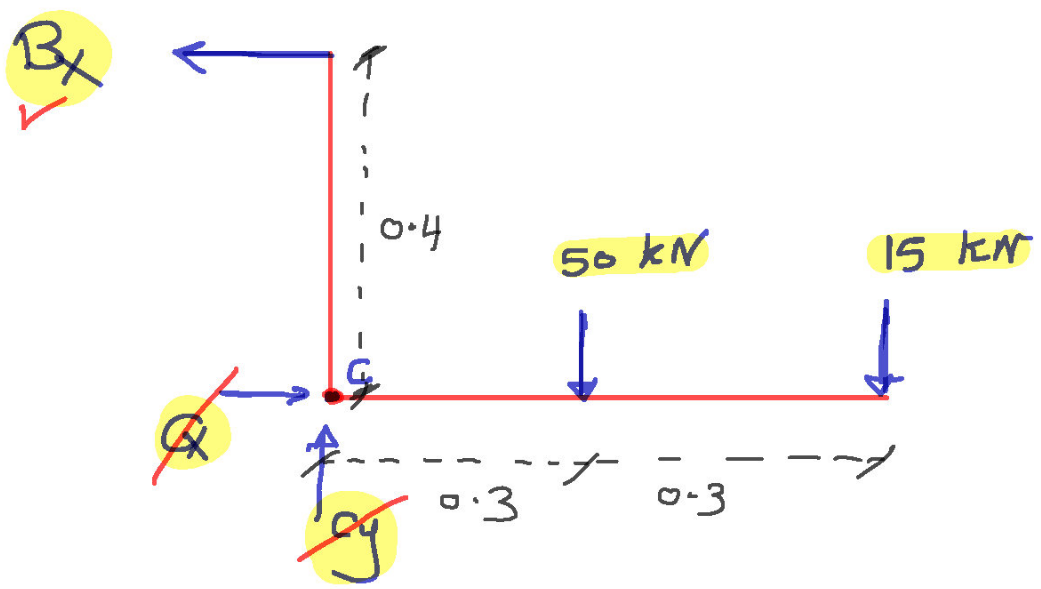
$$= 42.4 \text{ MPa}$$

Qatar University Department of Mechanical & Industrial Engineering	MECH 223 Solid Mechanics	TUTORIAL # 1



A pin C and a round aluminum rod at B support the rigid bar BCD. If the allowable shear stress for the pin is **50 MPa**, what in the minimum diameter required for the pin at C ?

$\tau_{all} = 50$ MPa



$\Sigma M_C = 0 \quad ↺+$

$B_x * 0.4 - 50 * 0.3 - 15 * 0.6 = 0$

$B_x = 60$ kN

$\Sigma F_x = 0 \quad \xrightarrow{+}$

$C_x - 60 = 0 \Rightarrow C_x = 60$ kN

$\Sigma F_y = 0 \quad \uparrow +$

$C_y - 50 - 15 = 0 \Rightarrow C_y = 65$ kN

$F_c = \sqrt{C_x^2 + C_y^2}$

$= \sqrt{(60)^2 + (65)^2} = 88.5$ kN

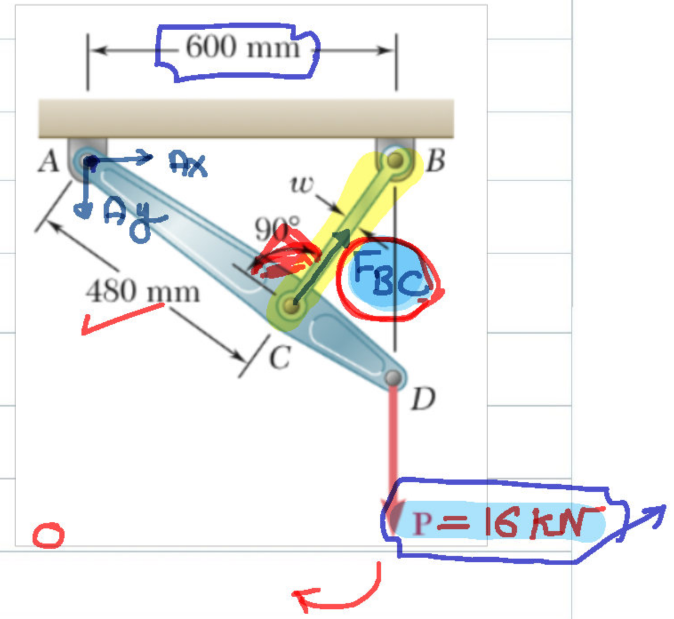
$\Rightarrow$ Double shear @ Pin (c) :—

$\tau_{all} = \frac{F_c}{2A} = \frac{F_c * \cancel{4}^{2}}{\cancel{2} \pi D^2}$

$D = \sqrt{\frac{2 F_c}{\pi \tau_{all}}} = \sqrt{\frac{2 * 88.5 * 10^3}{\pi * 50 * 10^6}}$

$= 33.6$ MM

1.37 Link BC is 6 mm thick, has a width w = 25 mm, and is made of a steel with a 480-MPa ultimate strength in tension. What is the safety factor used if the structure shown was designed to support a 16-kN load **P**?



* $\sigma_u = 480 \text{ MPa}$

$\Sigma M_A = 0$ ↻+

$F_{BC} * 480 - 16 * 600 = 0$

$F_{BC} = 20 \text{ kN}$

* $\sigma_u = \dfrac{F_{ult}}{A}$

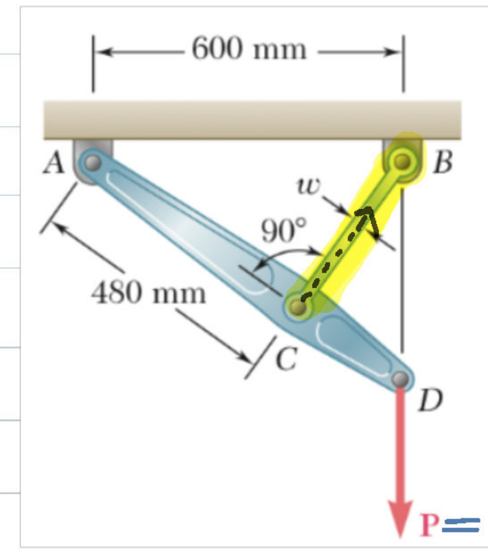
$F_{ult} = \sigma_u * A = 480 * 10^6 * 0.006 * 0.025$

$= 72 * 10^3 \text{ N}$

* $F.S = \dfrac{F_{ult}}{F_{BC}} = \dfrac{72 * 10^3}{20 * 10^3}$

$= 3.6$

1.38 Link *BC* is 6 mm thick and is made of a steel with a 450-MPa ultimate strength in tension. What should be its width *w* if the structure shown is being designed to support a 20-kN load **P** with a factor of safety of 3?

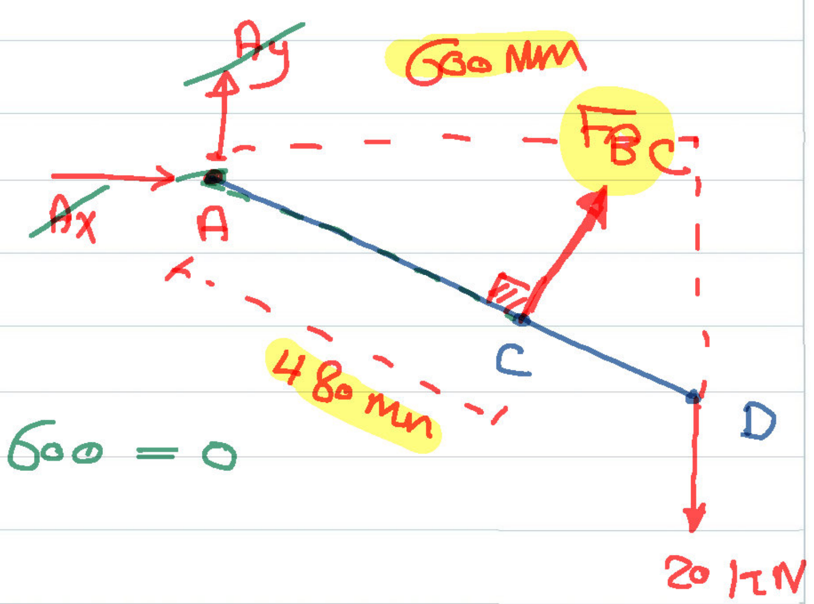


$\sigma_{ult} = 450$ MPa

$F.S = 3$

$$F.S = \frac{\sigma_{ult}}{\sigma_{all}}$$

$$\sigma_{all} = \frac{\sigma_{ult}}{F.S} = \frac{450}{3} = 150 \text{ MPa}$$



$\Sigma M_A = 0 \quad \circlearrowleft +$

$+ F_{BC} * 480 - 20 * 600 = 0$

$F_{BC} = 25$ kN

$$\text{Stress} = \frac{\text{Force}}{\text{Area}}$$

$$\sigma_{all} = \frac{F_{BC}}{A} \Rightarrow A = \frac{F_{BC}}{\sigma_{all}}$$

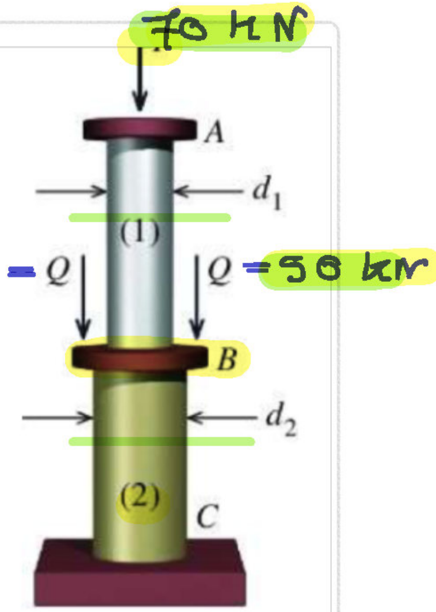
$$A = \frac{25 * 10^3}{150 * 10^6} = 1.67 * 10^{-4} \text{ M}^2$$

$A = w\,t$

$$w = \frac{A}{t} = \frac{1.67 * 10^{-4}}{0.006} = 0.0278 \text{ M}$$

$w = 27.8$ MM

P1.3 Two solid cylindrical rods (1) and (2) are joined together at flange *B* and loaded with loads of *P* = 70 kN and *Q* = 50 kN as shown in Figure. If the normal stress in each rod must be limited to 210 MPa, determine the minimum diameter required for each rod.



Solution

$$\sigma = \frac{P}{A} = \frac{4P}{\pi d^2}$$

$$d = \sqrt{\frac{4P}{\pi \sigma}}$$

Rod 1

$$P_{AB} = -70 \text{ kN}$$

$$\sigma_{AB} = -210 \text{ MPa}$$

$$d = \sqrt{\frac{4 * -70 * 10^3}{\pi * -210 * 10^6}}$$

$$= 20.6 \text{ mm}$$

Rod 2

$$P_{BC} = -70 - 50 - 50 = -170 \text{ kN}$$

$$\sigma_{BC} = -210 \text{ MPa}$$

$$d = \sqrt{\frac{4 * -170 * 10^3}{\pi * -210 * 10^6}}$$

$$= 31.1 \text{ mm}$$