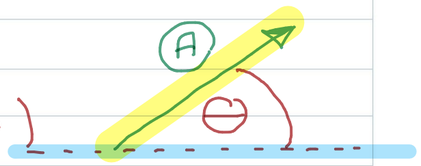
Statics of Particles 2

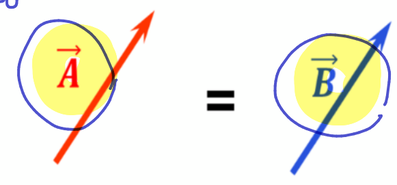
Vectors } Mag (A)
direction (θ)



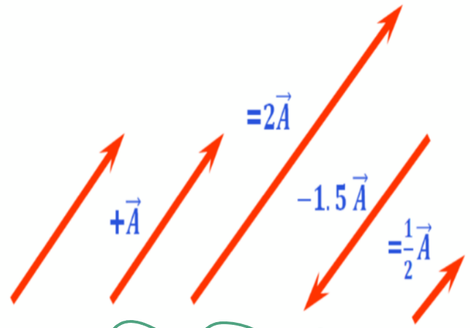
Ex Force , Velocity

* Scalar } only quantity
No - direction
Ex Mass , volume

- Vectors are **equal** when they have the same magnitude and same direction



- Vectors can be simply **added** or **subtracted**, if they have the same direction



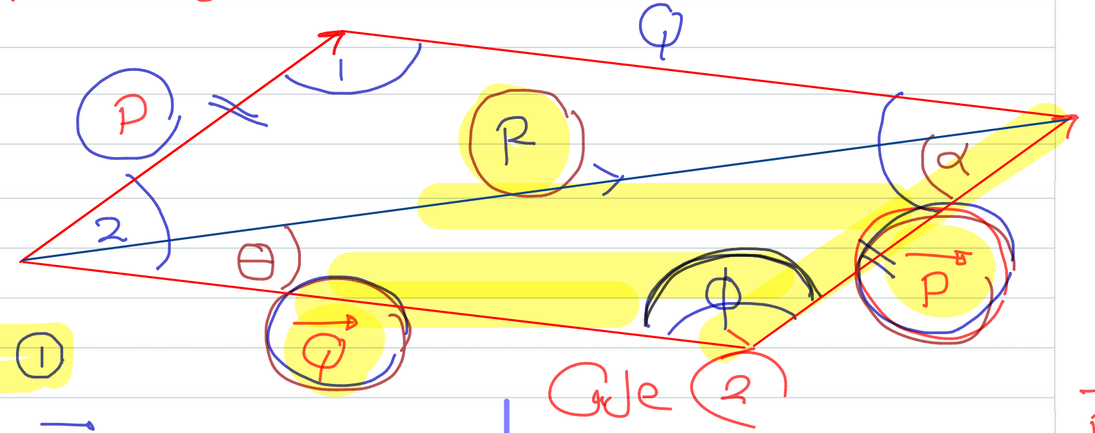
A=1 → B=4 → $\vec{A}+\vec{B}=5$

Parallelogram Law
Trigonometric method

} only 2- Forces

$\vec{R} = \vec{P} + \vec{Q}$

$\hat{1} + \hat{2} = 180^\circ$



Case ① Case ②

Given $\vec{P}$, $\vec{Q}$ Required $\vec{R}$ (Mag, direction)

Given R Required $\vec{P}$, $\vec{Q}$

① Conclude internal angle between $\vec{P}$ & $\vec{Q}$ (ϕ)

② Mag ⇒ Cos-law

$$R = \sqrt{P^2 + Q^2 - 2PQ\cos\phi}$$

③ direction ⇒ Sin law

$$\frac{R}{\sin\phi} = \frac{P}{\sin\theta} = \frac{Q}{\sin\alpha}$$

① Conclude all internal angles (θ, α, ϕ)

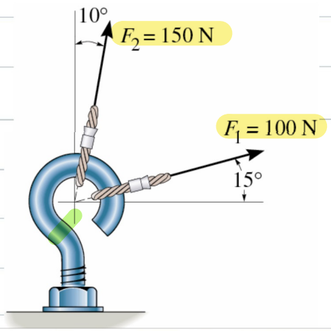
② Sin-law

$$\frac{R}{\sin\phi} = \frac{P}{\sin\theta} = \frac{Q}{\sin\alpha}$$

Example 1:-

The screw eye in the figure at the left is subjected to two forces $\vec{F}_1$ and $\vec{F}_2$.

Determine the magnitude and direction of the resultant force.

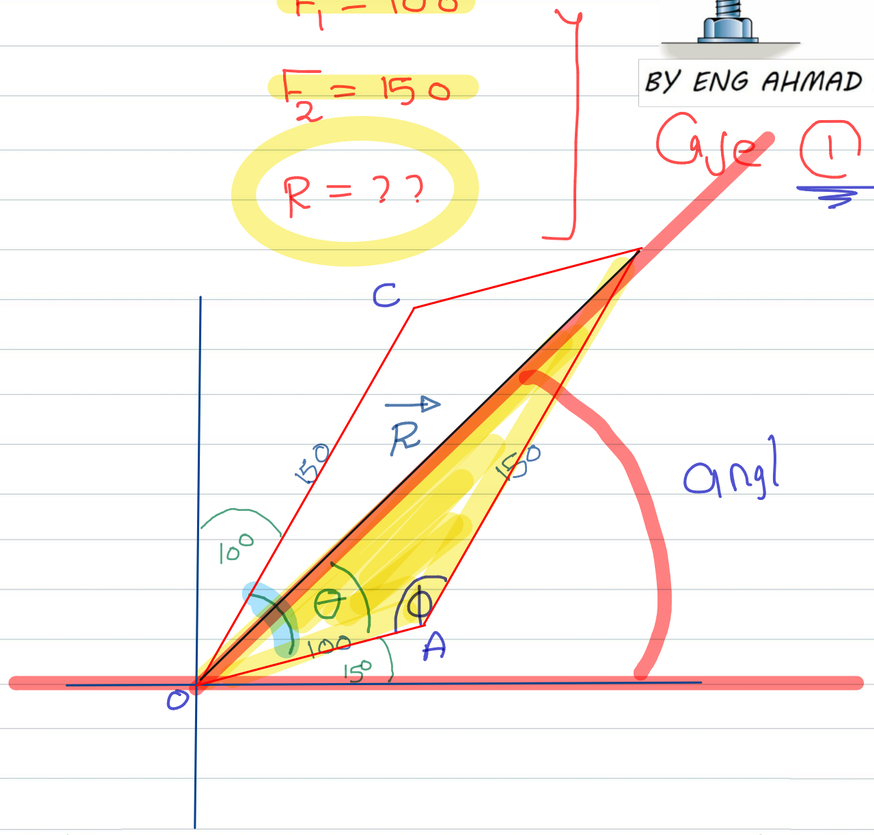


$F_1 = 100$

$F_2 = 150$

$R = ??$

Case (1)



(1) angle $COA = 90 - 15 - 10 = 65^\circ$

$\phi = 180 - 65 = 115^\circ$

(2) $R = \sqrt{100^2 + 150^2 - 2(100)(150)\cos 115}$

$R = 213$ N

(3) $\frac{150}{\sin\theta} = \frac{213}{\sin 115}$

$\theta = \sin^{-1}\left(\frac{150 \sin 115}{213}\right)$

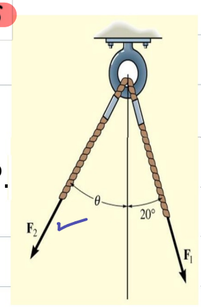
$= 39.7^\circ$

angle $= 15 + 39.7^\circ$

$= 54.7$

Example 2 :-

The ring below is subjected to $\mathbf{F}_1$ and $\mathbf{F}_2$. If we want a resultant force of 1kN and directed vertically downward, determine the magnitude of $\mathbf{F}_1$ and $\mathbf{F}_2$ if $\theta = 30^o$.

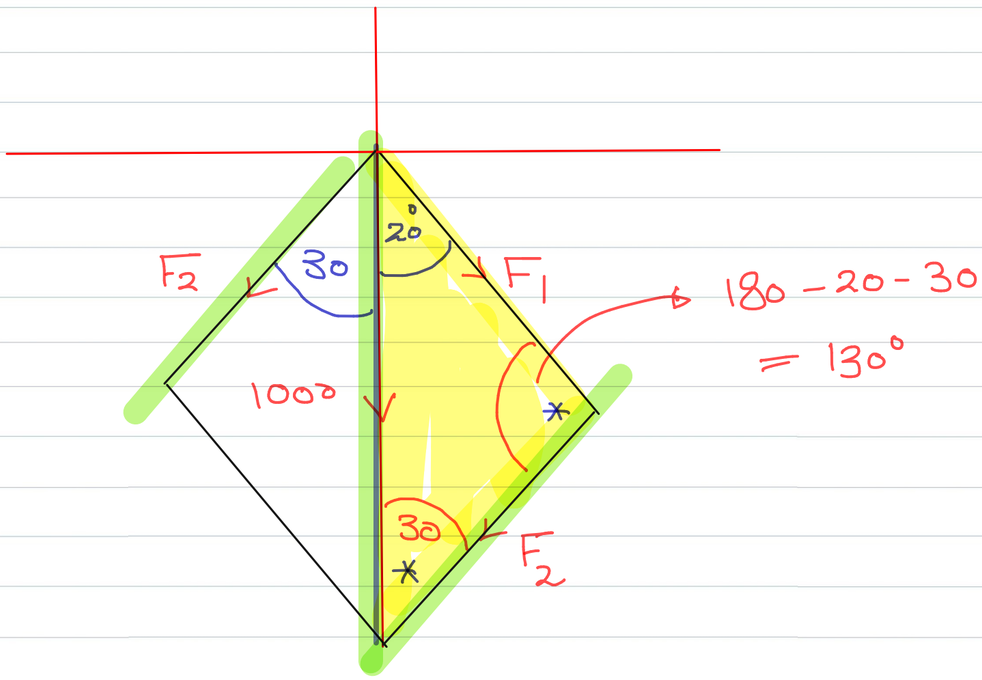


$R = 1000\ N$ ↓

$F_1 = ?$

$F_2 = ?$

Use ②

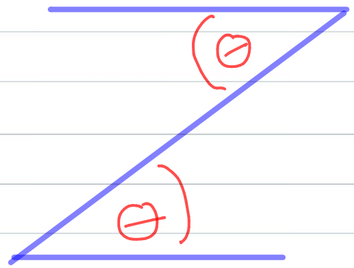


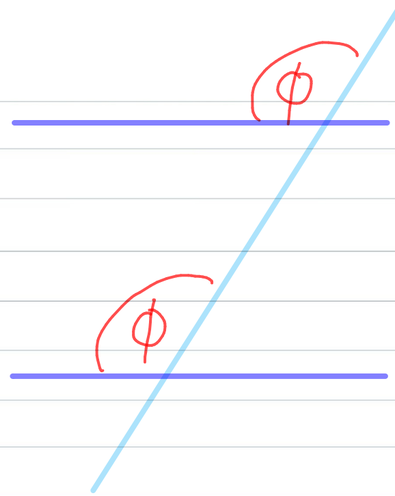
Sin - law

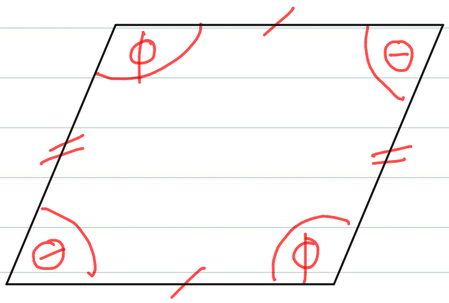
$$\frac{F_1}{\sin 30} = \frac{F_2}{\sin 20} = \frac{1000}{\sin 130}$$

$$F_1 = \frac{1000 \sin 30}{\sin 130} = 653\ N$$

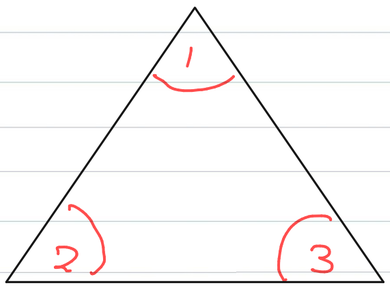
$$F_2 = \frac{1000 \sin 20}{\sin 130} = 446\ N$$







$$\theta + \phi = 180$$

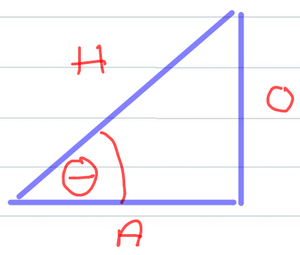


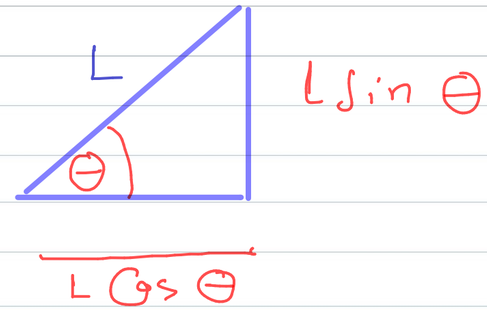
$$\hat{1} + \hat{2} + \hat{3} = 180^{\circ}$$

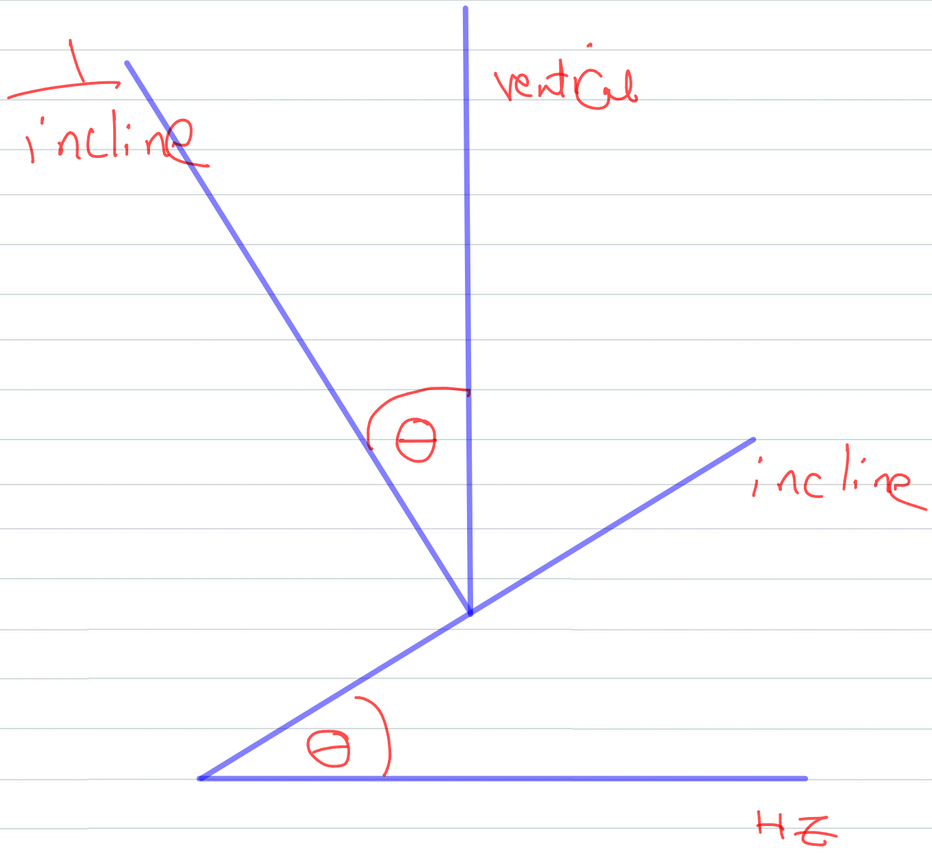
$$\sin\theta = \frac{O}{H}$$

$$\cos\theta = \frac{A}{H}$$

$$\tan\theta = \frac{O}{A}$$

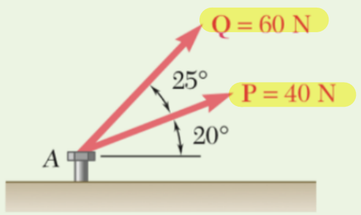


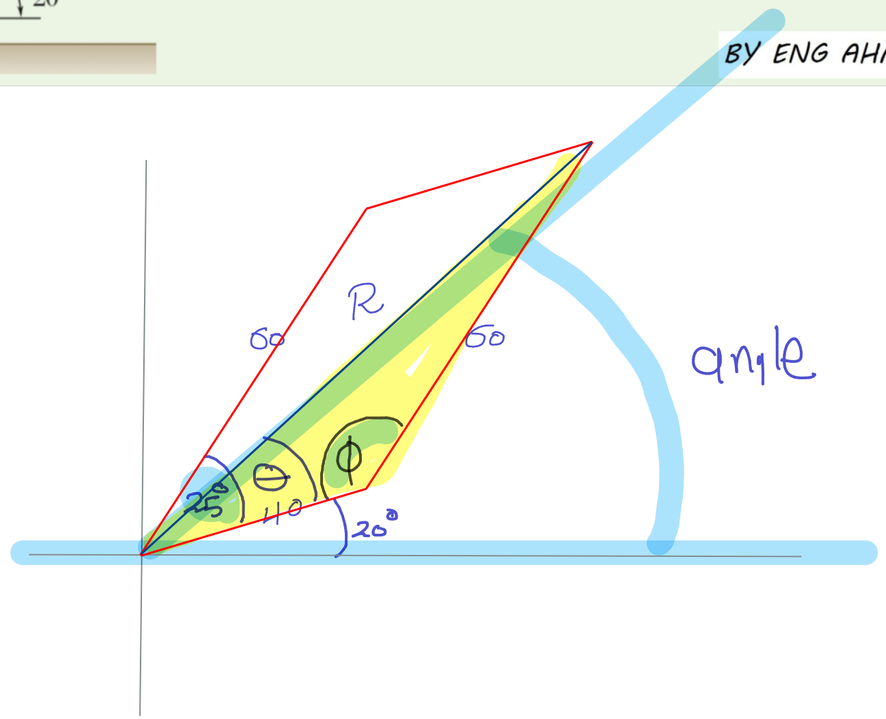




Sample Problem 2.1

Two forces **P** and **Q** act on a bolt *A*. Determine their resultant.





✓ $\phi = 180 - 25 = 155^\circ$

✓ $R = \sqrt{40^2 + 60^2 - 2(40)(60)\cos 155}$

$= 97.73$ (N)

Sin-law

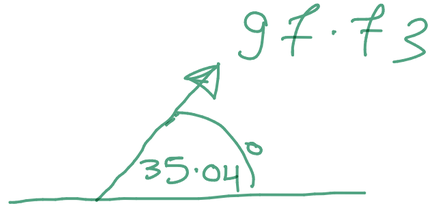
$$\frac{60}{\sin\theta} = \frac{97.73}{\sin 155}$$

$$\sin\theta = \frac{60 \sin 155}{97.73}$$

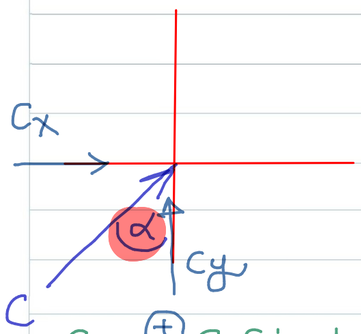
$$\theta = 15.04^\circ$$

$$\text{angle} = 15.04 + 20$$

$$= 35.04^\circ$$

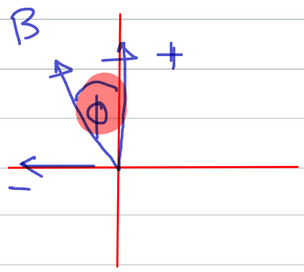


Rectangular/Cartesian Components Method



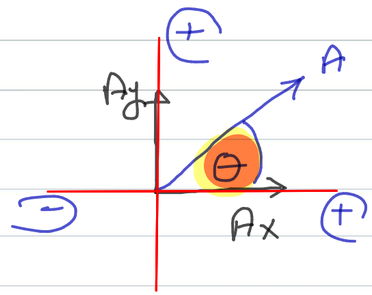
$C_x = (+) C \sin\alpha$

$C_y = (+) C \cos\alpha$



$B_x = (-) B \sin\phi$

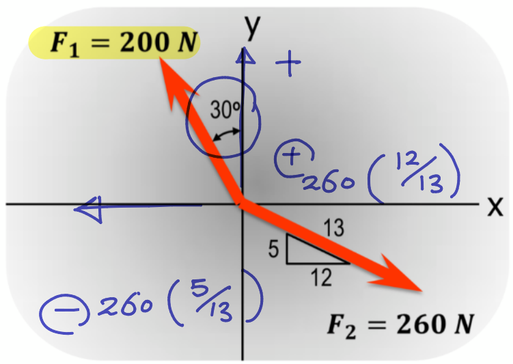
$B_y = (+) B \cos\phi$



$A_x = (+) A \cos\theta$

$A_y = (+) A \sin\theta$

Determine the x and y Cartesian components of the $\mathbf{F}_1$ and $\mathbf{F}_2$ forces acting on the boom. Put each force in the Cartesian vector form.



$F_{1x} = -200 \sin 30 = -100$

$F_{1y} = +200 \cos 30 = 173.2$

$F_1 = (-100)\,i + (173.2)\,j$

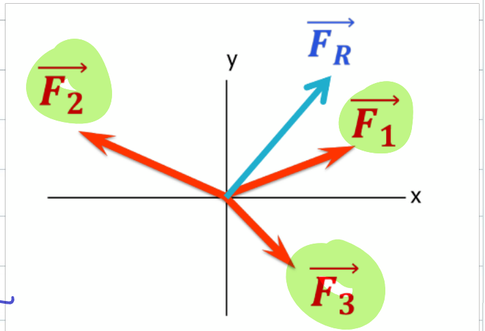
$F_{2x} = 260\,(12/13) = 240$

$F_{2y} = -260\,(5/13) = -100$

$F_2 = (240)\,i - 100\,j$

Coplanar Force Resultants

More than 2-Forces



① Resolve

F_{1x} F_{2x} F_{3x}

F_{1y} F_{2y} F_{3y}

② $R_x = \Sigma F_x$ $\pm\rightarrow$

$= F_{1x} + F_{2x} + F_{3x}$

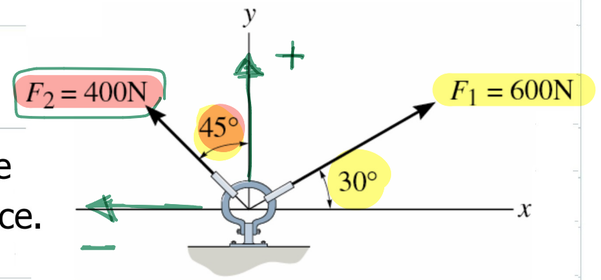
$R_y = \Sigma F_y$ $\uparrow +$

③ $R = \sqrt{R_x^2 + R_y^2}$ ✓

④ $\theta = \tan^{-1} \dfrac{R_y}{R_x}$ ✓

Example 3 :-

The link in the figure is subjected to two forces, $\mathbf{F_1}$ and $\mathbf{F_2}$. Determine the resultant magnitude and orientation of the resultant force.



* Resolve :-

$F_{1x} = 600 \cos 30 = 519.6$

$F_{1y} = 600 \sin 30 = 300$

$F_{2x} = -400 \sin 45 = -282.8$

$F_{2y} = {}^{+}400 \cos 45 = 282.8$

* $R_x = \Sigma F_x \quad \xrightarrow{+}$

$= 519.6 - 282.8 = 236.8$ (N)

$R_y = \Sigma F_y \quad \uparrow +$

$= 300 + 282.8 = 582.8$ (N)

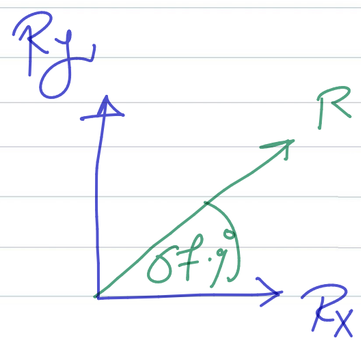
H.W ⇒ Method (1)

* $R = \sqrt{R_x^2 + R_y^2}$

$= \sqrt{236.8^2 + 582.8^2} = 629.1$ (N)

* $\theta = \tan^{-1} \frac{582.8}{236.8}$

$= 67.9°$

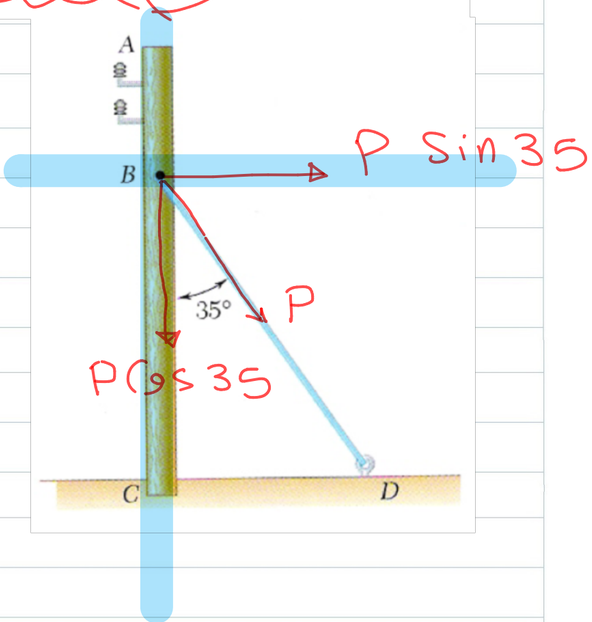


Problem # 2

The Guy wire *BD* exerts on the telephone pole *AC* a force **P** directed along *BD*. Knowing that **P** has a 450-N component along line *AC*,

Determine: (a) The magnitude of the force **P**,

(b)Its component in a direction perpendicular to *AC*.



(*) $P_y = 450$

$P \cos 35 = 450$

$P = \dfrac{450}{\cos 35}$

$= 549.3$

(*) $P_x = P \sin 35$

$= 549.3 \sin 35 = 315$ N

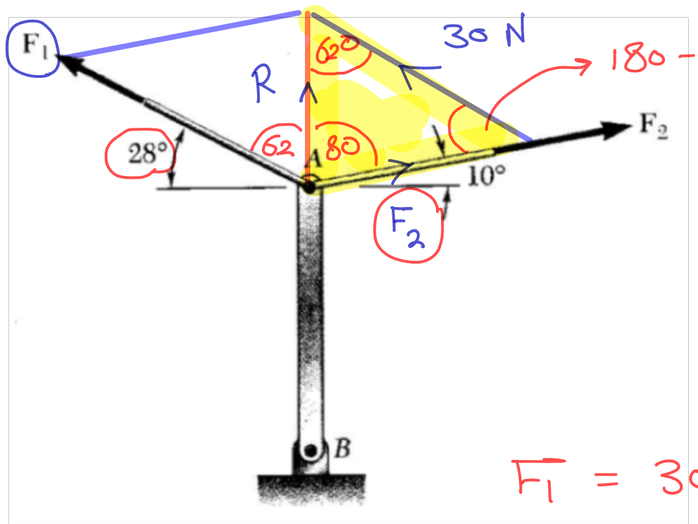
Problem # 1

Two control rods are attached at *A* to lever *AB* .

Using trigonometry and knowing that the force in the left-hand rod is **F1=30N**,

Determine: (a) The required force F2 in the right-hand rod if the resultant R of that forces exerted by the rods on the lever is to be vertical.

(b) The corresponding magnitude of R.



$F_1 = 30$

$F_2 = ??$

$R = ??$

Use sin law

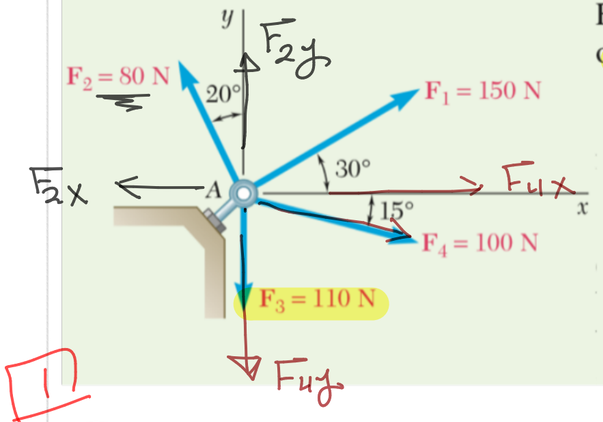
$$\frac{F_2}{\sin 62} = \frac{R}{\sin 38} = \frac{30}{\sin 80}$$

$$F_2 = 27 \quad N$$

$$R = 19 \quad N$$

Sample Problem 2.3

Four forces act on bolt *A* as shown. Determine the resultant of the forces on the bolt.



(1)

(*) $F_{1x} = 150 \cos 30 = 129.9$

$F_{1y} = 150 \sin 30 = 75$

(*) $F_{2x} = -80 \sin 20 = -27.4$

$F_{2y} = 80 \cos 20 = 75.2$

(*) $F_{3x} = 0$

$F_{3y} = -110$

* $F_{4x} = 100 \cos 15 = 96.6$

$F_{4y} = -100 \sin 15 = -25.9$

(2)

$R_x = \Sigma F_x$

$= 129.9 - 27.4 + 96.6$

$= 199.1$

$R_y = \Sigma F_y$

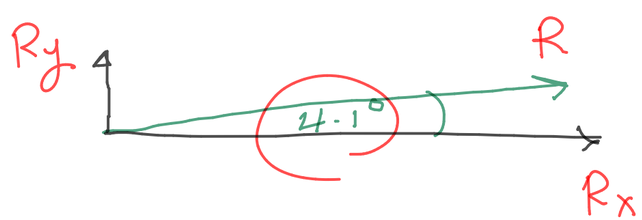
$= 75 + 75.2 - 110 - 25.9$

$= 14.3$

$R = 199.1\,\hat{i} + 14.3\,\hat{j}$

$R = \sqrt{199.1^2 + 14.3^2} = 199.6$ (N)

$\alpha = \tan^{-1}\left(\frac{14.3}{199.1}\right) = 4.1$



2.3 Equilibrium of a Particle

$\Sigma F = 0$

@ Rest

@ Moving with Constant velocity

① Resolve

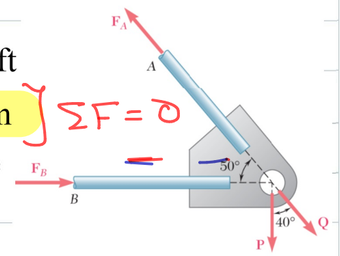
② $\Sigma F_x = 0$ $\quad +\rightarrow$

$\Sigma F_y = 0$ $\quad +\uparrow$

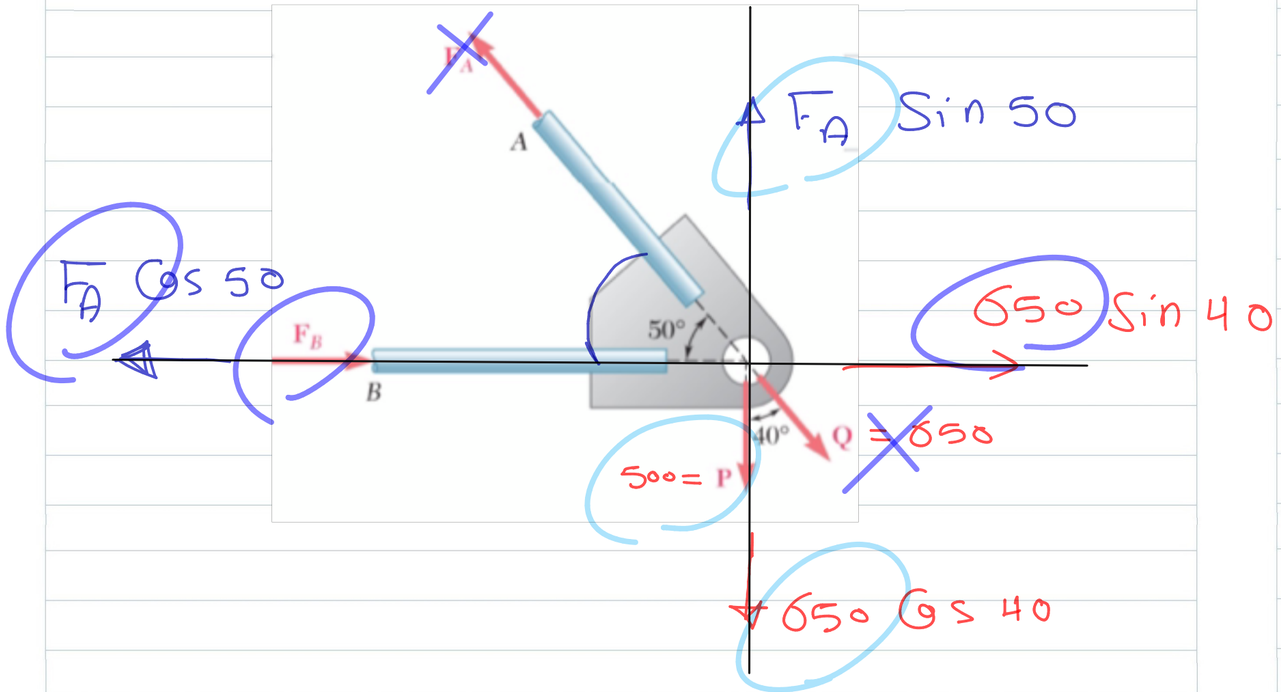
2 - Unknown

PROBLEM 2.51

Two forces **P** and **Q** are applied as shown to an aircraft connection. Knowing that the connection is in equilibrium and that $P = 500$ N and $Q = 650$ N, determine the magnitudes of the forces exerted on the rods A and B.



$\Sigma F = 0$



$\Sigma F_y = 0 \quad \uparrow +$

$F_A \sin 50 - 500 - 650 \cos 40 = 0$

$F_A \sin 50 = 997.9$

$F_A = 1303$ N

$\Sigma F_x = 0 \quad \rightarrow +$

$650 \sin 40 - 1303 \cos 50 + F_B = 0$

$F_B = 1303 \cos 50 - 650 \sin 40$

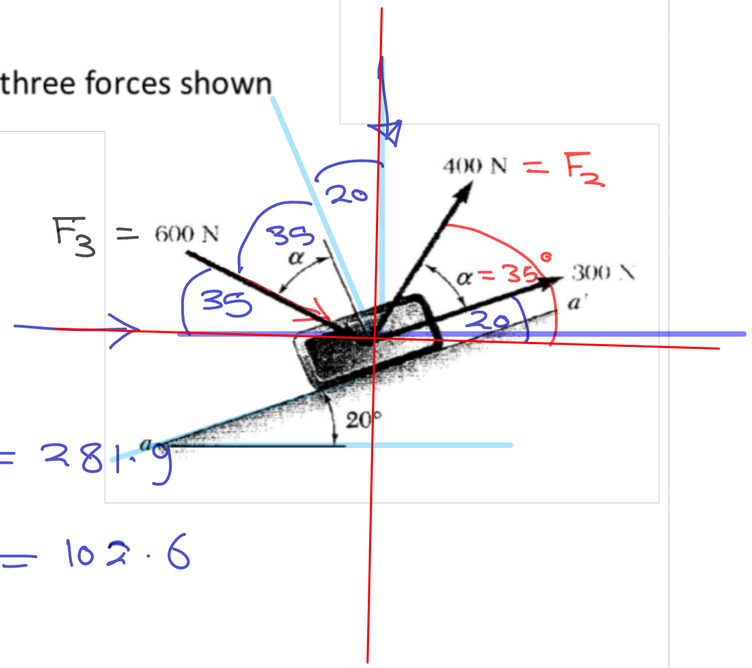
$= 420$ N

Problem # 3

BY ENG AHMAD SABRI ; +974 5572 4426

Knowing that $\alpha = 35°$,

Determine: The resultant of the three forces shown



Resolve

$F_1 = 300$

$F_{1x} = 300 \cos 20 = 281.9$

$F_{1y} = 300 \sin 20 = 102.6$

$F_2 = 400 \text{ N}$

$F_{2x} = 400 \cos 55 = 229.4 \text{ N}$

$F_{2y} = 400 \sin 55 = 327.7 \text{ N}$

$F_3 = 600 \text{ N}$

$F_{3x} = +600 \cos 35 = 491.5$

$F_{3y} = -600 \sin 35 = -344$

(*) $R_x = \Sigma F_x = 281.9 + 229.4 + 491.5$

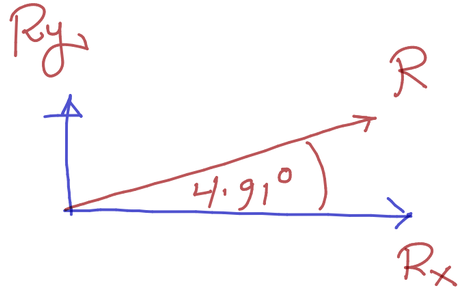
$= 1002.8$

$R_y = \Sigma F_y = 102.6 + 327.7 - 344$

$= 86.3$

(*) $R = \sqrt{1002.8^2 + 86.3^2} = 1006.5$ N

(*) $\ominus = \tan^{-1} \frac{86.3}{1002.8} = 4.91°$



Quiz

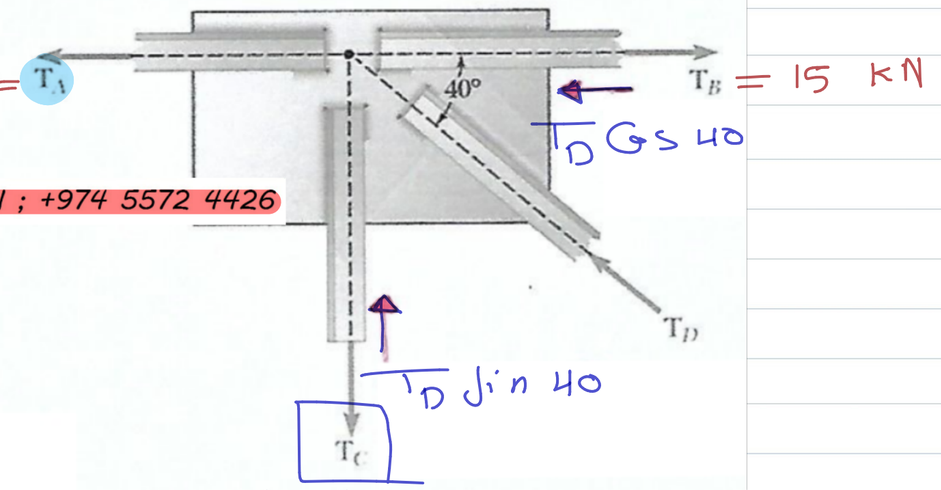
Quiz # 1

Fall 2016

Two forces of magnitude $T_A = 8$ KN and $T_B = 15$ KN are applied as shown to a welded connection. Knowing that the connection is in equilibrium, $\sum F = 0$

Determine: the magnitudes of the forces T_C and T_D.

$T_A = 8$



$T_B = 15$ kN

$T_D \cos 40$

$T_D \sin 40$

$\sum F_x = 0 \quad \xrightarrow{+}$

$15 - T_D \cos 40 - 8 = 0$

$T_D \cos 40 = 7$

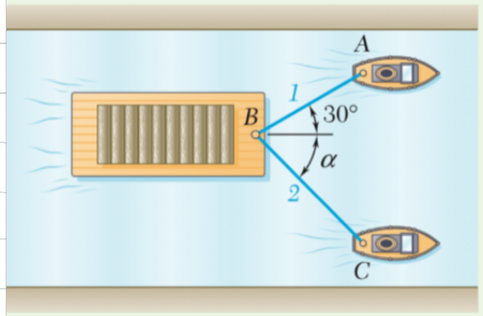
$T_D = 9.14$ kN

$\sum F_y = 0 \quad \uparrow +$

$9.14 \sin 40 - T_C = 0 \implies$

$T_C = 9.14 \sin 40 = 5.87$ (kN)

Sample Problem 2.2



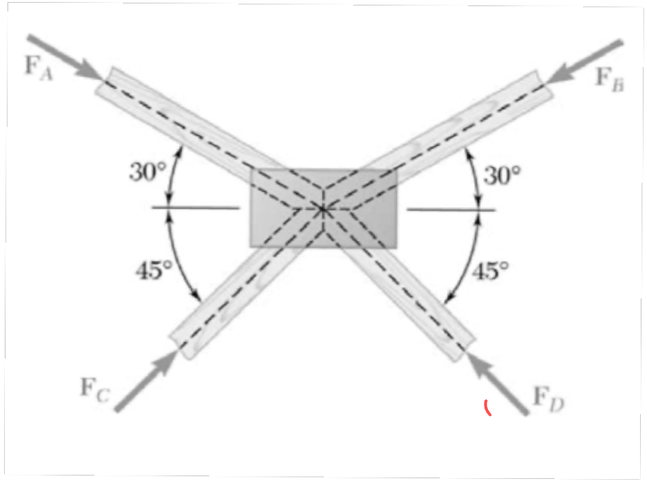
Two tugboats are pulling a barge. If the resultant of the forces exerted by the tugboats is a 5000-lb force directed along the axis of the barge, determine (*a*) the tension in each of the ropes, given that $\alpha = 45°$, (*b*) the value of α for which the tension in rope *2* is minimum.

H.W

Problem # 5

Four wooden members are joined with metal plate connectors and are in equilibrium under the action of the four forces shown. Knowing that $F_A = 2.3\ \text{kN}$ and $F_B = 2.1\ \text{kN}$,

Determine: the magnitudes of the other two forces.



H.W