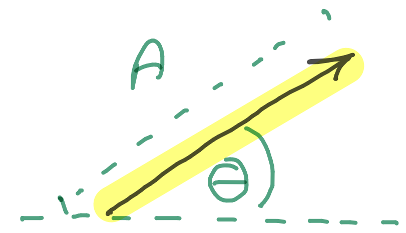
Class ①

lec ① → Force Vectors

*** Vectors** } Magnitude (A)
direction (θ)



Ex. Force, velocity

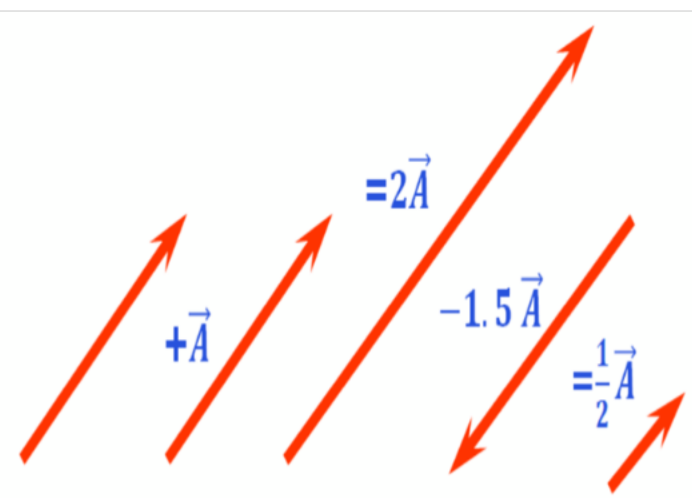
*** Scalar** } only Magnitude
No-direction

Ex. Mass, volume

Vectors are **equal** when they have the same magnitude and same direction

$\vec{A}$ = $\vec{B}$

Vectors can be simply **added** or **subtracted**, if they have the same direction



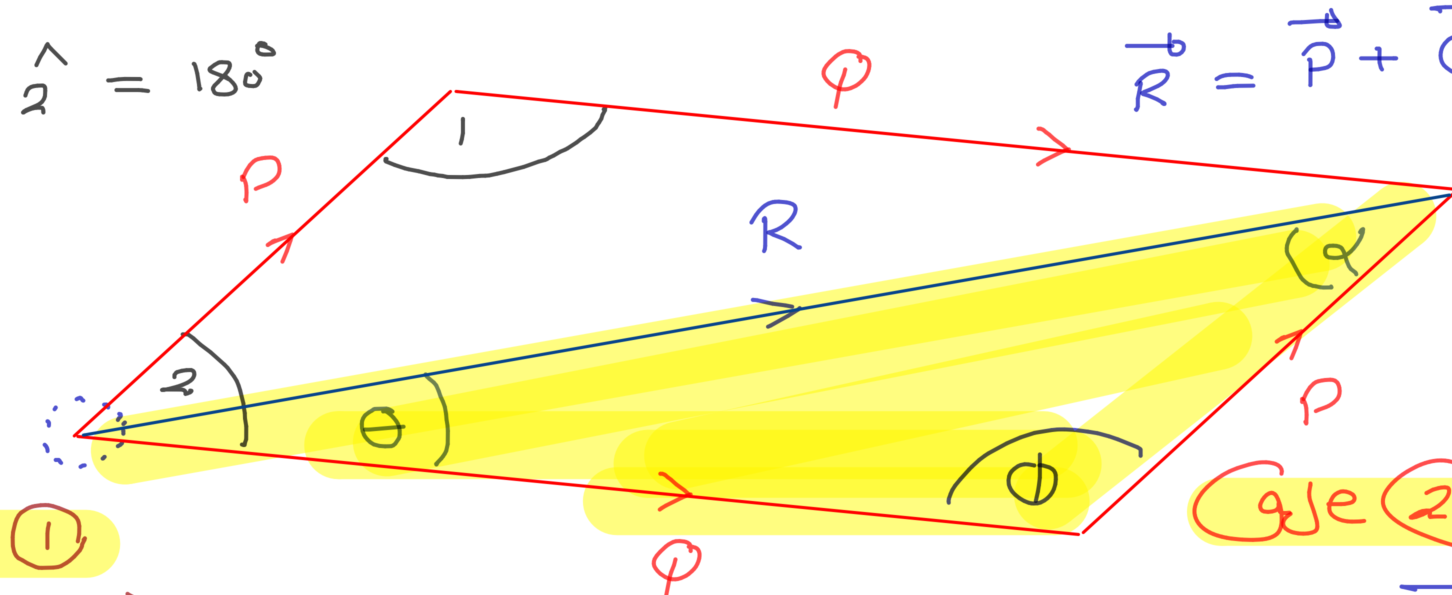
$\vec{A} = 1$ $\vec{B} = 3$ $A + B = 4$

Parallelogram Law
Trigonometric method

} only two Forces

$1 + 2 = 180°$

$\vec{R} = \vec{P} + \vec{Q}$



Case ①

Given $\vec{P}$, $\vec{Q}$ Required (R): Mag, direction

① Calculate internal angle between $\vec{P}$ & $\vec{Q}$ $\boxed{\phi}$

② Mag → Cos-law

$$R = \sqrt{P^2 + Q^2 - 2PQ\cos\phi}$$

③ Direction → sin-law

$$\frac{R}{\sin\phi} = \frac{P}{\sin\theta} = \frac{Q}{\sin\alpha}$$

Case ②

Given $\vec{R}$ Required $\vec{P}$, $\vec{Q}$

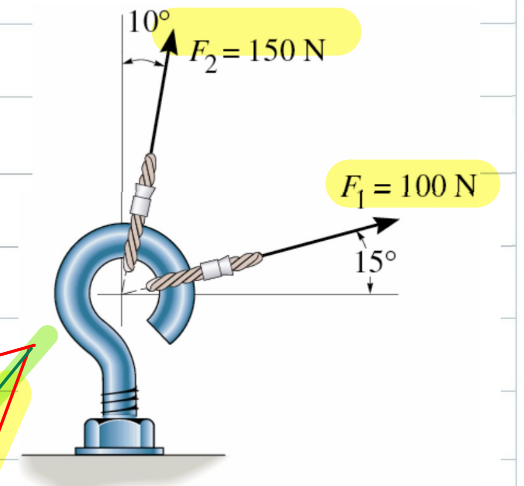
① Calculate all internal angles (ϕ, α, θ)

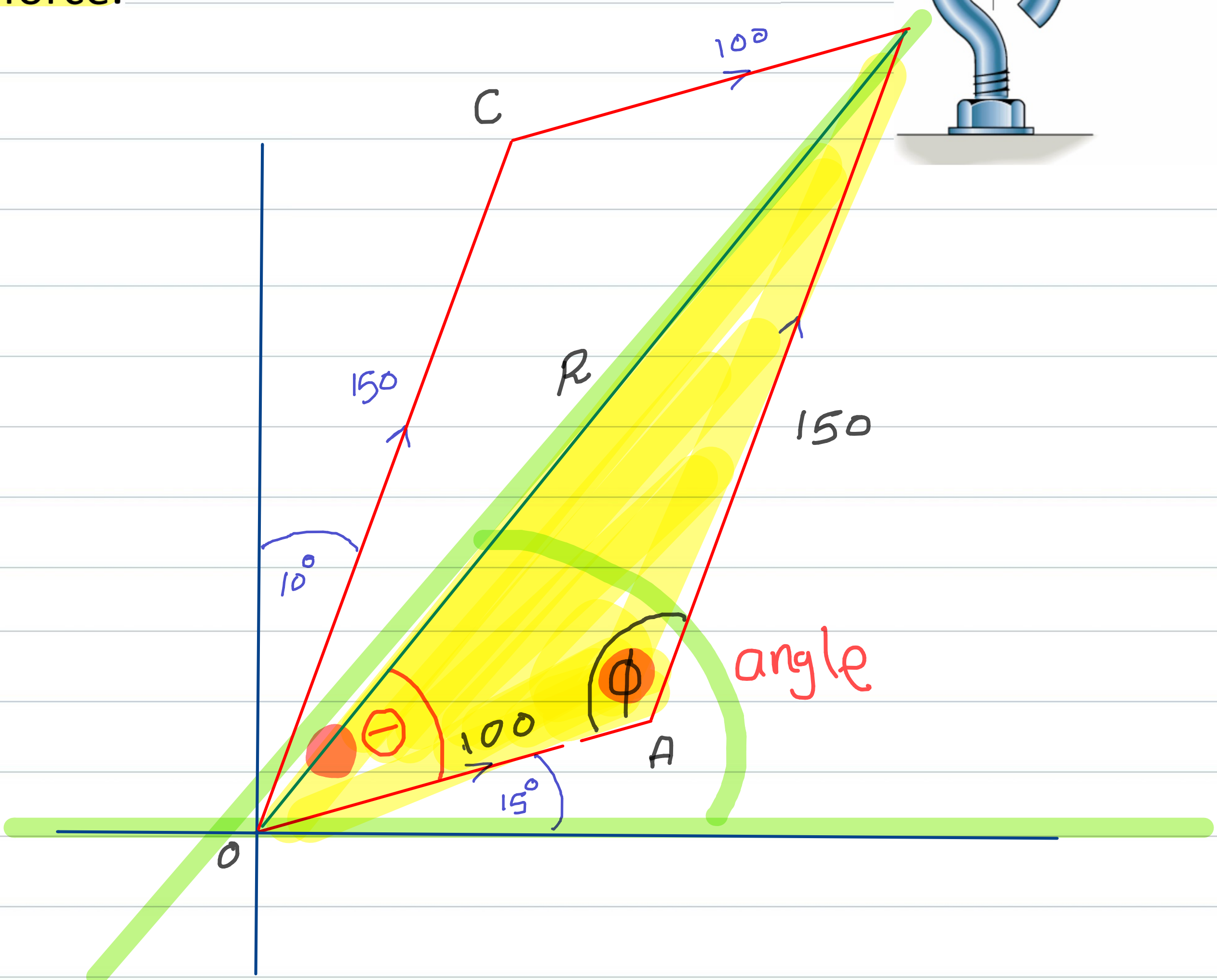
② sin-law :-

$$\frac{R}{\sin\phi} = \frac{P}{\sin\theta} = \frac{Q}{\sin\alpha}$$

Example 1:-

The screw eye in the figure at the left is subjected to two forces $\vec{F}_1$ and $\vec{F}_2$.
Determine the magnitude and direction of the resultant force.





① angle $CoA = 90 - 15 - 10 = 65^\circ$

$$\phi = 180 - 65 = 115^\circ$$

② $R = \sqrt{100^2 + 150^2 - 2(100)(150)\cos 115}$

$$R = 213 \text{ N}$$

③ $\Rightarrow$ sin-law :-

$$\frac{150}{\sin\theta} = \frac{213}{\sin 115}$$

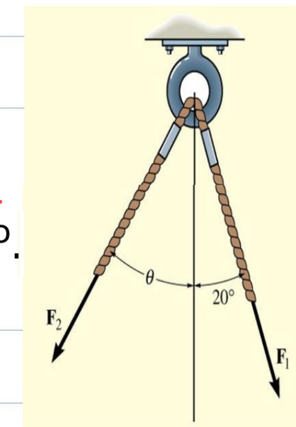
$$\theta = \sin^{-1}\left(\frac{150 \sin 115}{213}\right)$$

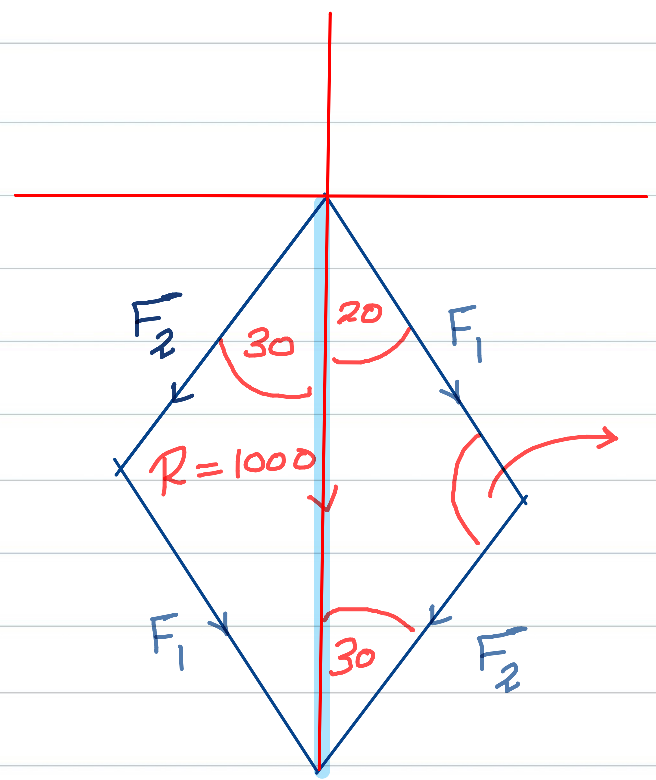
$$= 39.7^\circ$$

$$\text{angle} = 39.7^\circ + 15^\circ = 54.7^\circ$$

Example 2 :-

The ring below is subjected to F_1 and F_2. If we want a resultant force of 1kN and directed vertically downward, determine the magnitude of F_1 and F_2 if $\theta = 30^o$.





$180 - 20 - 30$
$= 130°$

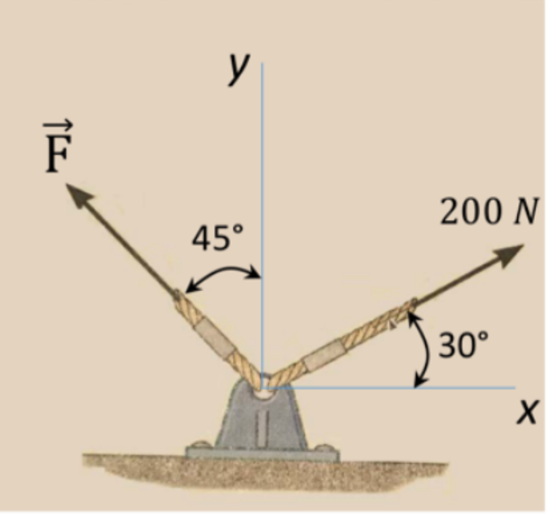
sin – law

$$\frac{F_1}{\sin 30} = \frac{F_2}{\sin 20} = \frac{1000}{\sin 130}$$

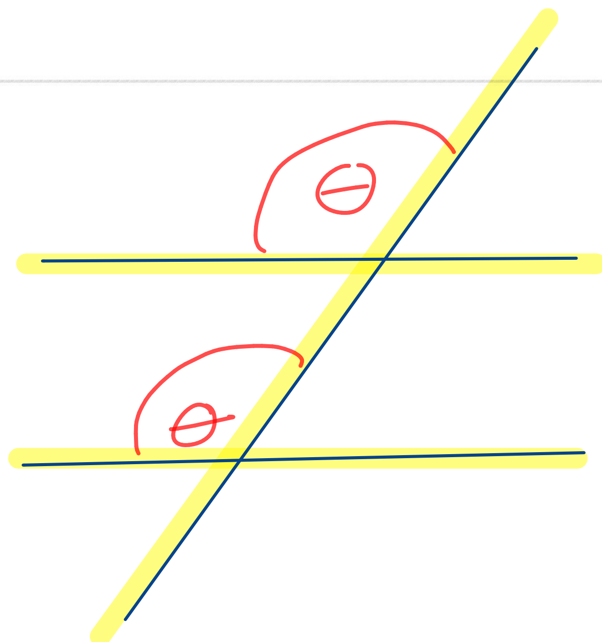
$$F_1 = \frac{1000 \sin 30}{\sin 130} = 653 \text{ N}$$

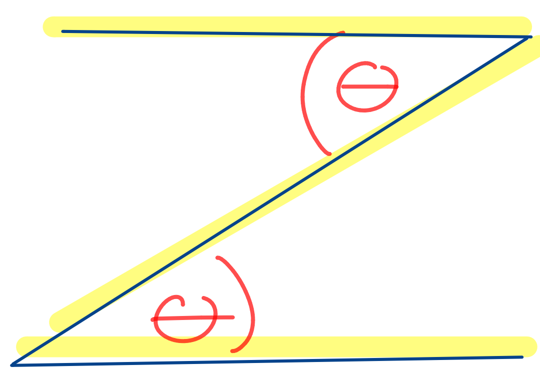
$$F_2 = \frac{1000 \sin 20}{\sin 130} = 446 \text{ N}$$

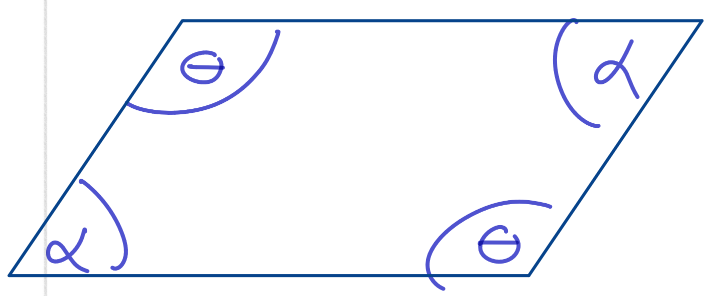
Example 2: Determine the magnitude of force $\vec{F}$ in the figure below and the magnitude of the resultant force $\vec{R}$, if $\vec{R}$ is along the positive y axis.



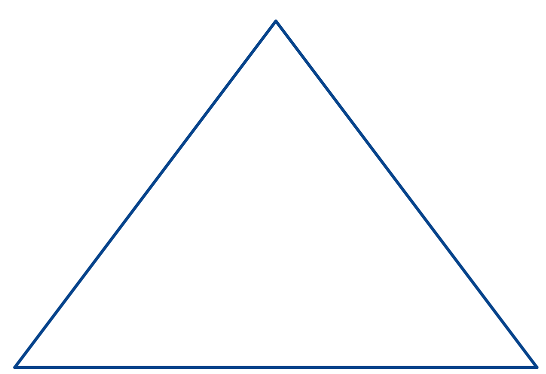
H.W







$\theta + \alpha = 180$

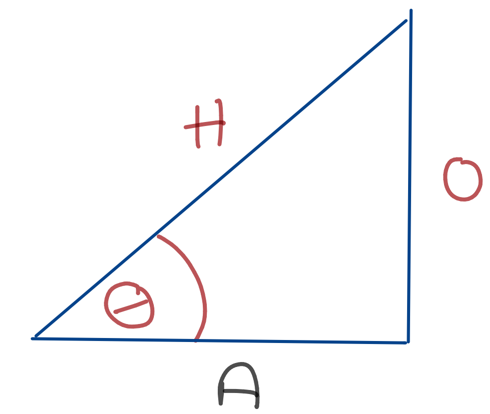


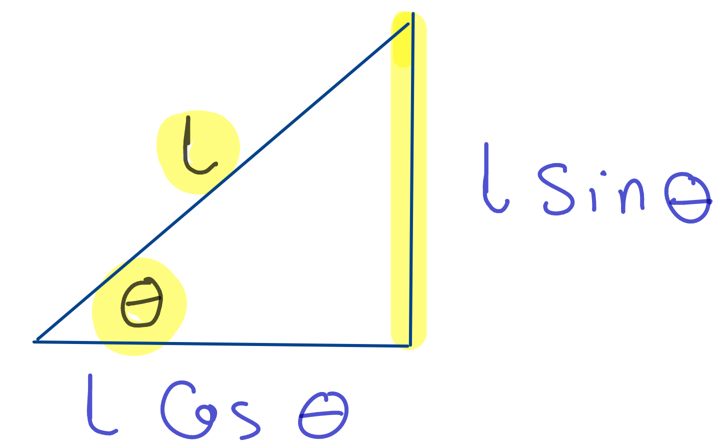
$\Sigma \Delta = 180$

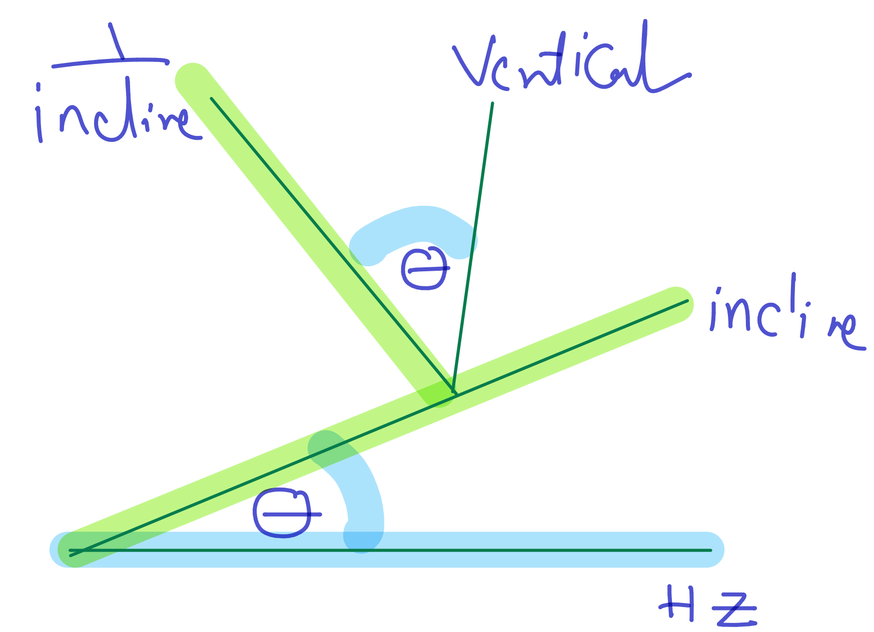
$\sin\theta = \frac{O}{H}$

$\cos\theta = A/H$

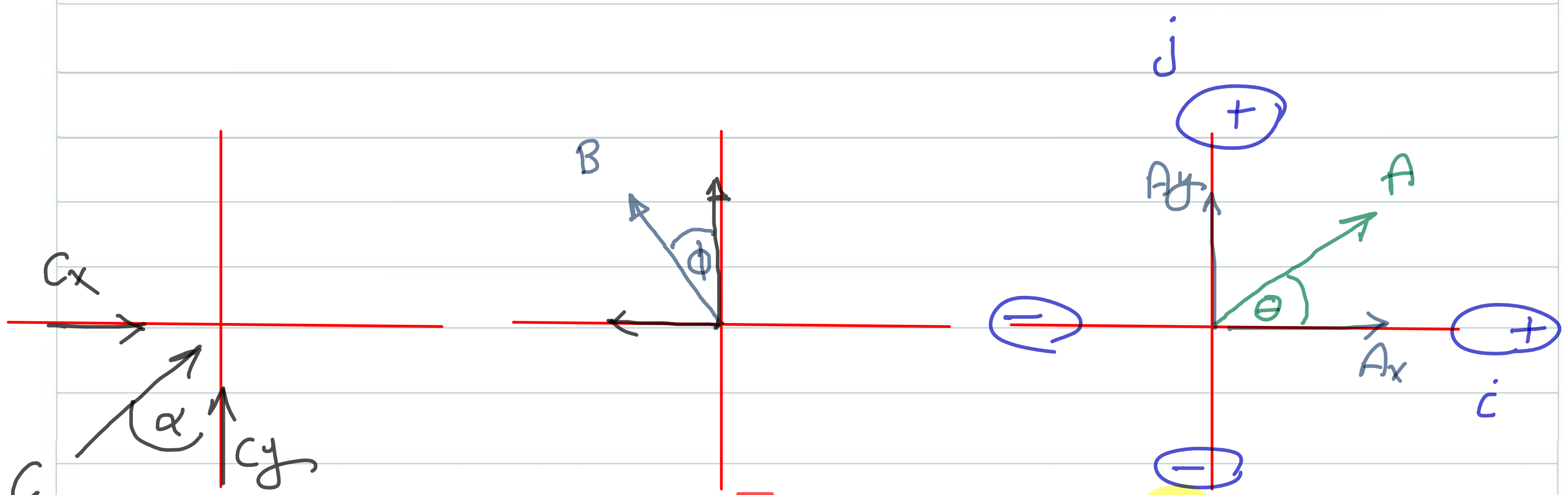
$\tan\theta = \frac{O}{A}$







Rectangular/Cartesian Components Method



$C_x = +C\sin\alpha$

$C_y = +C\cos\alpha$

$B_x = -B\sin\phi$

$B_y = +B\cos\phi$

$\vec{B} = B_x\hat{i} + B_y\hat{j}$

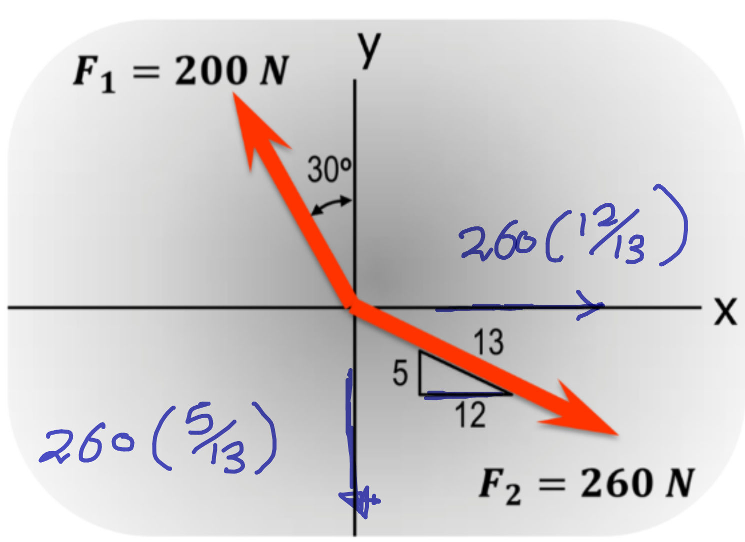
$A_x = A\cos\theta$

$A_y = A\sin\theta$

$A = \sqrt{A_x^2 + A_y^2}$

$\theta = \tan^{-1}\frac{A_y}{A_x}$

Determine the x and y Cartesian components of the $\mathbf{F}_1$ and $\mathbf{F}_2$ forces acting on the boom. Put each force in the Cartesian vector form.



$F_{1x} = -200\sin 30 = -100$

$F_{1y} = 200\cos 30 = 173$

$F_{2x} = 260\left(\frac{12}{13}\right) = 240$

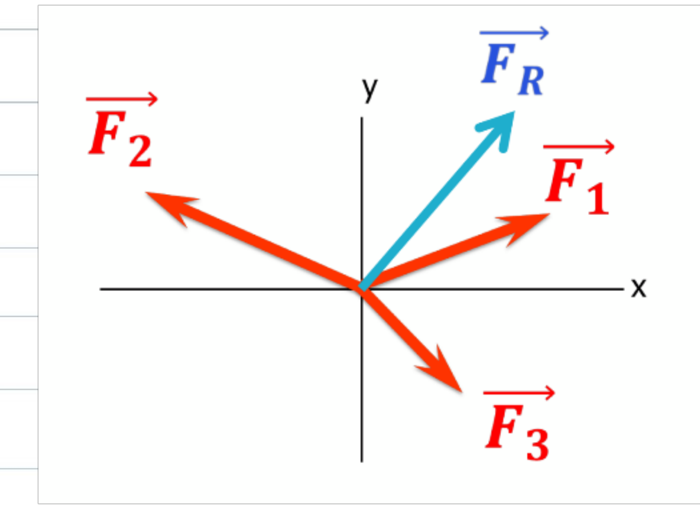
$F_{2y} = -260\left(\frac{5}{13}\right) = -100$

$\vec{F_1} = (-100)\hat{i} + 173\hat{j}$

$\vec{F_2} = (240)\hat{i} + (-100)\hat{j}$

Coplanar Force Resultants

More than 2-Forces



① $\vec{F_1} = F_{1x}\hat{i} + F_{1y}\hat{j}$

$\vec{F_2} = F_{2x}\hat{i} + F_{2y}\hat{j}$

$\vec{F_3} = F_{3x}\hat{i} + F_{3y}\hat{j}$

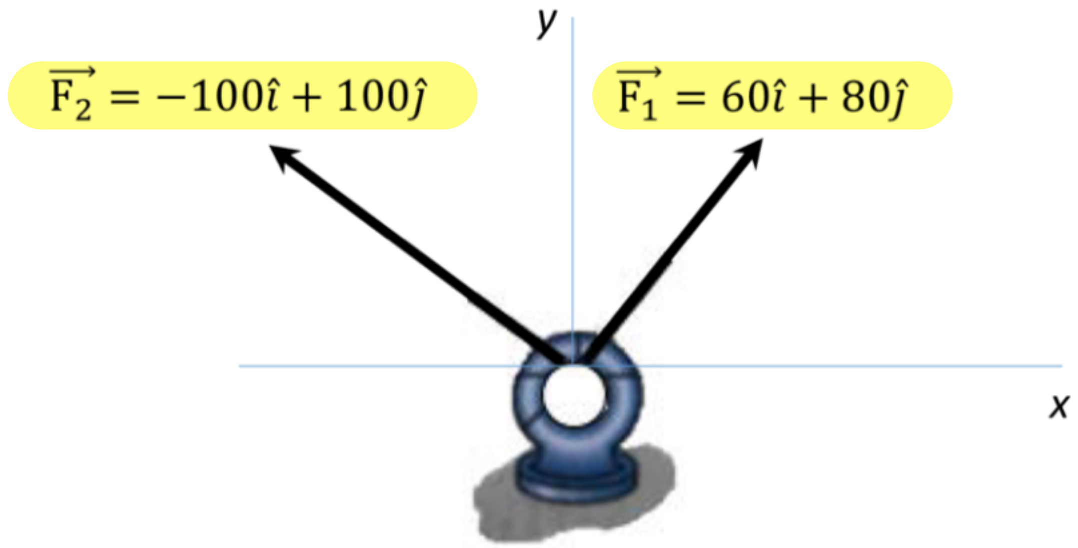
② $\vec{R} = \vec{F_1} + \vec{F_2} + \vec{F_3}$

$= \underbrace{(F_{1x} + F_{2x} + F_{3x})}_{R_x}\hat{i} + \underbrace{(F_{1y} + F_{2y} + F_{3y})}_{R_y}\hat{j}$

③ $R = \sqrt{R_x^2 + R_y^2}$

④ $\theta = \tan^{-1}\frac{R_y}{R_x}$

Example 3: Determine the magnitude and direction (angle) of the resultant force acting on the ring.



(2) $\vec{R} = \vec{F_1} + \vec{F_2}$

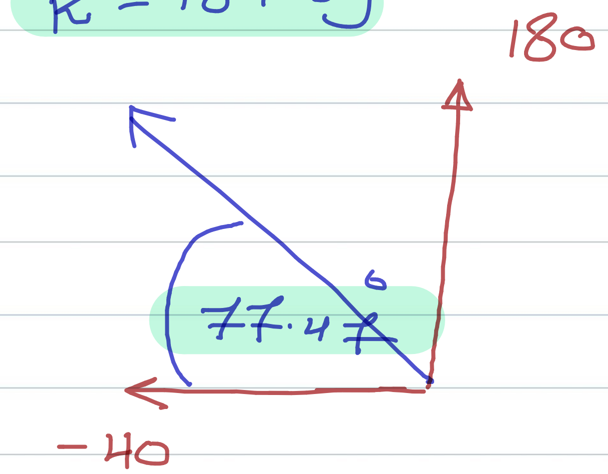
$= 60\,i + 80\,j - 100\,i + 100\,j$

$\vec{R} = -40\,i + 180\,j$

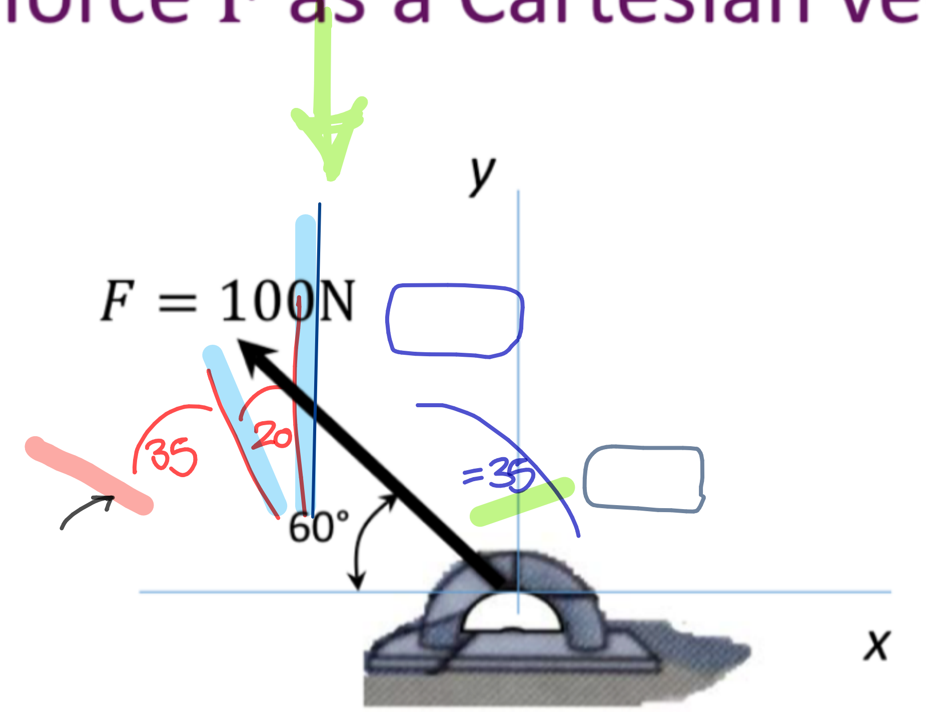
(3) $R = \sqrt{40^2 + 180^2} = 184.39$

(4) $\theta = \tan^{-1} \dfrac{180}{-40} = -77.47$

$R = 184.39$



Example 4: Express the force $\vec{F}$ as a Cartesian vector. F_3



Resolve :—

$F_1 = 300$ N

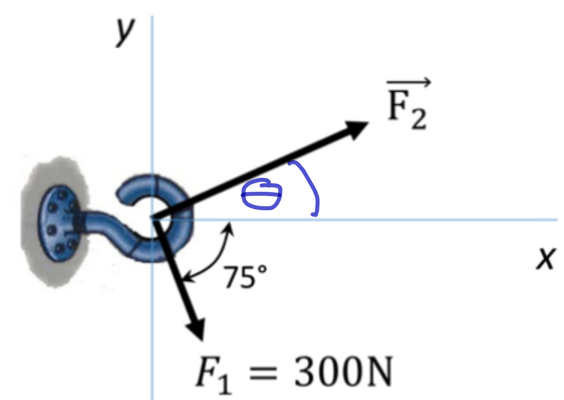
'' z =

$F_x = -100 \cos 60 = -50$

$F_y = 100 \sin 60 = 86.6$

$F = -50\,\hat{i} + 86.6\,\hat{j}$

Example 5: Two forces act on the hook shown in the figure below. Specify the components of $\overrightarrow{F_2}$ so that the resultant force $\overrightarrow{F_R}$ acts along the positive x axis and has a magnitude of 700 N.



$$\vec{R} = 700\,\hat{i}$$

$$F_{1x} = 300 \cos 75 = 77.65\,\hat{i}$$

$$F_{1y} = -300 \sin 75 = -289.78\,\hat{j}$$

$$\vec{F_1} = 77.65\,\hat{i} - 289.78\,\hat{j}$$

$$\vec{F_2} = F_2 \cos\theta\,\hat{i} + F_2 \sin\theta\,\hat{j}$$

$$\vec{R} = \vec{F_1} + \vec{F_2}$$

$$700\,\hat{i} = 77.65\,\hat{i} - 289.78\,\hat{j} + F_2 \cos\theta\,\hat{i} + F_2 \sin\theta\,\hat{j}$$

$$700\,\hat{i} = (77.65 + F_2 \cos\theta)\,\hat{i} + (-289.78 + F_2 \sin\theta)\,\hat{j}$$

$$77.65 + F_2 \cos\theta = 700$$

$$\Downarrow$$

$$F_2 \cos\theta = 622.35 \Rightarrow (1)$$

$$-289.78 + F_2 \sin\theta = 0$$

$$F_2 \sin\theta = 289.78 \Rightarrow (2)$$

Divide Eq" (2) by (1)

$$\frac{\cancel{F_2} \sin\theta}{\cancel{F_2} \cos\theta} = \frac{289.78}{622.35} = 0.466$$

$$\tan\theta = 0.466 \Rightarrow \theta = 25^\circ$$

in Eq" (1)

$$F_2 = \frac{622.35}{\cos 25} = 686.7\ (N)$$