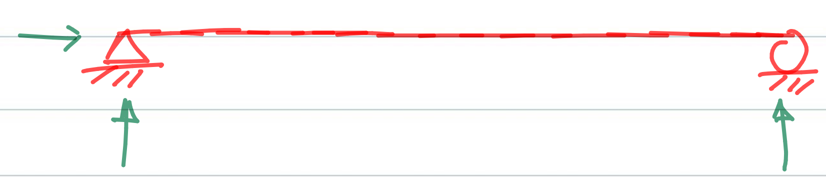
Review

* beam } to resist M

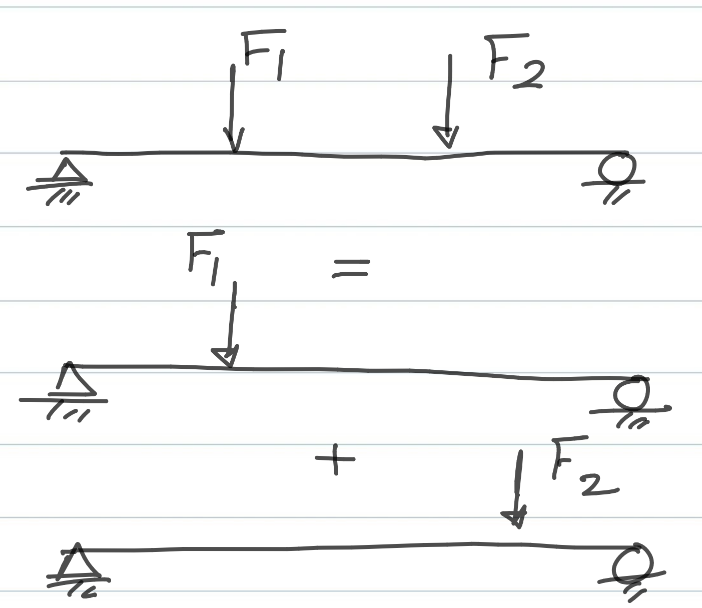
* Column } to resist axial Compressive Force

Simple supported beam

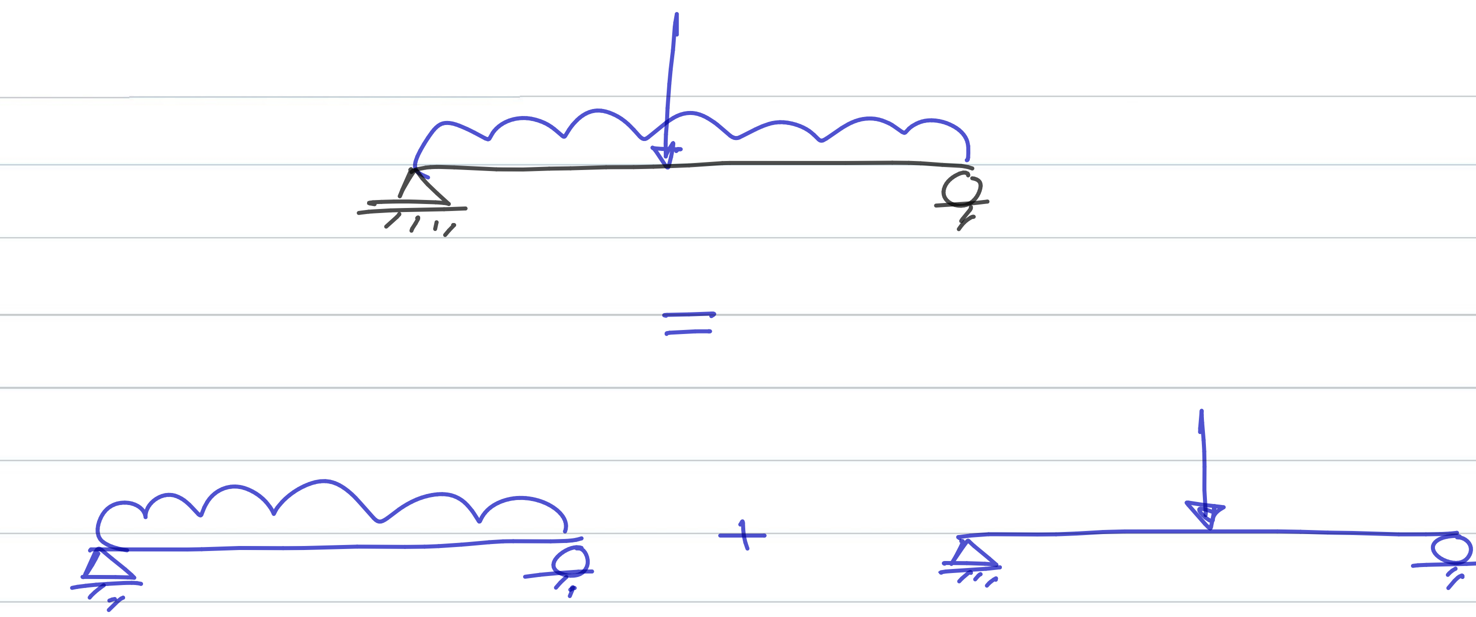


Cantilever beam

* Principal of Superposition :-



actual beam



* Eq of Equilibrium :-

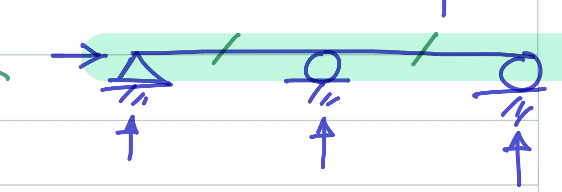
$$\Sigma F_x = 0 \quad \xrightarrow{+}$$

$$\Sigma F_y = 0 \quad \uparrow +$$

$$\Sigma M = 0 \quad \circlearrowleft +$$

4 Reaction
3 } $\boxed{4-3}$

beam } n ⇒ Members, r ⇒ reaction



$r = 3n$ } Statically determinate
Unknowns = 3

$r > 3n$ } Statically indeterminate
Unknowns > 3 — More Stable

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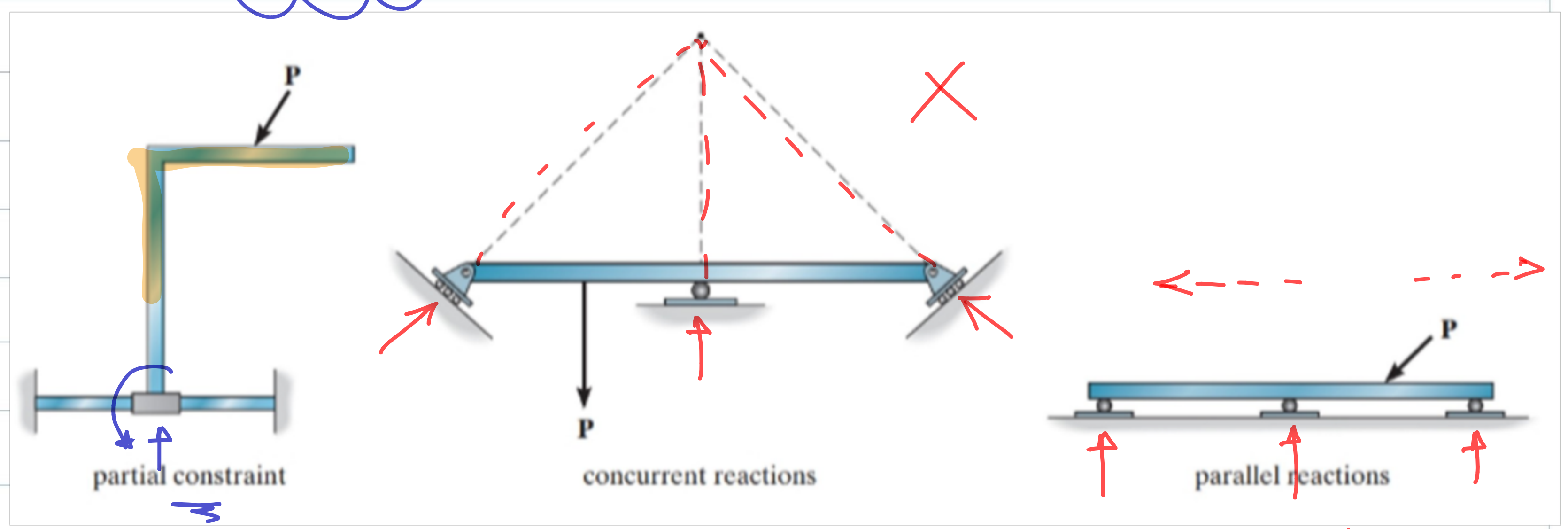
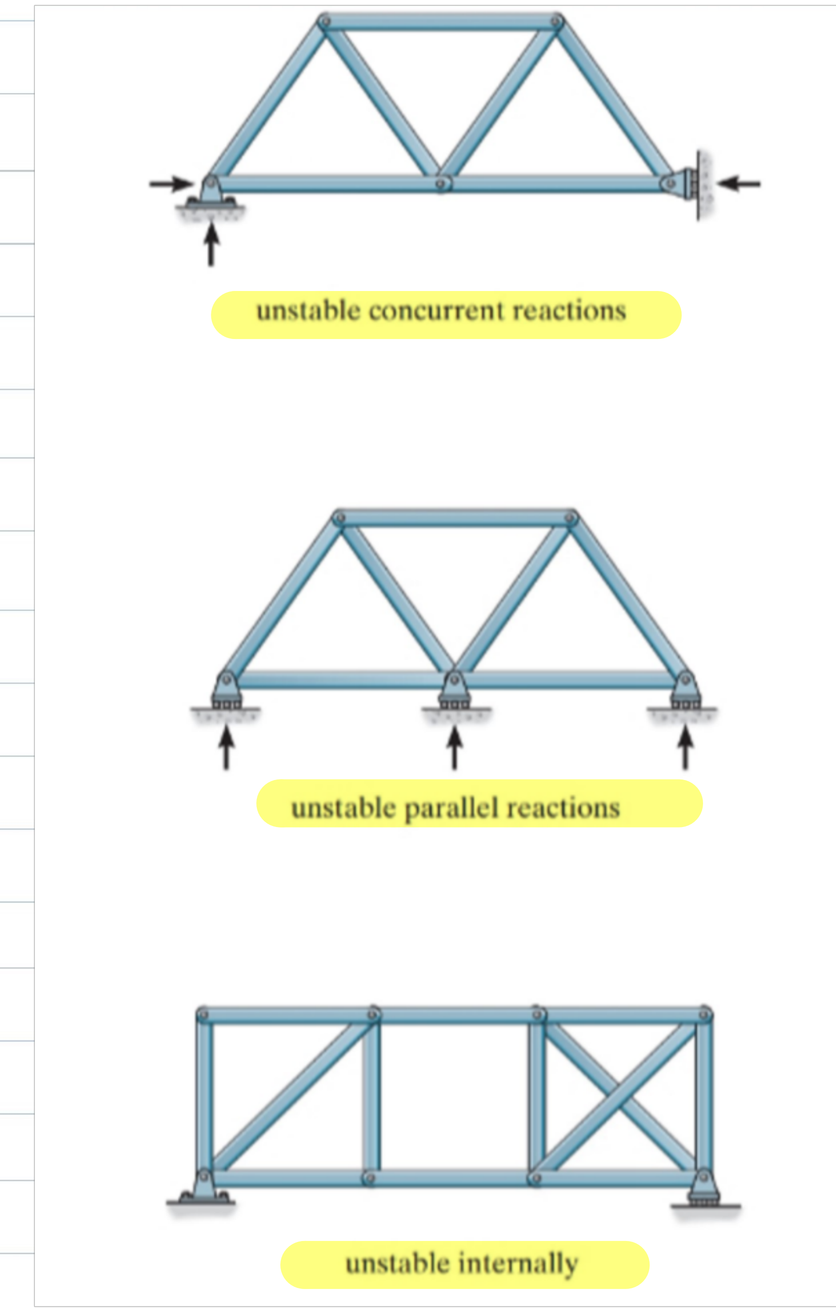
Stability



unstable

Reaction < Eq

unstable



$b + r < 2j$ unstable

Trusses

- # members = b
- # reaction = r
- # Joints = J

$b + r = 2j$ } statically determinate

$b + r > 2j$ } statically indeterminate

Methods of analysis

Equilibrium

$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma M = 0$

3-Eq's

Compatibility

①

②

1st degree ⇒ ① Compatibility Eq

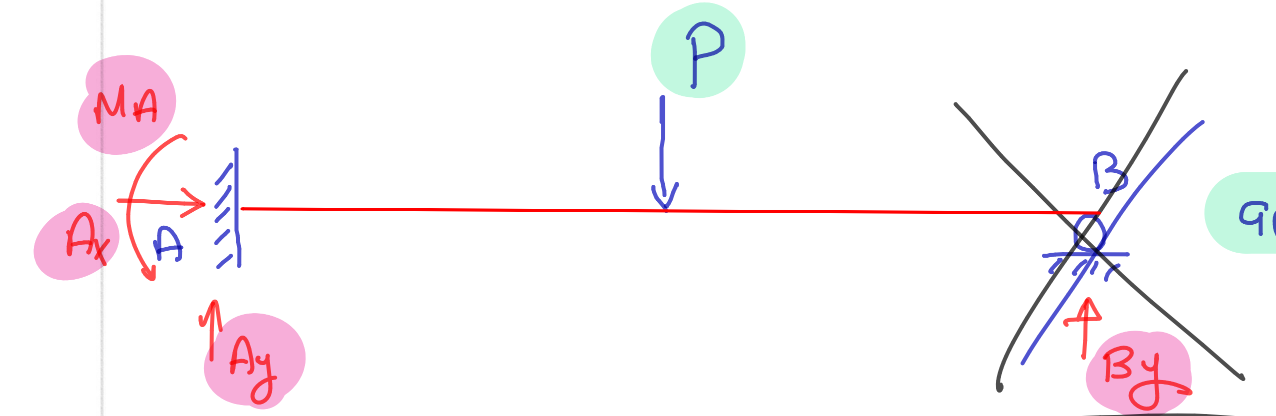
2nd degree ⇒ 2-Eq's Extra

Deflection @ support = 0

Slope @ Fixed = 0

Slope @ pin or roller = ✓

Ex



$\Delta_A = 0$

$\theta_A = 0$

$\boxed{\Delta_B = 0}$

$\theta_B = ✓$

4-unknowns

3-Eq's

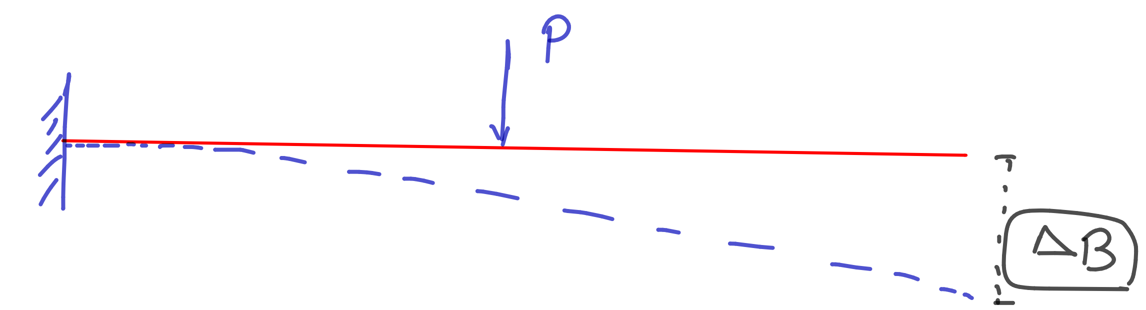
1st degree

⇓

1-redundant @ B

By

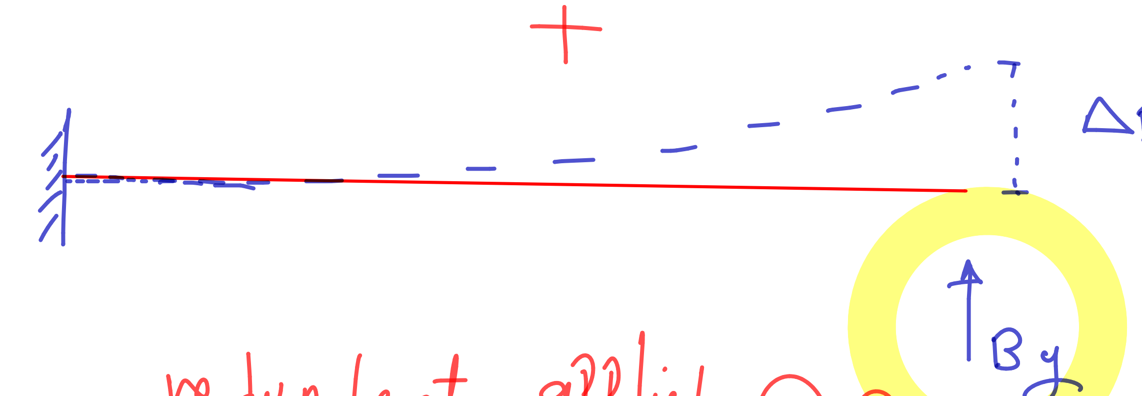
=



Primary Structure

↓ -

+



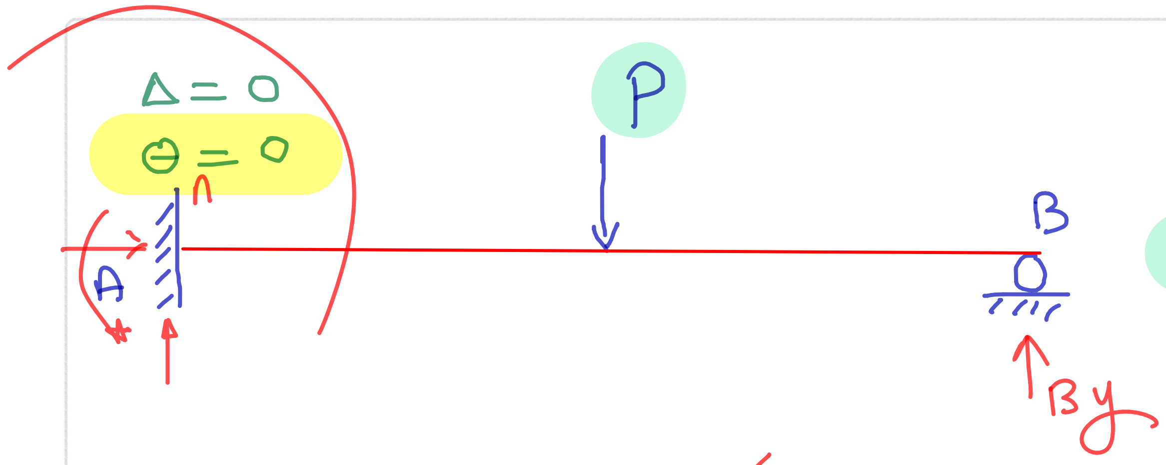
$\Delta_B' = B_y F_{BB}$

↑ +

redundant applied @ B

$0 = -\Delta_B + \Delta_B'$

$0 = -\Delta_B + B_y F_{BB}$



actual beam

4 - unknowns
3 - Equ's

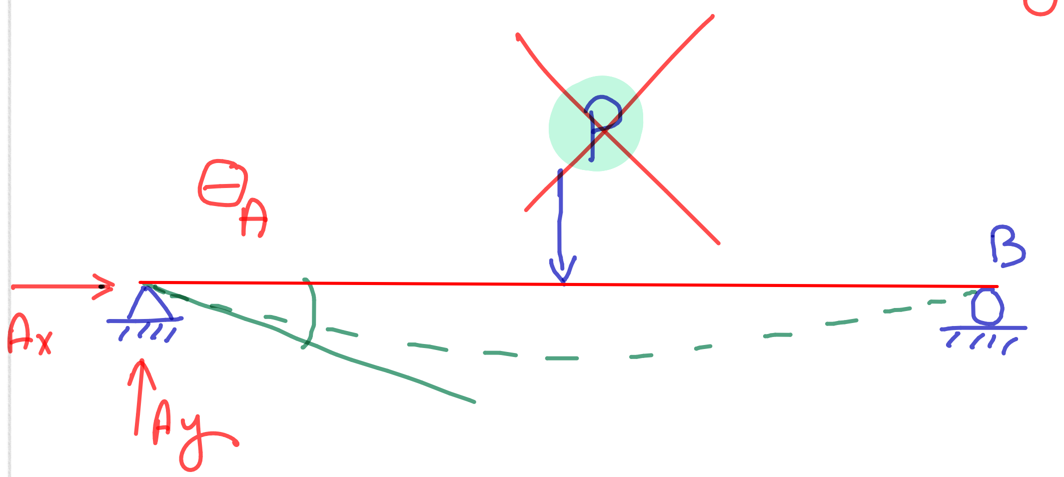
1st degree

$\Downarrow$

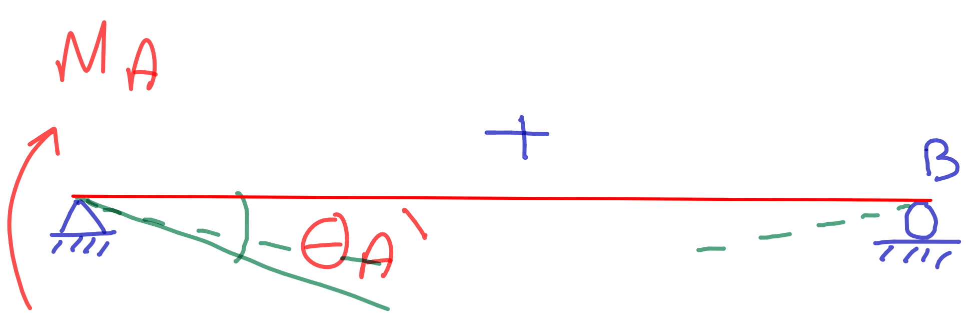
1 - redundant

@ A

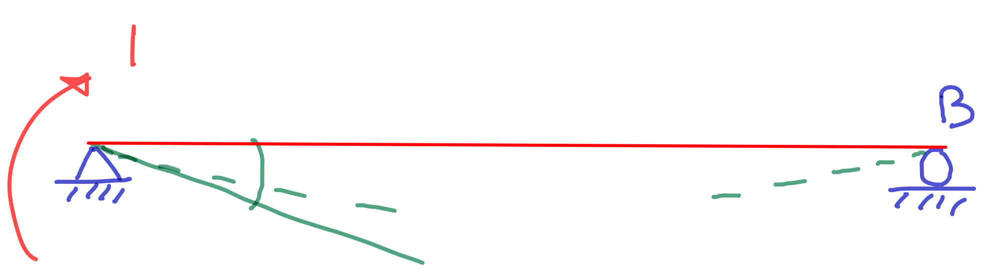
$\boxed{M_A}$



Primary structure



$$\theta_A' = M_A \, \alpha_{AA}$$



$$0 = \theta_A + \theta_A'$$

$$0 = \theta_A + M_A \, \alpha_{AA}$$

$\Downarrow$

to get Moment M_A

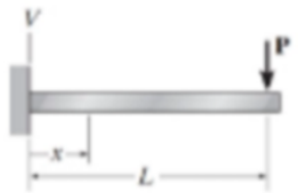
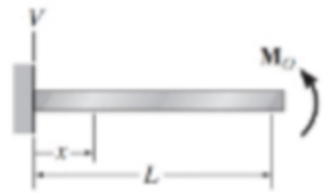
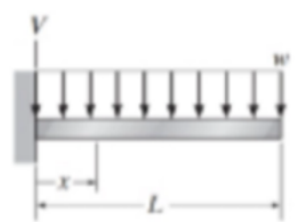
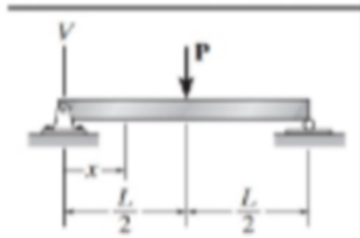
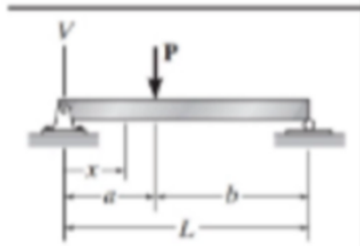
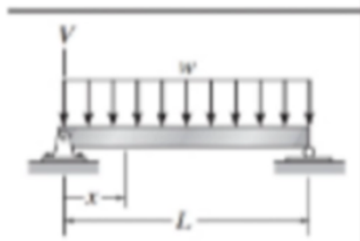
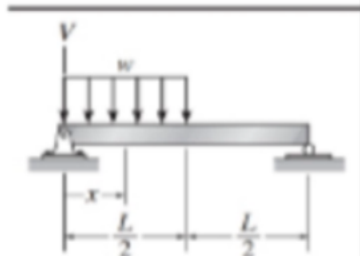
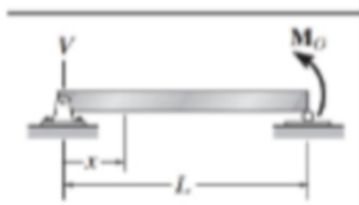
Then use

$\sum F_x = 0$ $\xrightarrow{+}$

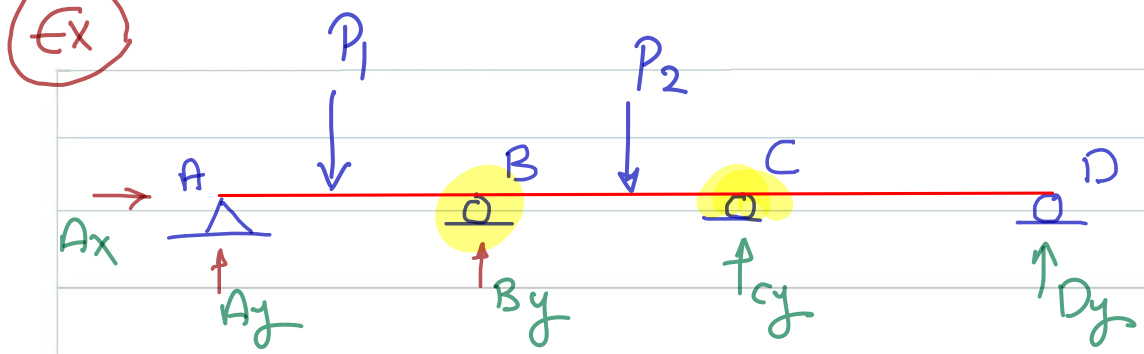
$\sum F_y = 0$ $\uparrow +$

$\sum M = 0$ $\circlearrowleft +$

Beam Deflections and Slopes

Loading	v + ↑	θ + ↰	Equation + ↑ + ↰
	$v_{max} = -\frac{PL^3}{3EI}$ at $x = L$	$\theta_{max} = -\frac{PL^2}{2EI}$ at $x = L$	$v = \frac{P}{6EI}(x^3 - 3Lx^2)$
	$v_{max} = \frac{M_O L^2}{2EI}$ at $x = L$	$\theta_{max} = \frac{M_O L}{EI}$ at $x = L$	$v = \frac{M_O}{2EI}x^2$
	$v_{max} = -\frac{wL^4}{8EI}$ at $x = L$	$\theta_{max} = -\frac{wL^3}{6EI}$ at $x = L$	$v = -\frac{w}{24EI}(x^4 - 4Lx^3 + 6L^2x^2)$
	$v_{max} = -\frac{PL^3}{48EI}$ at $x = L/2$	$\theta_{max} = \pm\frac{PL^2}{16EI}$ at $x = 0$ or $x = L$	$v = \frac{P}{48EI}(4x^3 - 3L^2x)$, $0 \le x \le L/2$
		$\theta_L = -\frac{Pab(L+b)}{6LEI}$ $\theta_R = \frac{Pab(L+a)}{6LEI}$	$v = -\frac{Pbx}{6LEI}(L^2 - b^2 - x^2)$ $0 \le x \le a$
	$v_{max} = -\frac{5wL^4}{384EI}$ at $x = \frac{L}{2}$	$\theta_{max} = \pm\frac{wL^3}{24EI}$	$v = -\frac{wx}{24EI}(x^3 - 2Lx^2 + L^3)$
		$\theta_L = -\frac{3wL^3}{128EI}$ $\theta_R = \frac{7wL^3}{384EI}$	$v = -\frac{wx}{384EI}(16x^3 - 24Lx^2 + 9L^3)$ $0 \le x \le L/2$ $v = -\frac{wL}{384EI}(8x^3 - 24Lx^2 + 17L^2x - L^3)$ $L/2 \le x \le L$
	$v_{max} = -\frac{M_O L^2}{9\sqrt{3}EI}$	$\theta_L = -\frac{M_O L}{6EI}$ $\theta_R = \frac{M_O L}{3EI}$	$v = -\frac{M_O x}{6EIL}(L^2 - x^2)$

Ex



actual beam

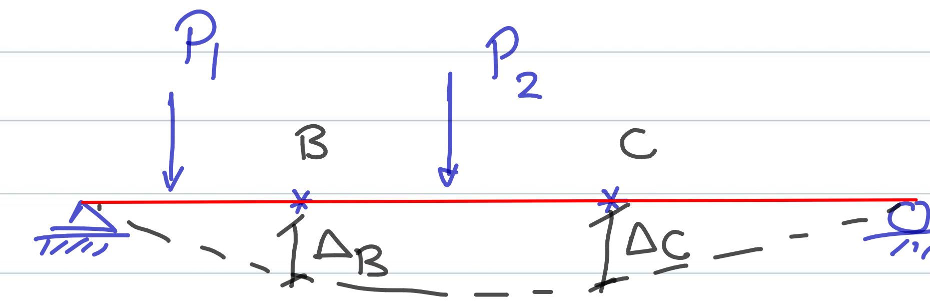
5-unknowns

3-Eq's

2nd-degree

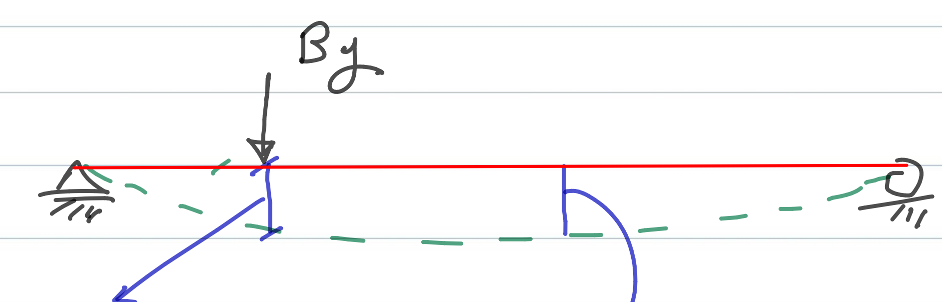
2-redundant

@ B & C



Primary Structure

+



Refundent By apply

$\Delta'_{BB} = B_y F_{BB}$ $\Delta'_{CB} = B_y F_{CB}$

Refundent Cy applied

$\Delta'_{BC} = C_y F_{BC}$ $\Delta'_{CC} = C_y F_{CC}$

$\Delta_B + \Delta'_{BB} + \Delta'_{BC} = 0$ (1)

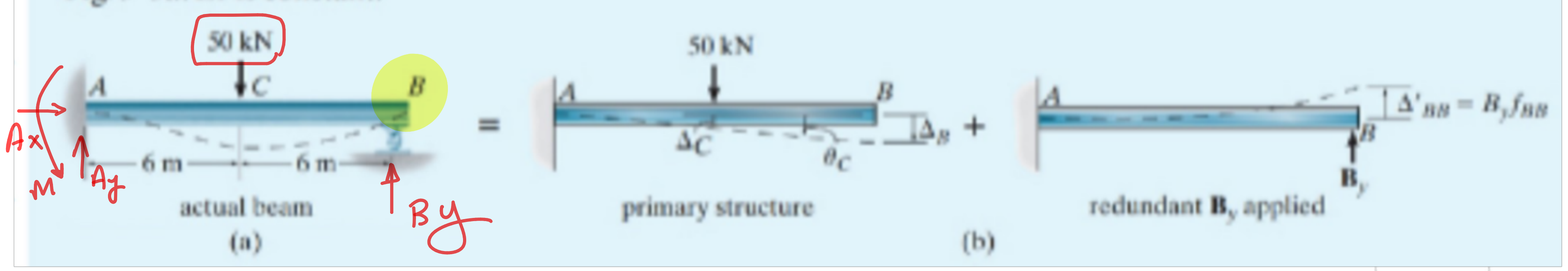
$\Delta_C + \Delta'_{CB} + \Delta'_{CC} = 0$ (2)

$\Delta'_{CB} = \Delta'_{BC}$

$F_{CB} = F_{BC}$

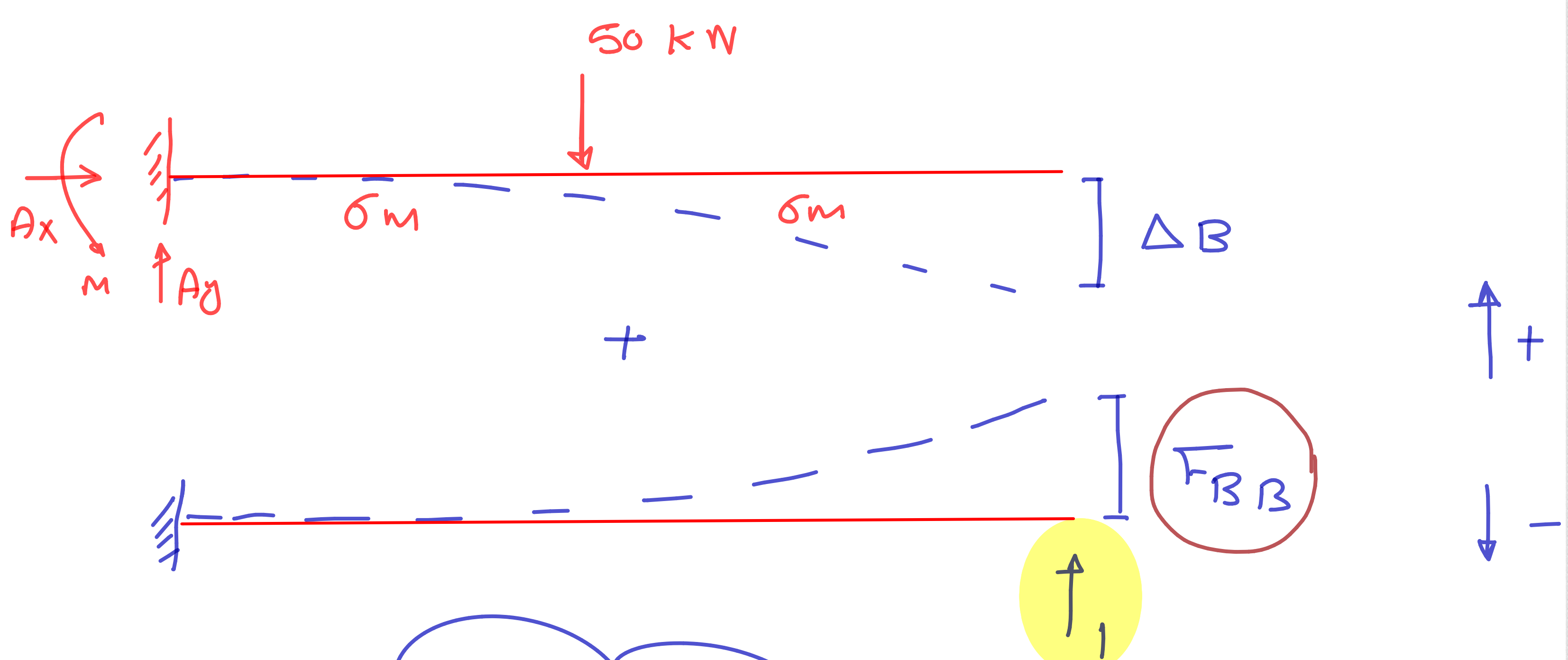
EXAMPLE 9.1

Determine the reaction at the roller support B of the beam shown in Fig. 9–9a. EI is constant.



4 - unknowns
3

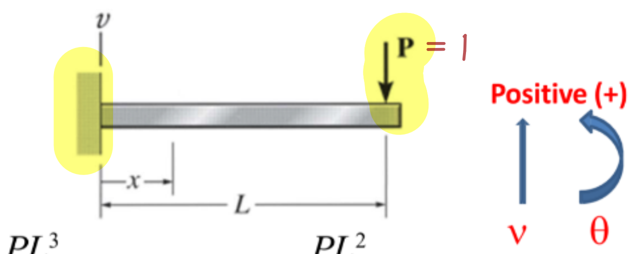
1- degree
Redundant @ By



$0 = -\Delta_B + \Delta_B'$

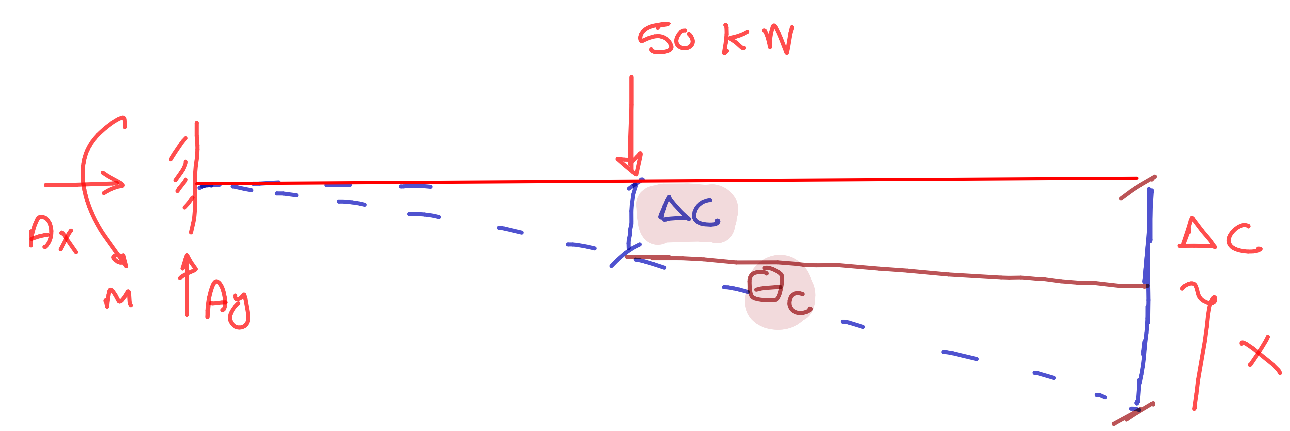
$0 = -\Delta_B + B_y F_{BB}$

$$v = \frac{P}{6EI}(x^3 - 3Lx^2)$$



$at\ x = L \qquad v_{max} = -\frac{PL^3}{3EI} \qquad \theta_{max} = -\frac{PL^2}{2EI}$

$$F_{BB} = \frac{PL^3}{3EI} = \frac{1 * 12^3}{3EI} = \frac{576}{EI}$$



$$\Delta_C = \frac{PL^3}{3EI} = \frac{50 * 6^3}{3EI} = \frac{3600}{EI}$$

$$\theta_C = \frac{PL^2}{2EI} = \frac{50 * 6^2}{2EI} = \frac{900}{EI}$$

$$\theta_C = \frac{x}{6} \Rightarrow x = \theta_C * 6$$

$$\Delta_B = \Delta_C + \theta_C * 6 = \frac{3600}{EI} + \frac{900}{EI} * 6$$

$$= \frac{9000}{EI}$$

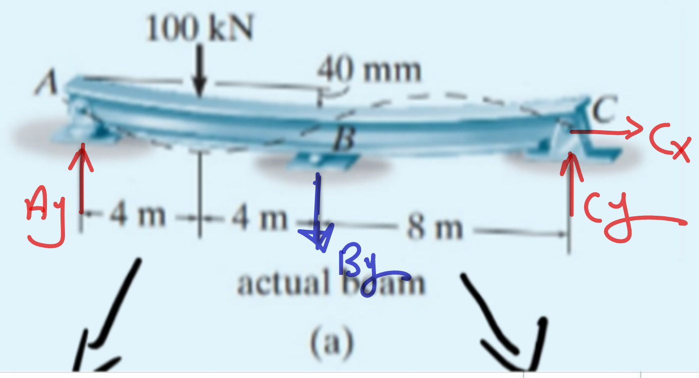
$$0 = -\frac{9000}{EI} + B_y \frac{576}{EI}$$

$B_y = 15.6$ kN

EXAMPLE 9.2

Draw the shear and moment diagrams for the beam shown in Fig. 9–10*a*. The support at *B* settles 40 mm. Take $E = 200$ GPa, $I = 500(10^6)$ mm^4.

40 mm



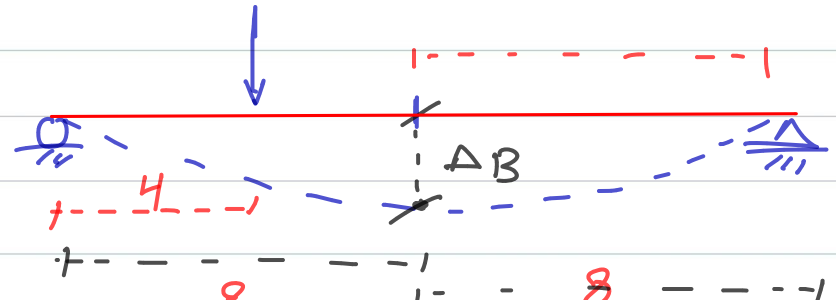
=

4 - unknowns
3 - Equations

1st - degree
⇓
1 - redundant @ B
By

Primary

100 KN

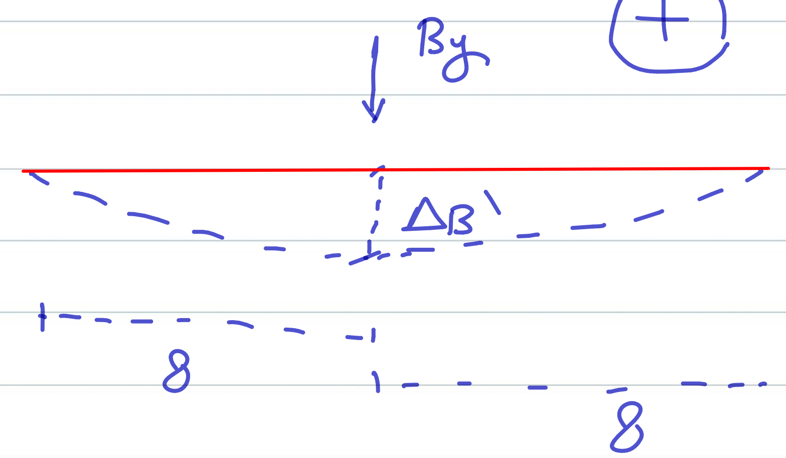


ΔB

↓+

Redundant
By applied

By
+

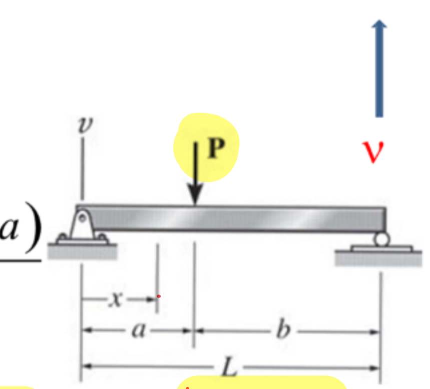


Compatibility Eq^n

$$0.04 = \Delta_B + B_y F_{BB} \Rightarrow (1)$$

$$v = -\frac{Pbx}{6LEI}(L^2 - b^2 - x^2), \quad 0 \le x \le a$$

$$\theta_L = -\frac{Pab(L+b)}{6LEI} \qquad \theta_R = \frac{Pab(L+a)}{6LEI}$$



$x = 8$ $\quad a = 12$ $\quad b = 4$

$$\Delta_B = \frac{100 * 4 * 8}{6 * 16 * EI}(16^2 - 4^2 - 8^2)$$

$$= \frac{5866.7}{EI}$$

$$F_{BB} = \frac{PL^3}{48EI} = \frac{1 * 16^3}{48EI} = \frac{85.33}{EI}$$

in Eq^n (1)

$$0.04 = \frac{5866.7}{EI} + B_y * \frac{85.33}{EI}$$

$$0.04 * 200 * 10^9 * 500 * 10^6 * 10^{-12}$$

$$= 5866.7 + 85.33 B_y$$

⇓

$$B_y = -21.88 \text{ kN}$$

$B_y = 21.88$ kN ↑

$\Sigma M_C = 0$ ↺ → A_y ✓

$\Sigma F_y = 0$ → C_y

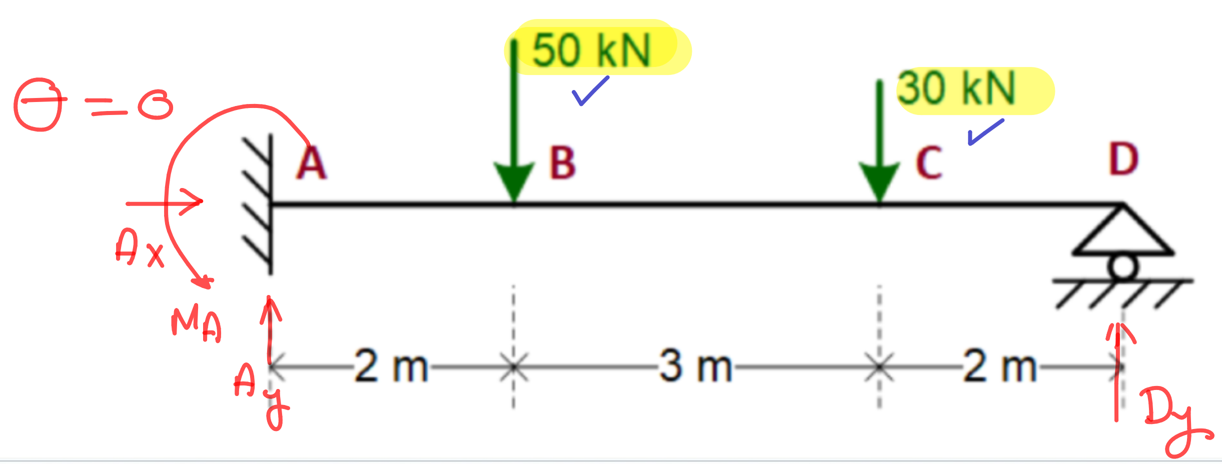
SFD
BMD

Solve the following exercises using the **Force Method**.

For the beams of the figures:

(1) Find all the support reactions.
(2) Draw the shear force diagram.
(3) Draw the bending moment diagram.

Exercise 1.



4- unknowns
3- Eq's

1st - degree
⇓
1 - redundant @ (A) (M_A)

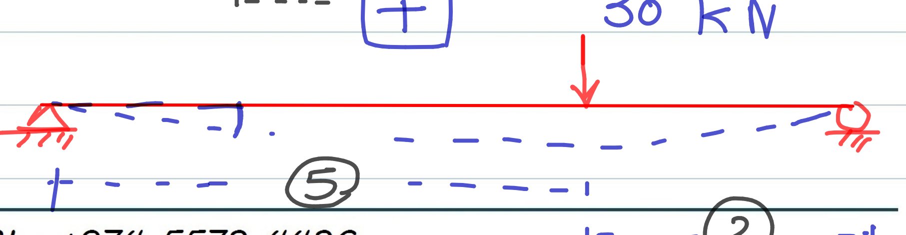
Primary structure

θ_{A_1}

a = 2
b = 5

+

θ_{A_2}



a = 5
b = 2

Redundant
MA applied

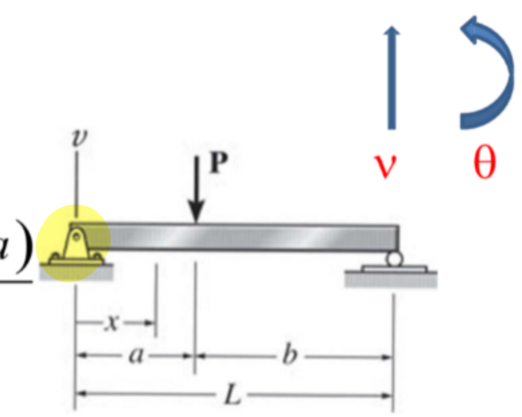
Compatibility Eq's

$$0 = \theta_{A_1} + \theta_{A_2} + \theta'_A$$

$$0 = \theta_{A_1} + \theta_{A_2} + M_A\,\alpha_{AA} \Rightarrow (1)$$

$$v = -\frac{Pbx}{6LEI}\left(L^2 - b^2 - x^2\right), \quad 0 \le x \le a$$

$$\theta_L = -\frac{Pab\left(L+b\right)}{6LEI} \qquad \theta_R = \frac{Pab\left(L+a\right)}{6LEI}$$



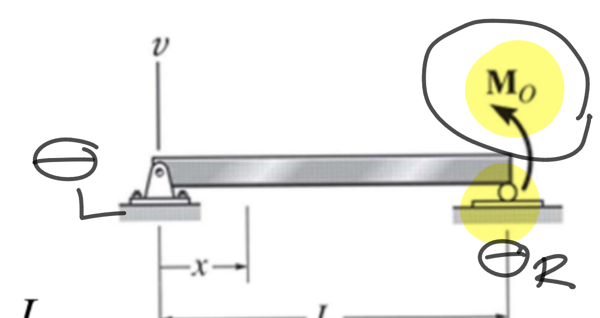
$$\theta_{A_1} = \frac{50 * 2 * 5\,(7+5)}{6 * 7\,EI} = \frac{142.86}{EI} \ \text{kN-m}^3$$

$$\theta_{A_2} = \frac{30 * 5 * 2\,(7+2)}{6 * 7\,EI} = \frac{64.28}{EI}$$

$$v = \frac{M_0 x}{6EIL}\left(x^2 - 3Lx + 2L^2\right)$$

$$v_{max} = -\frac{M_0 L^2}{9\sqrt{2}EI}$$

$$\theta_L = -\frac{M_0 L}{6EI} \qquad \theta_R = \frac{M_0 L}{3EI}$$



$$\alpha_{AA} = \frac{M_0 L}{3EI}$$

$$\alpha_{AA} = \frac{M_o L}{3EI} = \frac{1 * 7}{3EI} = \frac{2.33}{EI}$$

$$0 = \theta_{A_1} + \theta_{A_2} + M_A \, \alpha_{AA} \Rightarrow 1$$

$$0 = \frac{142.86}{\cancel{EI}} + \frac{64.28}{\cancel{EI}} + M_A * \frac{2.33}{\cancel{EI}}$$

$$M_A = -88.78 \ kN \cdot m$$

$$M_A = 88.78 \ kN \cdot m$$