Fundamental principles of traffic flow

DR DEEPTI MULEY

CVEN-461-L01

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Source: https://www.123rf.com/photo_11786145_motorway-with-light-traffic.html

Source: https://coronaviruslink.com/a-positive-from-coronavirus-pandemic-less-traffic-especially-in-these-major-cities.html

Source: https://qatar.yallamotor.com/car-news/abu-dhabi-police-extends-discount-period-on-traffic-fines-by-50--for-an-additional-three-months-6666

Learning outcomes

- Become familiar with the different elements of traffic flow
- Become familiar with interrelationships between various elements of traffic flow
- Become familiar with the fundamental principles of traffic flow models, gap and gap acceptance, and queuing

Study of traffic flow

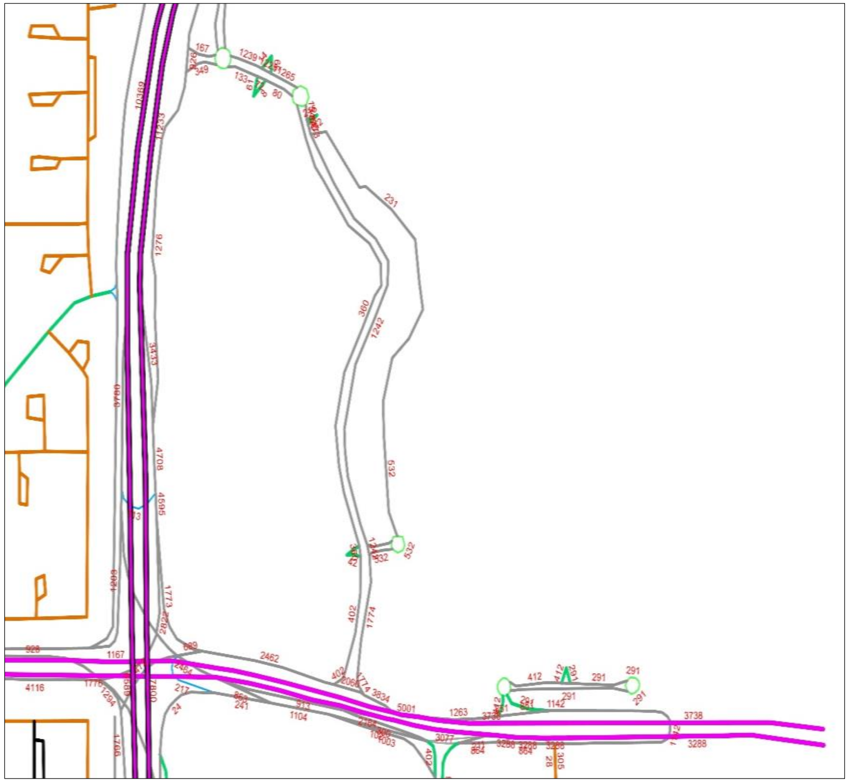
- Mathematical way to describe traffic movements
- Basic elements: Queuing, car following, gap acceptance, lane changing
- Useful in conducting operational analysis of the facility
- Used in simulation for designing new systems and scenario testing

Microscopic traffic simulation

Software: VISSIM, AIMSUN, Paramics

https://www.youtube.com/watch?v=W7ZUqDNWoYs

Examples of flow bundle analysis



Traffic flow elements

- Flow
- Density
- Speed
- Headway

Time-Space diagram

Traffic flow elements: Flow (q)

Rate of vehicles passing a reference point on highway during given period. (n) (T)

Expressed as {Vec/hr}

$$q = \frac{n}{T} * 3600 \quad \text{Veh/hr} \qquad (6.1)$$

where

n = the number of vehicles passing a point in the roadway in T sec

q = the equivalent hourly flow

Average Annual Daily Traffic (AADT), Average Annual Weekday Traffic (AAWT), Average Daily Traffic (ADT), Average Weekday Traffic (AWT)

Traffic flow elements: Density (k)

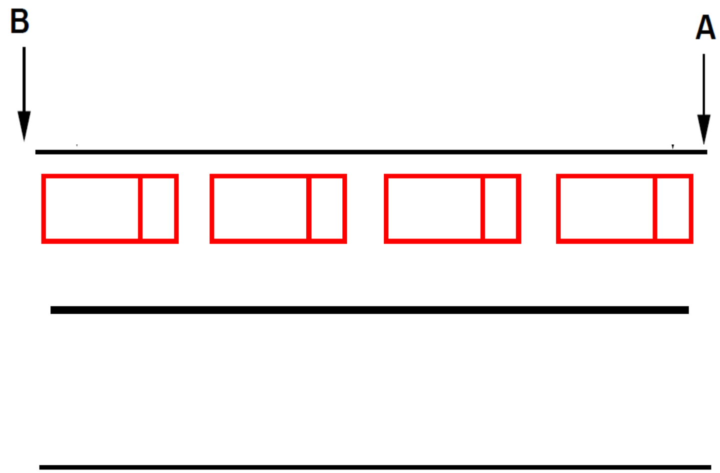
Number of vehicles traveling over a unit length of highway at an instant in time

$k = n / L$

Where *n* = no of vehicles in given length

L = length of section considered

Expressed as veh/KM



Traffic flow elements: Speed (u or v)

Distance traveled by a vehicle during a unit of time

Expressed as km/hr or m/s

Time mean speed

Space mean speed

The time mean speed is always higher than the space mean speed

Page 255-256

Traffic flow elements: Speed (u or v)

Time mean speed: arithmetic mean of the speeds of vehicles passing a point on a highway during an interval of time.

$$\bar{u}_t = \frac{\sum u_i}{n} \qquad (6.2)$$

where

n = number of vehicles passing a point on the highway
u_i = speed of the ith vehicle (m/sec)

Space mean speed: harmonic mean of the speeds of vehicles passing a point on a highway during an interval of time

$$\bar{u}_s = \frac{n}{\sum \frac{1}{u_i}} \qquad (6.3)$$

$$= \frac{nL}{\sum t_i}$$

where

$\bar{u}_s$ = space mean speed (m/sec)
n = number of vehicles
t_i = the time it takes the *i*th vehicle to travel across a section of highway (sec)
u_i = speed of the *i*th vehicle (m/sec)
L = length of section of highway (m)

Traffic flow elements: Headway

Page 256

Space/gap/distance between two vehicles

Time headway (h)

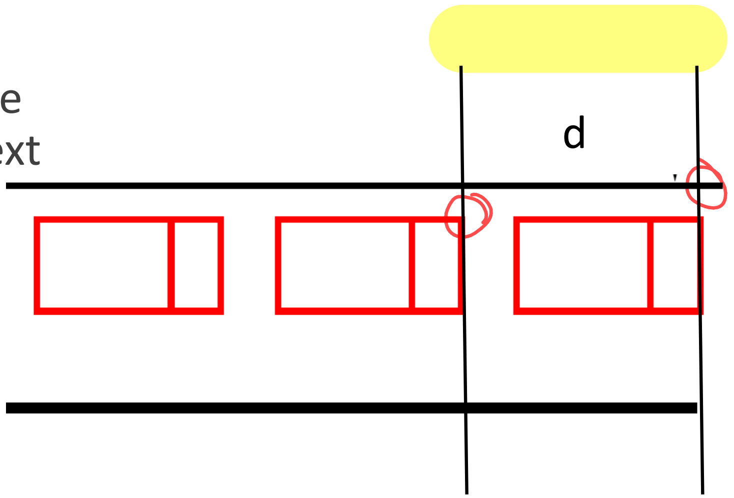
Difference between the time the front of a vehicle arrives at a point and the time the front of the next vehicle arrives at that same point

Usually expressed in seconds, h= $(t_2 - t_1)$

Space headway (d)

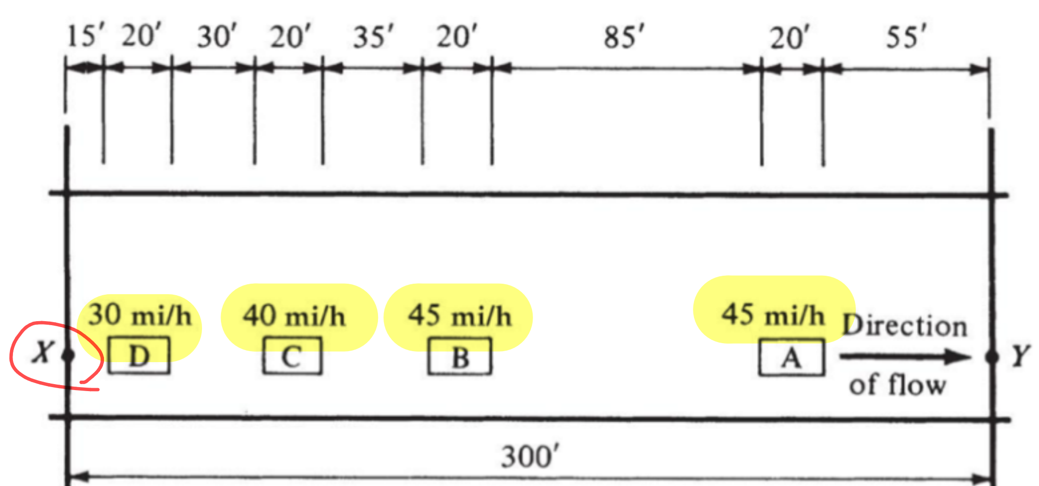
Distance between the front of a vehicle and the front of the following vehicle

Usually expressed in meters



Example 6.1

Figure shows vehicles traveling at constant speeds on a two-lane highway between sections X and Y with their positions and speeds obtained at an instant of time by photography. An observer located at point X observes the four vehicles passing point X during a period of T sec. The velocities of the vehicles are measured as 70, 70, 65, and 50 (km/h), respectively. Calculate the flow, density, time mean speed, and space mean speed.



$$q = \frac{n * 3600}{T}$$

$$= \frac{4 * 3600}{10} = 1440 \text{ Vec/hr}$$

$$K = n/L = \frac{4}{90} * 1000$$

$$= 44.4 \text{ Vec/KM}$$

$$U_t = \frac{\sum U_i}{n}$$

$$= \frac{70 + 70 + 65 + 50}{4} = 64 \text{ KM/hr}$$

$$U_s = \frac{n}{\sum 1/U_i}$$

$$= \frac{4}{1/70 + 1/70 + 1/65 + 1/50}$$

$$= 62.54 \text{ KM/hr}$$

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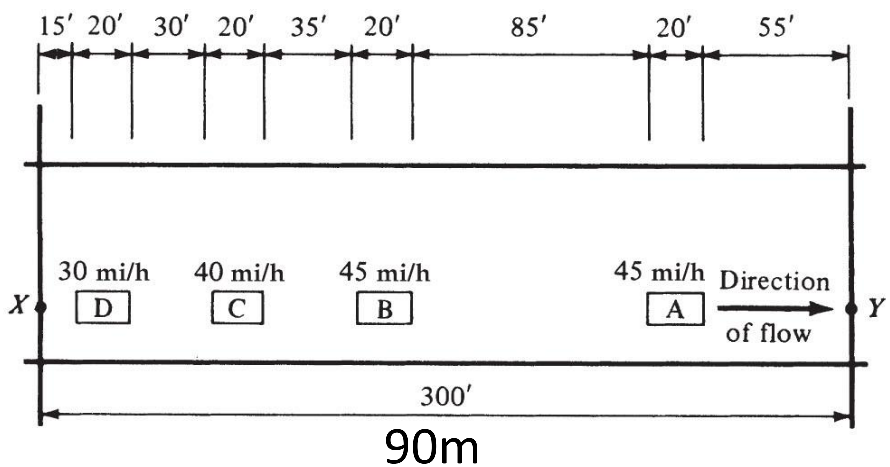


Figure 6.3 Locations and Speeds of Four Vehicles on a Two-Lane Highway at an Instant of Time

Example 6.1

Page 257

Solution:

Solution: The flow is calculated by

$$q = \frac{n \times 3600}{T} \tag{6.6}$$

$$= \frac{4 \times 3600}{T} = \frac{14{,}400}{T} \text{ veh/h}$$

If T = 60 sec,
q = 240 veh/h

With L equal to the distance between X and Y (m), density is obtained by

$$k = \frac{n}{L}$$

$$= \frac{4}{90} \times 1000 = 44.4 \text{ veh/km}$$

Page 257

Example 6.1

The time mean speed is found by

$$u_t = \frac{1}{n}\sum_{i=1}^{n} u_i$$

$$= \frac{50 + 65 + 70 + 70}{4} = 64 \text{ km/h}$$

The space mean speed is found by

$$\bar{u}_s = \frac{n}{\sum_{i=1}^{n}(1/u_i)}$$

$$= \frac{nL}{\sum_{i=1}^{n} t_i}$$

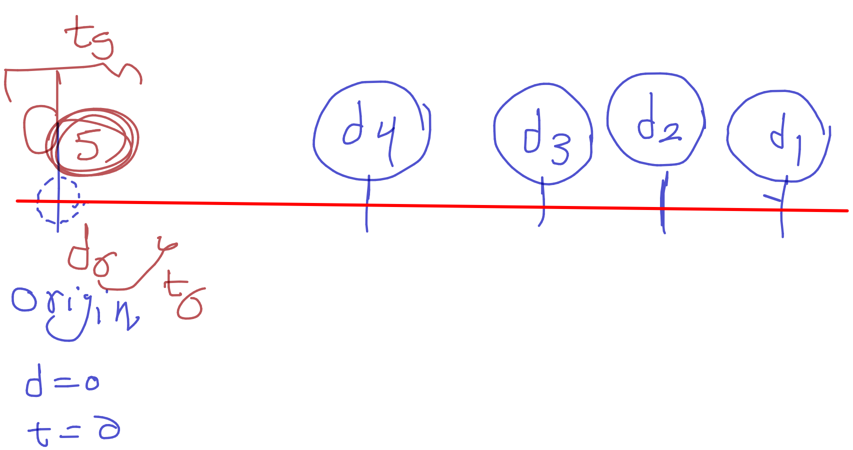
62.54 km/h

Time – space diagram

- shows trajectory of vehicles in the form of two- dimensional plot
- tool to understand movement of vehicles
- can be plotted for a single vehicle as well as multiple vehicles

$v = D/t$

Slope = Vertical / HZ



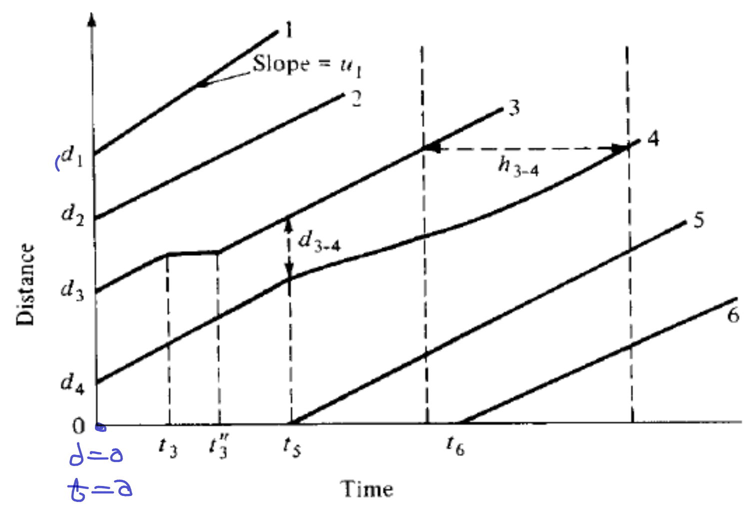


Figure 6.1 Time-Space Diagram

Fundamental Diagram of Traffic Flow

Page 262

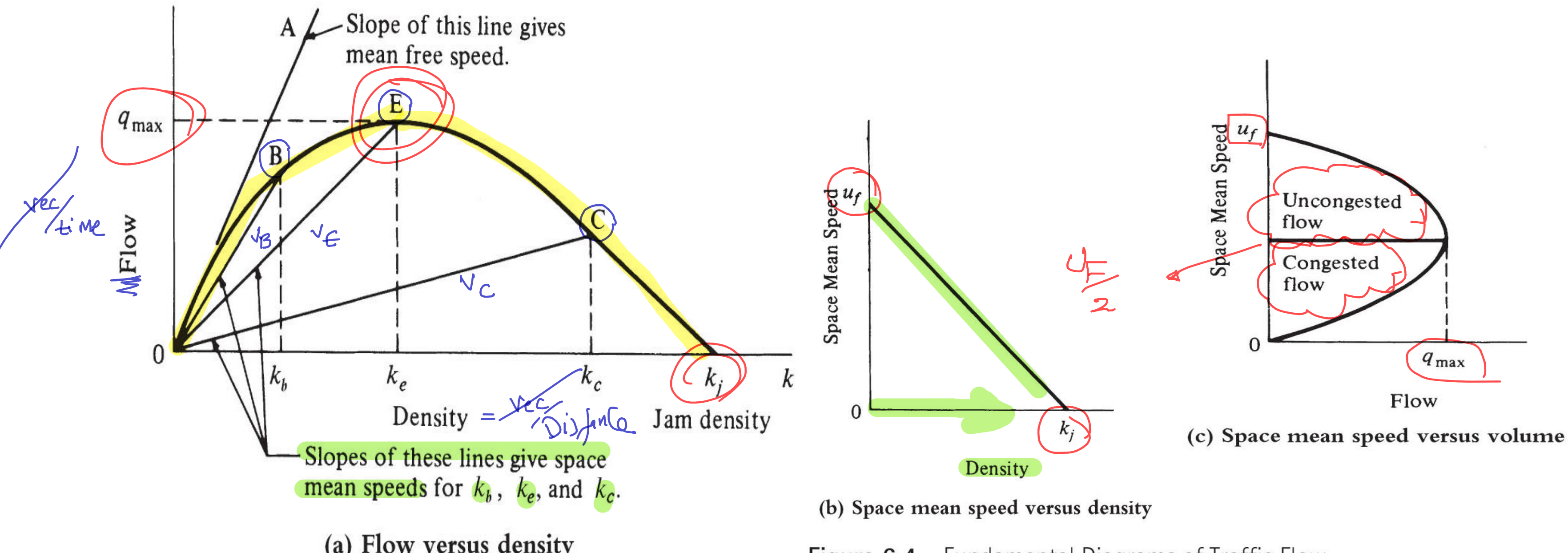


Figure 6.4 Fundamental Diagrams of Traffic Flow

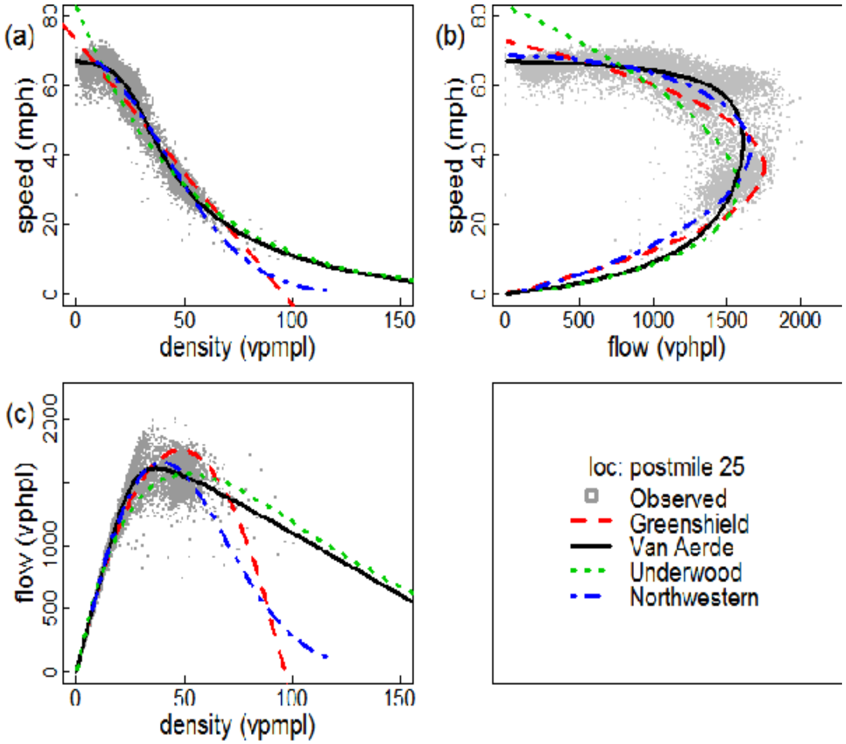
$V_A > V_B > V_E > V_C$ of K increase

Fundamental equation of traffic flow

Page 258

Flow = Density X Space Mean Speed

q = $k \times u_s$



Source: https://www.semanticscholar.org/paper/Estimating-Traffic-Flow-Rate-on-Freeways-from-Probe-Anuar-Habtemichael/ece7275a20ab1bc901bbe2055fd0bc417d665ad1/figure/1

Mathematical Relationships Describing Traffic Flow

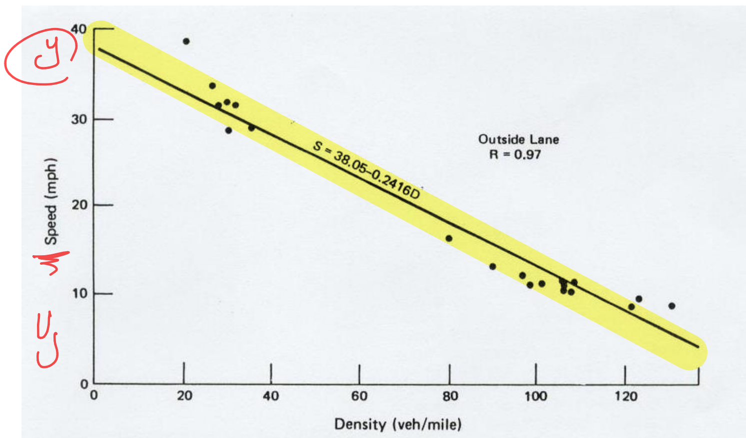
Page 263-273

- Macroscopic Approach: Considers traffic streams and develops algorithms that relate the flow to the density
  - Greenshields Model: linear relationship between speed and density
  - Greenberg Model: fluid-flow analogy, logarithmic relationship

- Microscopic Approach (Car-Following Theory): Considers spacing between consecutive vehicles and speeds of individual vehicles

Greenshield model (Linear relationship)

Page 261-262



$$y = a_0 + a_1 x$$

a b

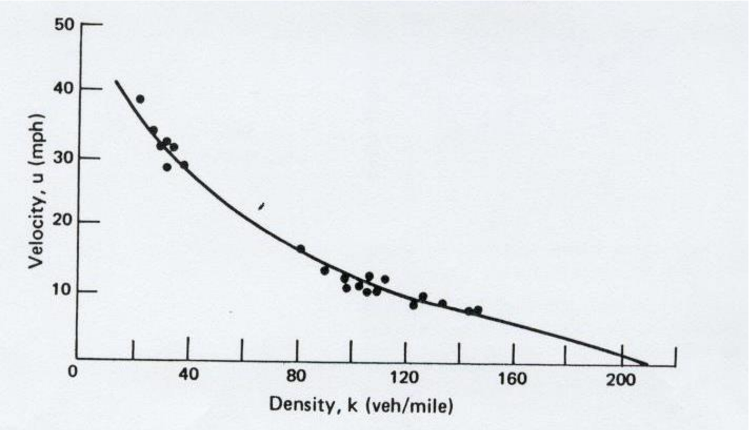
$$U_s = U_F - \frac{U_F}{k_j} k$$

Characteristics of the Greenshields model:

$$U_o = \frac{U_F}{2} \qquad k_o = \frac{k_j}{2} \qquad q_{max} = \frac{k_j U_F}{4}$$

This model works for all $k = 0$ to $k = k_j$

Greenburg model (Logarithmic relationship)



$$\bar{u}_s = c \ln \frac{k_j}{k}$$

Characteristics of the Greenburg model:

$$u_o = c \qquad \ln \frac{k_j}{k_o} = 1 \quad \text{or} \quad \frac{k_j}{k_o} = e^1 \quad \text{or} \quad k_0 = \frac{k_j}{e}$$

$$q_{\max} = u_o k_o$$

This model does not work near k = 0

Example 6.7

The data shown below were obtained by time-lapse photography on a highway. Use regression analysis to fit these data to the Greenshields model and determine (a) the mean free speed, (b) the jam density, (c) the capacity, and (d) the speed at maximum flow.

Speed (km/h) y_i	Density (veh/km) x_i
14.2	85
24.1	70
30.3	55
40.1	41
50.6	20
55.0	15

$y = a_0 + a_1 x$ ($a_0 \to a$, $a_1 \to b$)

$U_s = U_f - \frac{U_f}{K_j} K$

x_i	y_i	x_i^2	$x_i y_i$
85	14.2	7225	1207
70	24.1	4900	1687
55	30.3	3025	1666.5
41	40.1	1681	1644.1
20	50.6	400	1012
15	55	225	825
Σ 286	214.3	17456	8041.6

$$b = \frac{n \sum x_i y_i - \sum x_i \sum y_i}{n \sum x_i^2 - (\sum x_i)^2}$$

$$= \frac{6 * 8041.6 - 286 * 214.3}{6 * 17456 - (286)^2}$$

$$b = -0.56845$$

$$a = \bar{y} - b\bar{x}$$

$$= \frac{\sum y_i}{n} - b \frac{\sum x_i}{n}$$

$$= \frac{214.3}{6} - (-0.56845)\frac{286}{6}$$

$$a = 62.8124$$

(a) $U_F = a = 62.8$ km/hr

(b) $b = \frac{-U_F}{K_j}$

$0.56845 = \frac{62.8}{K_j}$

$K_j = 110.49 \ Vec/KM$

(c) $q = K \times U_S$

$= K\left(U_F - \frac{U_F}{K_j} K\right)$

$q = 62.8 K - \frac{62.8}{110.49} K^2$

@ q_{Max} $\frac{dq}{dK} = 0$

$\frac{dq}{dK} = 62.8 - 1.136 K = 0$

$K = 55.25$ when $q = q_{Max}$

$q_{Max} = 62.8 * 55.25 - \frac{62.8}{110.49} (55.25)^2$

$= 1735 \ Vec/hr$

or

$q_{Max} = \frac{K_j \cdot U_F}{4} = 1735 \ vec/hr$

(d)

$U_S = U_F - \frac{U_F}{K_j} (K)$ when $q = q_{Max}$

$= 62.8 - \frac{62.8}{110.49} * 55.25$

$= 31.4 \ KM/hr$

(*)

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Example 6.7

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14.2	85
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55.0	15

Example 6.7

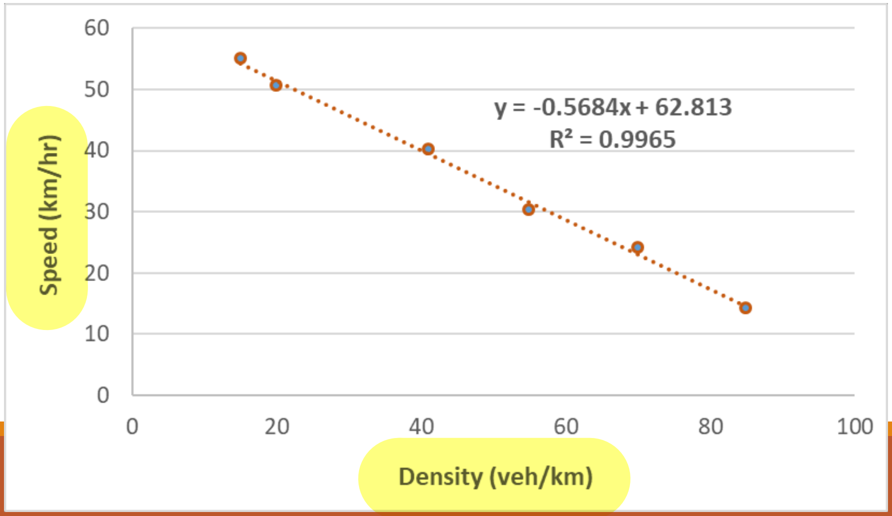
Page 299

The linear regression analysis can be applied to the given data to estimate parameters in Greenshields' model of traffic flow. Greenshields' model

$$\bar{u}_s = u_f - \frac{u_f}{k_j} k$$

Linear regression analysis yields values of

a = 62.8124 and b = –0.56845



Example 6.7

(a) Mean free flow speed, $u_f = a = 62.8$ km/h

(b) Jam density, k_j

In the regression model, $b = u_f / k_j$

$b = 0.56845$

$k_j = 62.8 / 0.56845 = 110.49$

$k_j = 110$ veh/km

Page 299

Example 6.7

(c) Capacity, q_{max}

Capacity occurs at maximum flow. State flow in terms of density.

$$q = k \times \bar{u}_s = k\left(u_f - \frac{u_f}{k_j} k \right)$$

$$q = 62.8k - (62.8/110.49)k^2$$

Take the derivative and set equal to zero to maximize flow; solve for density.

$$0 = 62.8 - 1.1368k$$

$$k = 55.25 \text{ when } q = q_{max}$$

Solve for *q*

$$q_{max} = 62.8(55.25) - 0.5684(55.25)^2$$

$$q_{max} = 1735 \text{ veh/h}$$

Example 6.7

Page 299

(d) Speed at maximum flow

Solve for mean speed using k when $q = q_{max}$

$$\bar{u}_s = u_f - \frac{u_f}{k_j} k$$

$u_s = 62.8 – 0.5684(55.25)$

$u_s = 31.4$ km/h

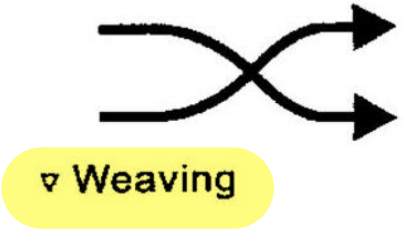
Page 299

Traffic maneuvers

- Merging: Vehicle in one traffic stream joins another stream moving in the same direction (i.e., on-ramps)

- Diverging: Vehicle leaves the traffic stream (i.e., off-ramps)

- Weaving: Vehicle first merges into a stream and crosses that stream, then merges into a second stream in the same direction

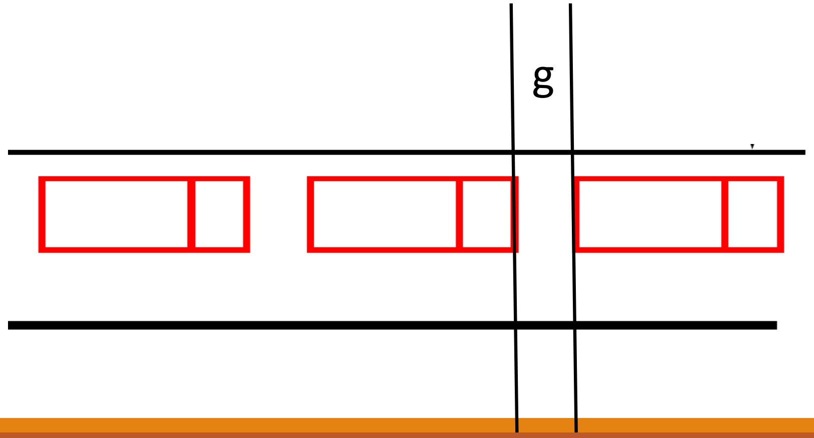


Page 286

Gap (g) and gap acceptance

- Gap: Distance between the rear bumper of a vehicle and the front bumper of the following vehicle
- A driver who intends to merge must evaluate the available gaps to determine which gap (if any) is large enough to accept the vehicle

Driver feels that the merging maneuver can be completed safely to join the new stream. This phenomenon is called gap acceptance



Page 291

Queuing theory

- Important to study congestion during peak hours
- Applied on queues in expressway on-ramps and off-ramps, signalized and unsignalized intersections, and on arterials
- Mathematical algorithms to study formation of queues, delays due to queuing, service method, and discharge of queues

Shock Waves in Traffic Streams

- Indicates transition between two traffic states
- Can be seen using time space diagram

The figure below describes the phenomenon of backups and queuing on a highway (bottleneck condition):

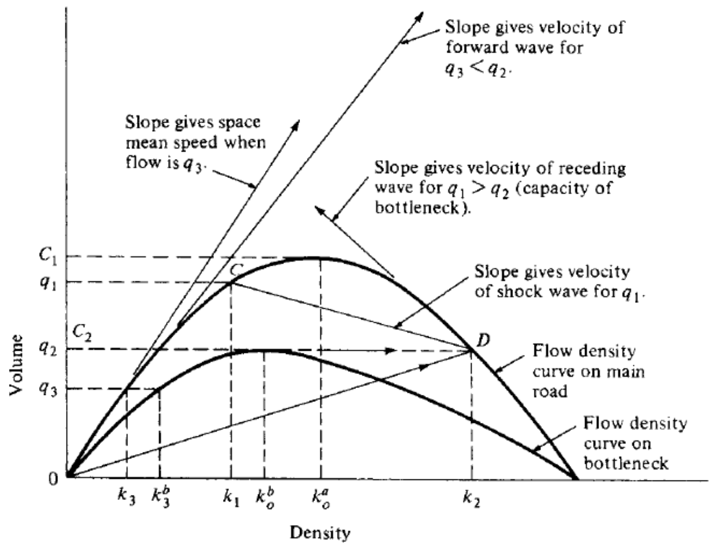


Figure 6.8 Kinematic and Shock Wave Measurements Related to Flow-Density Curve

Page 272-273

Types of Shock Waves

Frontal Stationary: Occurs when the capacity suddenly reduces to zero (i.e., closed lanes because of an accident)

Backward Forming: Formed when the capacity is reduced below the demand flow rate. Results in an upstream queue at the bottleneck (i.e., signal indication on an interchange becomes red)

Backward Recovery: Formed when the demand flow rate becomes less than the capacity of the bottleneck (i.e., signal indication on an interchange becomes green)

Rear Stationary and Forward Recovery: Occurs when the demand flow rate upstream of a bottleneck is first higher than the capacity of the bottleneck and then the demand flow rate reduces to the capacity of the bottleneck (i.e., peak hours in a tunnel)

Traffic flow insights: traffic waves and jams

Shock waves and phantom traffic jam

https://www.youtube.com/watch?v=19S3OdK6710

https://www.youtube.com/watch?v=goVjVVaLe10

Tutorial

- What is the difference between headway and gap?
- Are time mean speed and space mean speed same?
- Name different types of shock waves formed in traffic flow.

Reference

Nicholas J Garber, Lester A. Hoel, Traffic and Highway Engineering, SI Version, 5th edition, 2014, CL Engineering

Upcoming lecture

Capacity and Level of Service (LOS) Analysis

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LOS