Surveying for Construction / CVEN 270

Topic 2

Units, Precision, Accuracy and Errors

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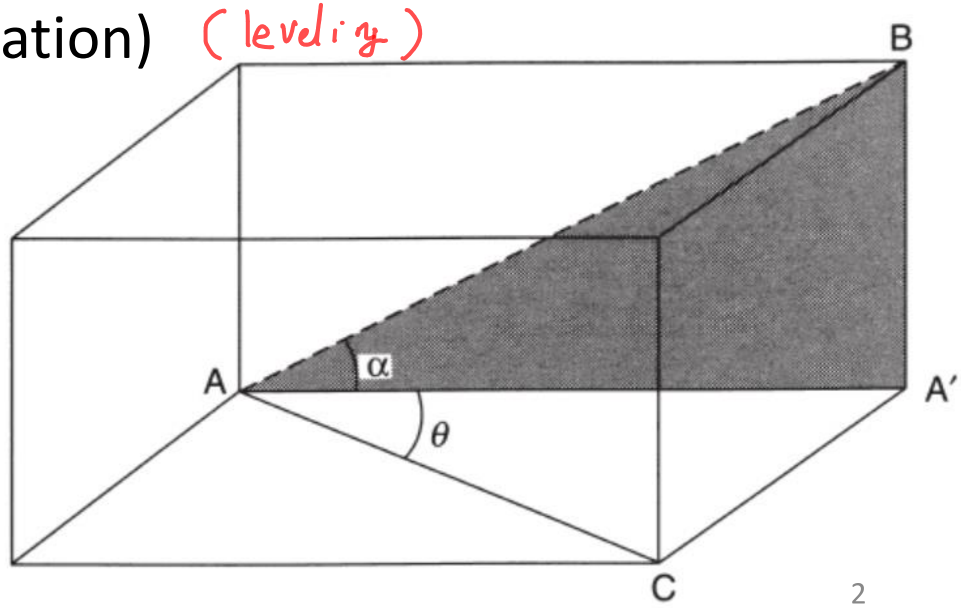
Surveying measurements

- Measuring distances and angles from a known reference are fundamental surveying operations.
- Five common types of surveying measurements

1. Distance
    1.1. Horizontal distance + Error
    1.2. Vertical distance (elevation) (leveling)
    1.3. Slope distance
2. Angle
    2.1. Horizontal angle
    2.2. Vertical angle



Measurement units

In surveying, the most commonly employed units are for:

- Length or distance
- Area
- Volume
- Angle

Distance

English Prefixes

US - system

- ✓ 1 foot = 12 inches
- ✓ 1 yard = 3 feet
- ✓ 1 inch = 2.54 cm
- ✓ 1 rod = 1 pole = 1 perch = 16.5 feet
- ✓ 1 vara = 33 inches
- ✓ 1 Gunter’s Chain = 66 feet = 100 links = 4 rods
- ✓ 1 mile = 5280 feet = 80 Gunter’s Chain
- ✓ 1 fathom = 6 ft

Metric Prefixes

SI - system

- ✓ 1 kilometer (km) = 1000 meters (m)
- ✓ 1 millimeter (mm) = 0.001 meter
- ✓ 1 centimeter (cm) = 0.01 meter
- ✓ 1 decimeter (dm) = 0.1 meter
- ✓ 1 m = 10 dm = 100 cm = 1000 mm

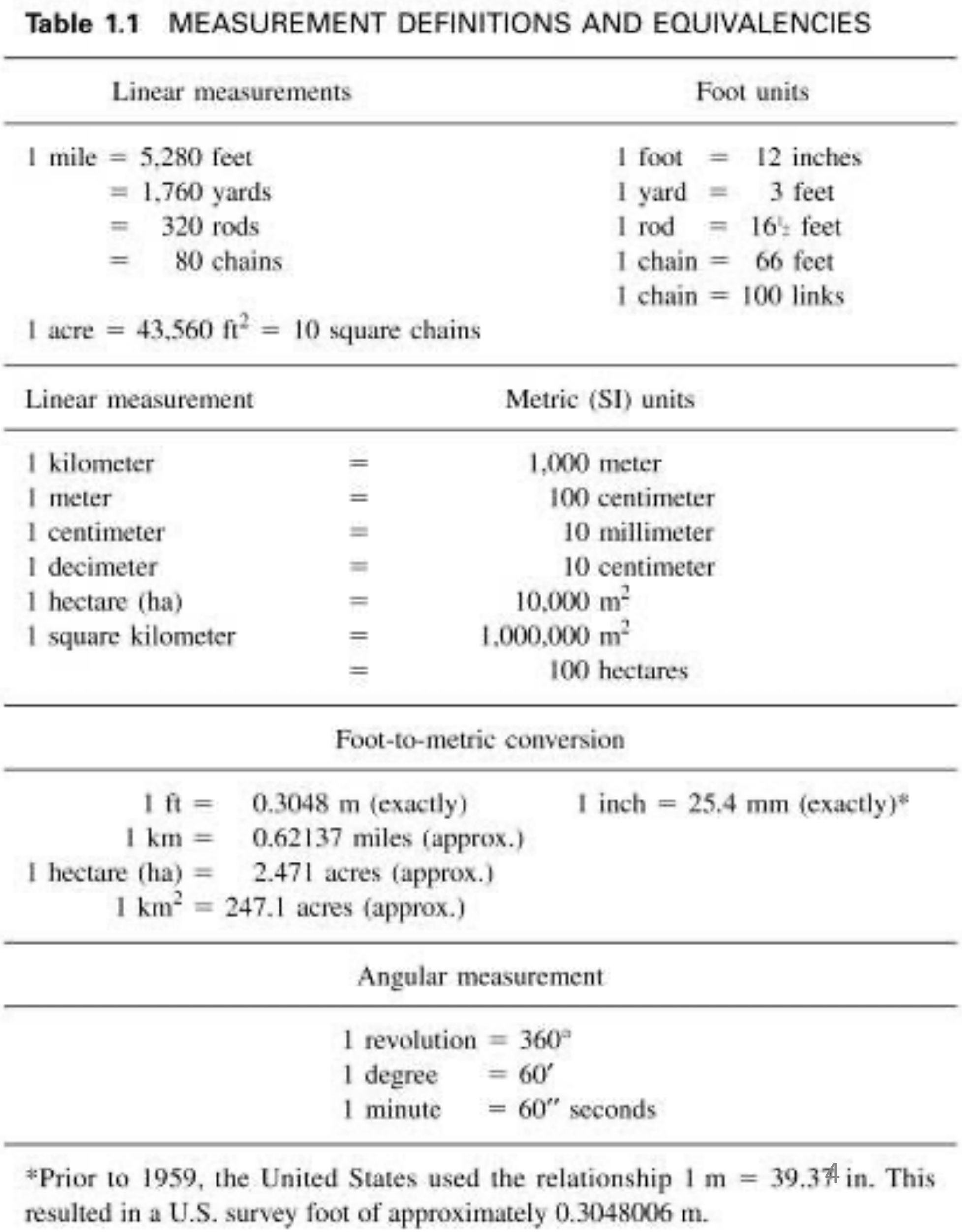


Table 1.1 MEASUREMENT DEFINITIONS AND EQUIVALENCIES

Linear measurements	Foot units
1 mile = 5,280 feet	1 foot = 12 inches
= 1,760 yards	1 yard = 3 feet
= 320 rods	1 rod = 16½ feet
= 80 chains	1 chain = 66 feet
	1 chain = 100 links
1 acre = 43,560 ft² = 10 square chains	

Linear measurement		Metric (SI) units
1 kilometer	=	1,000 meter
1 meter	=	100 centimeter
1 centimeter	=	10 millimeter
1 decimeter	=	10 centimeter
1 hectare (ha)	=	10,000 m²
1 square kilometer	=	1,000,000 m²
	=	100 hectares

Foot-to-metric conversion

1 ft = 0.3048 m (exactly)   1 inch = 25.4 mm (exactly)*

1 km = 0.62137 miles (approx.)

1 hectare (ha) = 2.471 acres (approx.)

1 km² = 247.1 acres (approx.)

Angular measurement

1 revolution = 360°

1 degree = 60′

1 minute = 60″ seconds

*Prior to 1959, the United States used the relationship 1 m = 39.37 in. This resulted in a U.S. survey foot of approximately 0.3048006 m.

Area

Metric Prefixes

- ✓ 1 hectare = 10000 m^2

English Prefixes

- ✓ 1 acre = 43560 ft^2 = 66*660 ft^2 = 10 square chain
- ✓ 1 ft^2 = 144 $inch^2$
- ✓ 1 $yard^2$ = 9 ft^2

Volume

- **Metric Prefixes**

✓1 m^3 = 1000000 cm^3 = 1000 liters

- **English Prefixes**

✓1 $yard^3$ = 27 ft^3

✓1 acre-foot = 43,560 ft^3

Angles

Degree < Minutes < Seconds

rad

Units

- ✓ Degrees (°), Minutes (‘), and Seconds (“)
- ✓ Radian (rad or ^{c})
- ✓ Gradian (^{g} or gon)
- ✓ *Turn/ revolution*

$1^\circ = 60$ Minutes

$1 \text{ Min} = 60^\circ$ Sec

$1^\circ = 1 * \pi/180$ rad

$1 \text{ rad} = 1 * \frac{180}{\pi}^\circ$

$1 \text{ rev} = 2\pi = 360^\circ$

2π rad = 360° = 400 gon = 1 Turn

1 rad ≈ 57° 17’ 44.8” ≈ 57.2958°

0.01745 rad ≈ 1°

Measurement Errors

true ~~value~~

observed value

average (Mean)

$$e_i = \underset{(x_i)}{\text{observed value}} - \underset{(x)}{\text{true value}}$$

$$v_i = e_i = x_i - \bar{x} \longrightarrow \text{Mean}$$

$$\text{Mean} = \bar{x} = \frac{\sum x_i}{n}$$

What is error?

Definition of error (e_i): difference between an observed value (x_i) for a quantity and its true value (x)

e_i = observed value − true value = $x_i - x$

The true value can never be determined and thus the true error as well. Thus the true value is replaced with the mean (average) of several measurements ($\hat{x}$). Then, the error is estimated as:

$v_i = x_i - \bar{x}$

$\bar{x} = \frac{\sum x_i}{n}$

v_i is called "residual"

$\hat{x}$ is called "most probable value"

For reading

Sources of Errors

- ***Natural errors:*** Due to variations in wind, temperature, humidity, atmospheric pressure, atmospheric refraction, gravity, and magnetic declination etc.
  E.g., Length of a steel tape can be varied with changes in temperature

- ***Instrumental errors:*** Due to any imperfection in the instrument

- ***Personal errors (human errors):*** Due to the mistakes by humans.

Blunders

- ***A blunder (or gross error)*** is a significant, <u>unpredictable mistake</u> caused by human error due to carelessness, miscommunication, fatigue, or poor judgment.

E.g.,

  - Improperly or incorrectly leveling the surveying instrument
  - Entering an incorrect control point number in the data collector

- Entering wrong values, such as, “15.13” as “50.13”

Types of Errors

Reading

- ***Systematic error (bias):*** caused by the equipment, observation methods, and certain environmental factors. As long as system conditions remain constant, the systematic errors will remain constant.

  → Can be mathematically adjusted

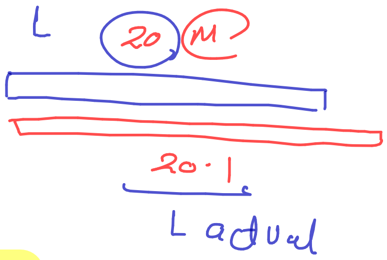
- Can be minimized by;

  Properly leveling survey instrument and targets

  Entering the appropriate environmental correction factors in the data collector

  Periodically calibrating the surveying equipment

Types of Errors



E.g.: Correcting error in the length of a tape:

$$Correct\ length\ of\ a\ line = \text{Measured length} * \frac{\text{actual tape length}}{\text{Nominal tape length}}$$

$$Correct\ area =$$

Example:

a line was found to be 376.4 m when measured with a tape of 20 m length. However after checking the tape length was found to be 20.04 m. What is the correct length of the line?

Answer: 377.2 m

Example:

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$L_{Measured} = 376.4$

$L_{Tape - NoMial} = 20$ M

$L_{Tape - actual} = 20.04$ M

$$L_{Corrected} = 376.4 * \frac{20.04}{20}$$

$= 377.2$ M

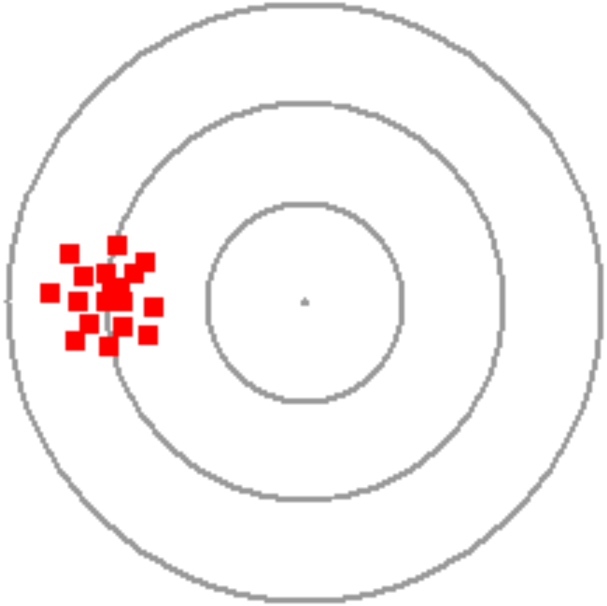
Types of Errors

Reafing

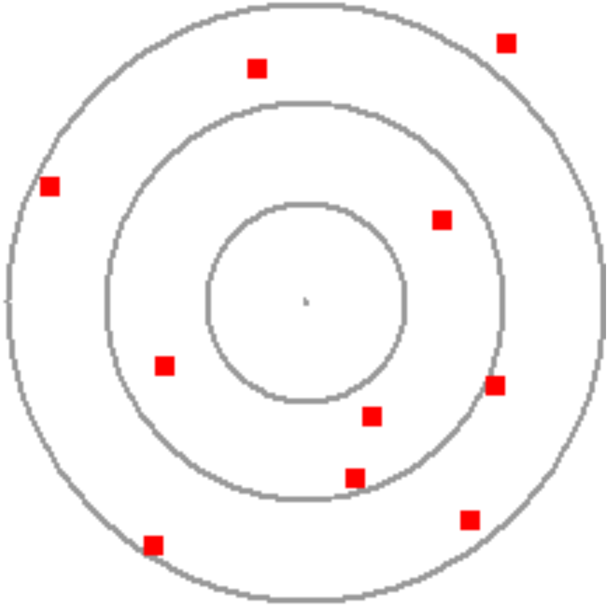
- ***Random error* (Accidental error):** are those remain in measured values after mistakes and systematic errors have been eliminated. They are caused by factors beyond the control of the observer.
- Obey the laws of probability.
- Correction factors cannot be computed and applied
- Can be minimized by;
  - Better instruments
  - Properly designed field procedures
  - **Making repeated measurements**

Types of Errors

Reading



Systematic Error



Random Error

Occurrence of Random Errors

Probability and Statistics

$$\bar{y} = \frac{\sum y_i}{n}$$

$$\upsilon = y_i - \bar{y}$$

$$\sigma^2 = \frac{\sum (y_i - Y)^2}{n}$$

$$s^2 = \frac{\sum (y_i - \bar{y})^2}{n-1}$$

y_i : Observation *i*

$\bar{y}$: Sample average

Y : Population average(unknown)

υ : Residual

σ^2 : Population variance

s^2 : Sample variance

Probability and Statistics

- Standard deviation / standard error (SE) of one measurement:

$$\nu = x_i - \bar{x} = y_i - \bar{y}$$

$$\sigma = \frac{\nu}{\sqrt{n-1}} = \sqrt{\frac{\sum (y_i - \bar{y})^2}{n-1}}$$

- Standard error (SE) of the mean (RMS):

$$\epsilon_M = \hat{\sigma} = \frac{\sigma}{\sqrt{n}} = \sqrt{\frac{\sum (y_i - \bar{y})^2}{n(n-1)}}$$

Occurrence of Random Errors

Example

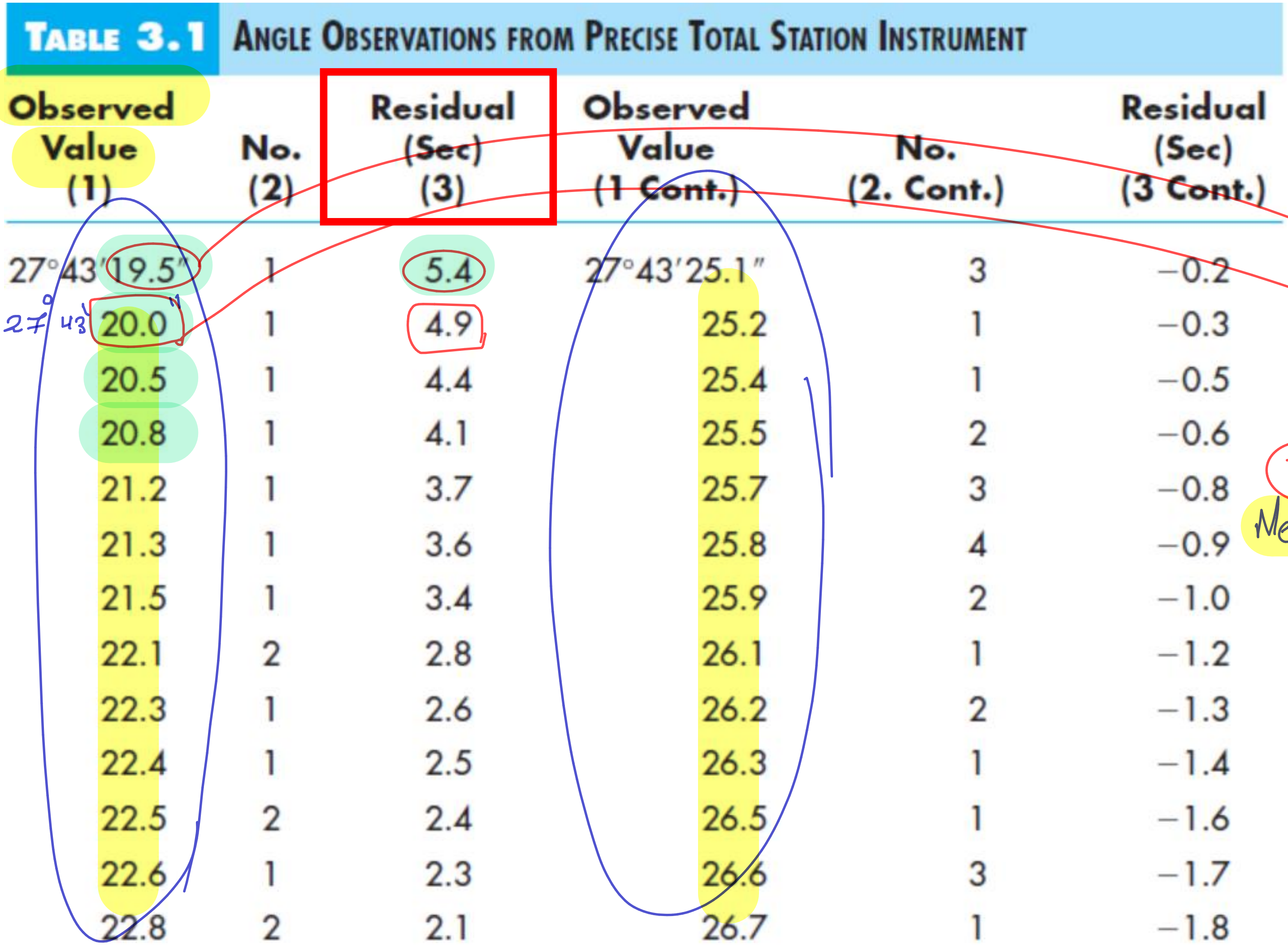


TABLE 3.1 ANGLE OBSERVATIONS FROM PRECISE TOTAL STATION INSTRUMENT

Observed Value (1)	No. (2)	Residual (Sec) (3)	Observed Value (1 Cont.)	No. (2. Cont.)	Residual (Sec) (3 Cont.)
27°43′19.5″	1	5.4	27°43′25.1″	3	−0.2
20.0	1	4.9	25.2	1	−0.3
20.5	1	4.4	25.4	1	−0.5
20.8	1	4.1	25.5	2	−0.6
21.2	1	3.7	25.7	3	−0.8
21.3	1	3.6	25.8	4	−0.9
21.5	1	3.4	25.9	2	−1.0
22.1	2	2.8	26.1	1	−1.2
22.3	1	2.6	26.2	2	−1.3
22.4	1	2.5	26.3	1	−1.4
22.5	2	2.4	26.5	1	−1.6
22.6	1	2.3	26.6	3	−1.7
22.8	2	2.1	26.7	1	−1.8

$27^{\circ} 43'$ $20.0''$

$n = 100$

$$\bar{x} = \frac{\Sigma x_i}{n}$$

$$= \frac{2494}{100}$$

$$= 24.9''$$

Mean $\bar{x}$ $= 27^{\circ}\ 43'\ 24.9''$

$$v = x_i - \bar{x}$$

23.0	1	1.9	26.8	2	−1.9
23.1	2	1.8	26.9	1	−2.0
23.2	2	1.7	27.0	1	−2.1
23.3	3	1.6	27.1	3	−2.2
23.6	2	1.3	27.4	1	−2.5
23.7	2	1.2	27.5	2	−2.6
23.8	2	1.1	27.6	1	−2.7
23.9	3	1.0	27.7	2	−2.8
24.0	5	0.9	28.0	1	−3.1
24.1	3	0.8	28.6	2	−3.7
24.3	1	0.6	28.7	1	−3.8
24.5	2	0.4	29.0	1	−4.1
24.7	3	0.2	29.4	1	−4.5
24.8	3	0.1	29.7	1	−4.8
24.9	2	0.0	30.8	1	−5.9
25.0	2	−0.1	$\Sigma = 2494.0$	$\Sigma = 100$	

Mean = 2494.0/100 = 24.9″

Most Probable Value = 27°43′24.9″

Frequency table of residuals

Class interval: 0.7”

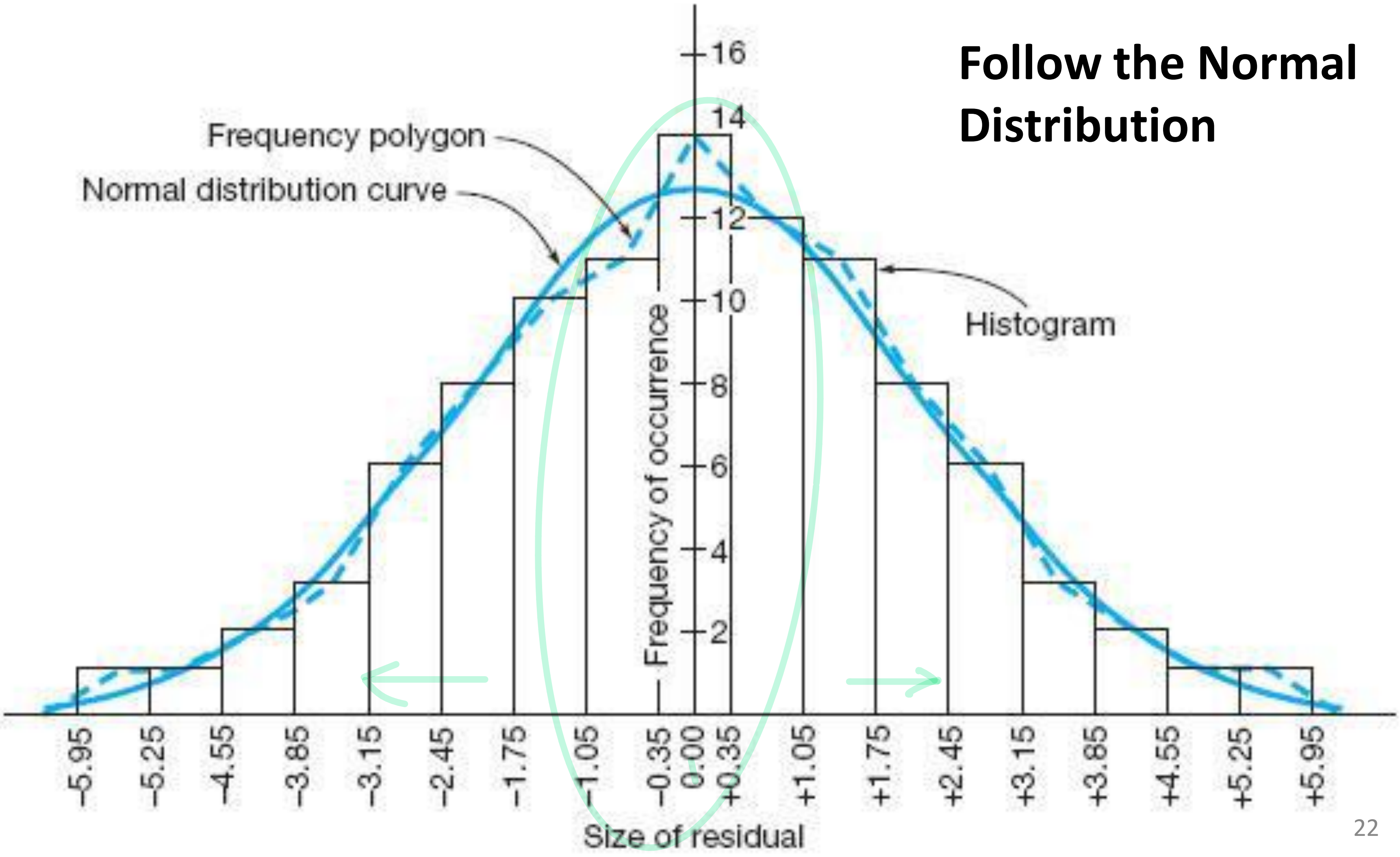
add 0.7

Histogram Interval (Sec)	Number of Residuals in Interval
−5.95 to −5.25	1
−5.25 to −4.55	1
−4.55 to −3.85	2
−3.85 to −3.15	3
−3.15 to −2.45	6
−2.45 to −1.75	8
−1.75 to −1.05	10
−1.05 to −0.35	11
−0.35 to +0.35	14
+0.35 to +1.05	12
+1.05 to +1.75	11
+1.75 to +2.45	8
+2.45 to +3.15	6
+3.15 to +3.85	3
+3.85 to +4.55	2
+4.55 to +5.25	1
+5.25 to +5.95	1
	$\Sigma = 100$

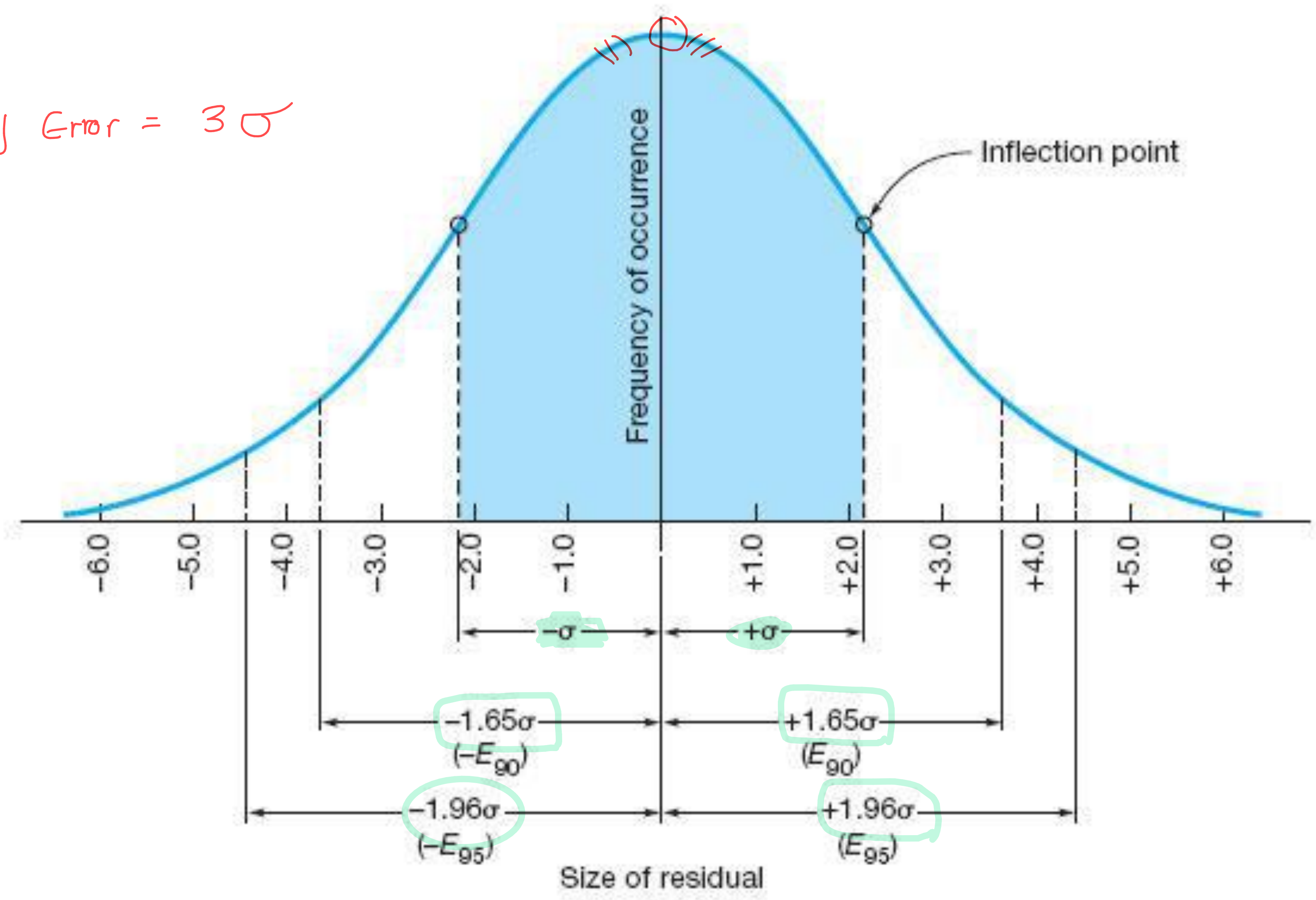
Histogram and frequency polygon of residuals

ERRorJ

Follow the Normal Distribution

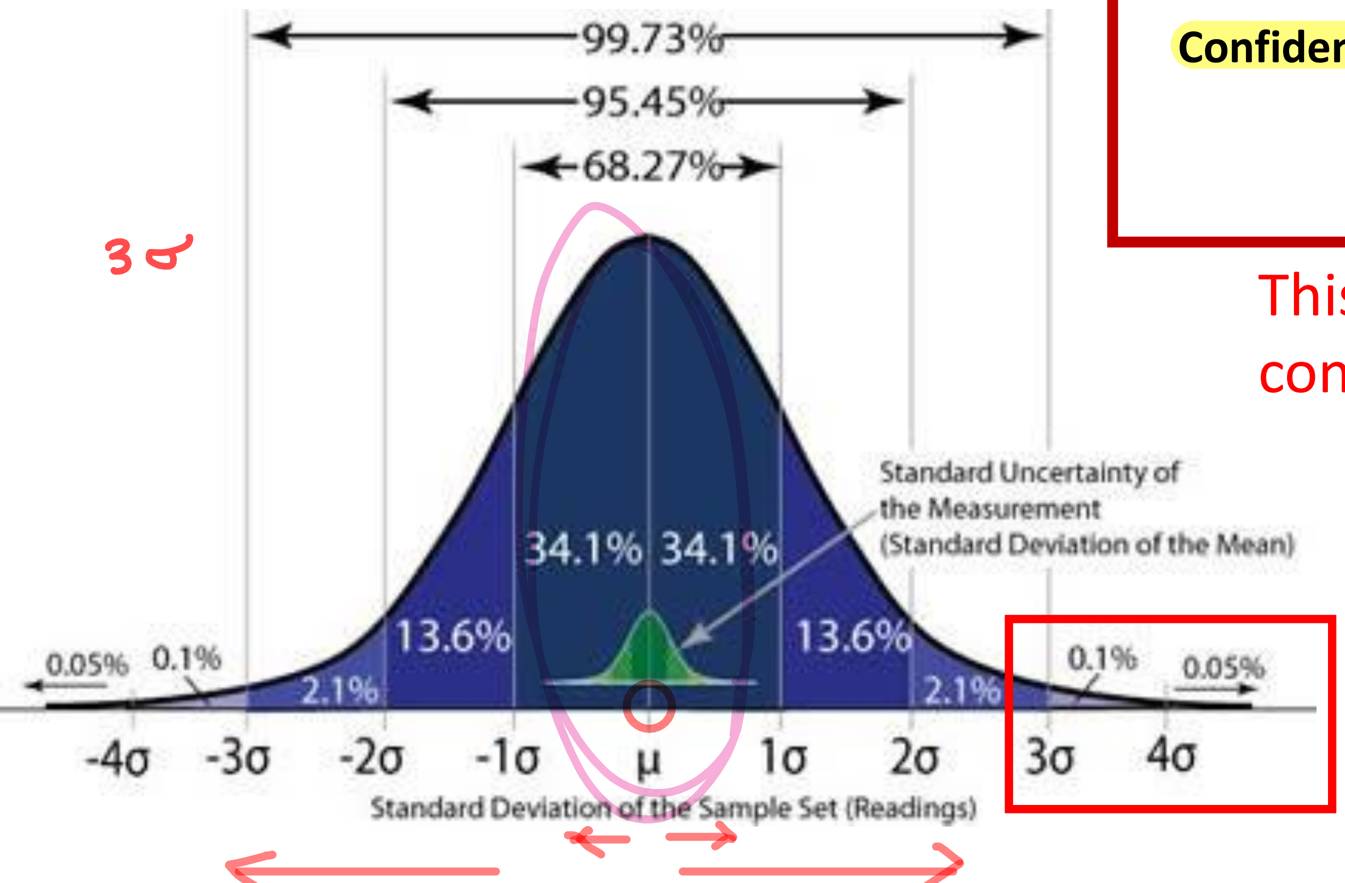


Max Error = 3σ



Confidence Interval

3σ



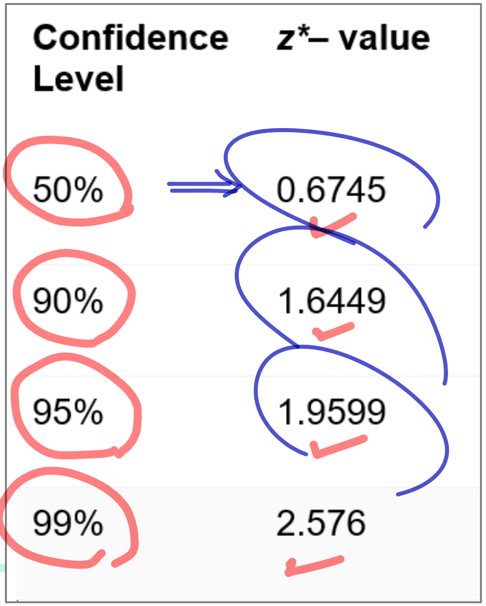
Confidence interval of an observation:

$$x = \bar{x} \pm z\sigma$$

Confidence interval of the mean:

$$x = \bar{x} \pm z\frac{\sigma}{\sqrt{n}}$$

This interval may contain the true value



Confidence Level	z*– value
50%	0.6745
90%	1.6449
95%	1.9599
99%	2.576

Most Probable Error = 50 % Error = ± 0.6745σ

Nowadays, surveyors are interested in the maximum error (90% ~ 95%) = (± 1.6449σ ~ ± 1.9599σ)

Example 1

Blunders = ??

Measurement (m)	First Iteration $v_i = d_i - \bar{d}$ (m)	Second Iteration $v_i = d_i - \bar{d}$ (m)
58.78	✓0.03	−0.03
58.83	✓0.08	0.02
58.80	✓0.05	−0.01
58.85	✓0.1	0.04
~~58.18~~	~~−0.57~~	Rejected
58.77	✓0.02	−0.04
58.79	✓0.04	−0.02
58.80	✓0.05	−0.01
58.81	✓0.06	0
58.82	✓0.07	0.01
58.79	✓0.04	−0.02
58.82	✓0.07	0.01

1st Iteration $n = 12$

$$\text{Mean} = \bar{y} = \frac{58.78 + 58.83 + \cdots}{12}$$

$$= 58.75 \text{ (M)}$$

$$\sigma = \sqrt{\frac{\Sigma (y_i - \bar{y})^2}{n-1}}$$

$$\sigma = \sqrt{\frac{0.03^2 + 0.08^2 + 0.05^2 + \cdots}{12-1}} = \pm 0.18$$

Estimated standard error of Mean

$$\hat{\sigma} = \frac{\sigma}{\sqrt{n}} = \frac{\pm 0.18}{\sqrt{12}} = \pm 0.05 \text{ M}$$

$$\text{Maximum Error} = 3\sigma = \pm 3 * 0.18$$

$$= \pm 0.54 \text{ (M)}$$

Reject Measurement = 58.18 M

2nd Iteration $n = 11$

$$\text{Mean} = \frac{58.78 + 58.83 + \cdots}{11} = 58.81$$

$$\sigma = \sqrt{\frac{0.03^2 + 0.02^2 + \cdots}{11-1}} = \pm 0.02$$

$$\text{Estimated Standard Error of Mean} = \frac{\pm 0.02}{\sqrt{11}} = \pm 0.006$$

$$\text{Max Error} = 3\sigma = \pm 3 * 0.02 = \pm 0.06 \text{ M}$$

No More Measured are rejected

Example 2

Determine the 50th and 95th percentile error in distance?

Measured Value (ft)	v	v²
152.93	+0.02	0.0004
153.01	+0.10	0.0100
152.87	-0.04	0.0016
152.98	+0.07	0.0049
152.78	-0.13	0.0169
152.89	-0.02	0.0004
Sum = 917.46		Sum = 0.0342

$n = 6$

$\Sigma x_i = 917.46$

$\bar{x} = \frac{\Sigma x_i}{n} = \frac{917.46}{6}$

$= 152.91$

$\sigma = \sqrt{\frac{\Sigma (x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{0.0342}{6-1}}$

$= 0.0824$

$E_{50} = \pm 0.6745 * \sigma$

$= \pm 0.6745 * 0.0824 = \pm 0.056$ ft

$E_{95} = \pm 1.9599\sigma = \pm 1.9599 * 0.0824$

$= \pm 0.1615$

What is the standard error of the mean and what is its 95th percentile confidence interval?

$E_M = \hat{\sigma} = \frac{\sigma}{\sqrt{n}}$

$= \frac{0.0824}{\sqrt{6}} = 0.0336$

$CI_M = X = \bar{x} \pm Z\, E_M$

$= 152.91 \pm 1.9599 * 0.0336$

$= 152.91 \pm 0.065$

$= (152.84 , 152.98)$