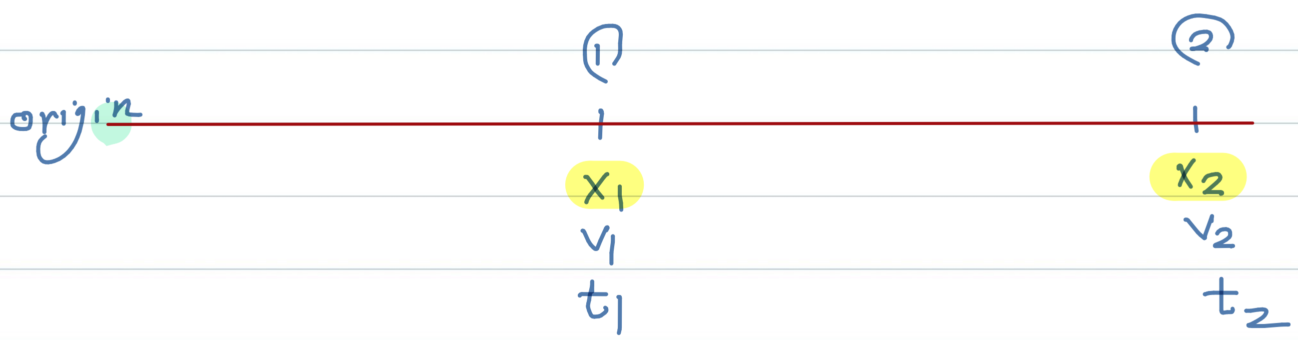
Kinematics of Particles

1 Chapter

Rectilinear Motion :-

- position
- velocity
- acceleration



Position :-

x_1 } Right of origin (+)

x_2 } left of origen (−)

Displacement :- (vector)

$$\Delta x = x_2 - x_1$$

+ →

← −

Distance travelled :- (scalar)

average velocity :- [Vector]

$$v_{av} = \frac{\Delta x}{\Delta t}$$

+ →

← −

Instantaneous velocity :- (vector)

$$v = \frac{dx}{dt}$$

+ →

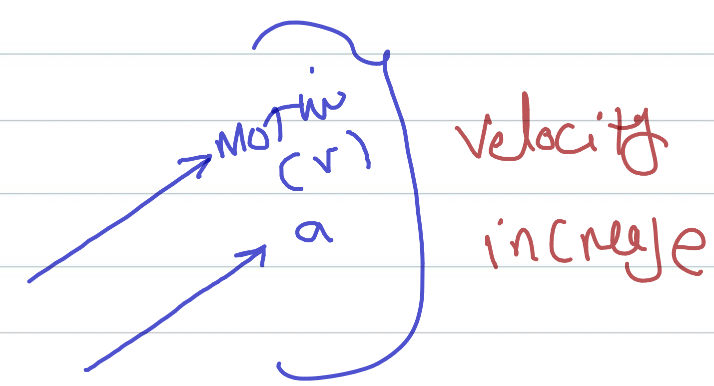
← −

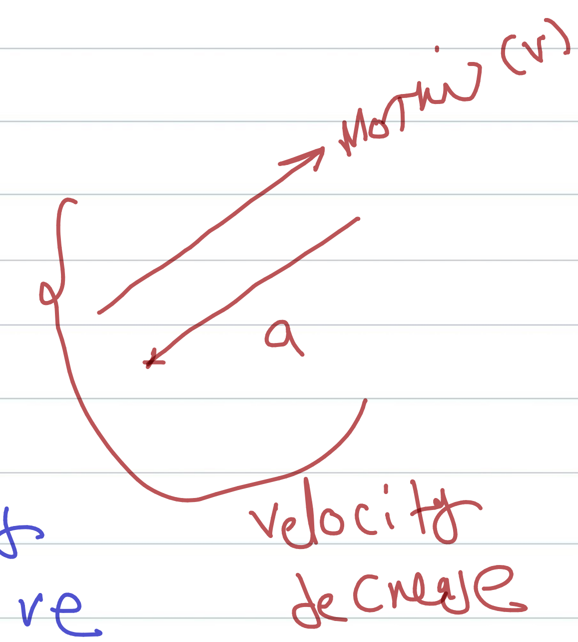
Instantaneous acceleration :- (vector)

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

+ →

← −





Speed = |v| } always Positive

$v = \frac{dx}{dt}$

$a = \frac{dv}{dt}$

derivative ↓ { x, v, a }

$$\int_{x_0}^{x} dx = \int_{0}^{t} v \, dt$$

$$\int_{v_0}^{v} dv = \int_{0}^{t} a \, dt$$

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PROBLEM 11.1

A snowboarder starts from rest at the top of a double black diamond hill. As he rides down the slope, GPS coordinates are used to determine his displacement as a function of time: $x = 0.5t^3 + t^2 + 2t$ where x and t are expressed in ft and seconds, respectively. Determine the position, velocity, and acceleration of the boarder when $t = 5$ seconds.

$$x = 0.5t^3 + t^2 + 2t$$

$$v = \frac{dx}{dt} = 1.5t^2 + 2t + 2$$

$$a = \frac{dv}{dt} = 3t + 2$$

@ t = 5 Sec

$$x = 0.5(5)^3 + (5)^2 + 2(5) = 97.5 \text{ Ft}$$

$$v = 1.5(5)^2 + 2(5) + 2 = 49.5 \text{ Ft/s}$$

$$a = 3(5) + 2 = 17 \text{ Ft/s}^2$$

Example Problem 1.1

- The curvilinear motion of a particle is represented by the equation :

$$s = 20t + 4t^2 - 3t^3$$

- What is the particle's initial velocity?

(A) 20 m/s

(B) 25 m/s

(C) 30 m/s

(D) 32 m/s

@ t=0

$$v = \frac{ds}{dt} = 20 + 8t - 9t^2$$

$$@\ t=0 \Rightarrow v = 20 + 8(0) - 9(0)^2$$

$$v = 20 \text{ m/s}$$

Example Problem 1.2

- The curvilinear motion of a particle is represented by the equation

$$s = 20t + 4t^2 - 3t^3$$

- What is the acceleration of the particle at time t=0?

(A) 2 m/s^2

(B) 3 m/s^2

(C) 5 m/s^2

(D) 8 m/s^2

$$a = \frac{dv}{dt} = 8 - 18t$$

$$@\ t=0 \Rightarrow a = 8 \text{ m/s}^2$$

Example Problem 1.3

- The curvilinear motion of a particle is represented by the equation

  $s = 20t + 4t^2 - 3t^3$
- What is the maximum speed reached by the particle?
  - (A) 21.8 m/s
  - (B) 27.9 m/s
  - (C) 34.6 m/s
  - (D) 48.0 m/s

$$S = 20t + 4t^2 - 3t^3$$

$$v = \frac{dS}{dt} = 20 + 8t - 9t^2$$

$$\boxed{a = \frac{dv}{dt} = 8 - 18t}$$

when $a = 0$ $\quad 8 - 18t = 0$

$$t = \frac{8}{18} = 0.444 \text{ sec}$$

$$\boxed{V_{Max} \text{ @ } a = 0}$$

$$V_{Max} = 20 + 8(0.444) - 9(0.444)^2$$

$$= 21.8 \text{ M/S}$$

Example Problem 11.1

- The position of a particle which moves along a straight line is defined by the relation

$$x = t^3 - 6t^2 - 15t + 40,$$

where x is expressed in meters and t in seconds.

- Determine
  - (a) the time at which the velocity will be zero,
  - (b) the position and distance traveled by the particle at that time,
  - (c) the acceleration of the particle at that time,
  - (d) the distance traveled by the particle from t=4 s to 6 s.

$$x = t^3 - 6t^2 - 15t + 40$$

$$v = \frac{dx}{dt} = 3t^2 - 12t - 15$$

$$a = \frac{dv}{dt} = 6t - 12$$

(a) $t = ??$ @ $v = 0$ } Changing direction

$3t^2 - 12t - 15 = 0$ } By Calculator

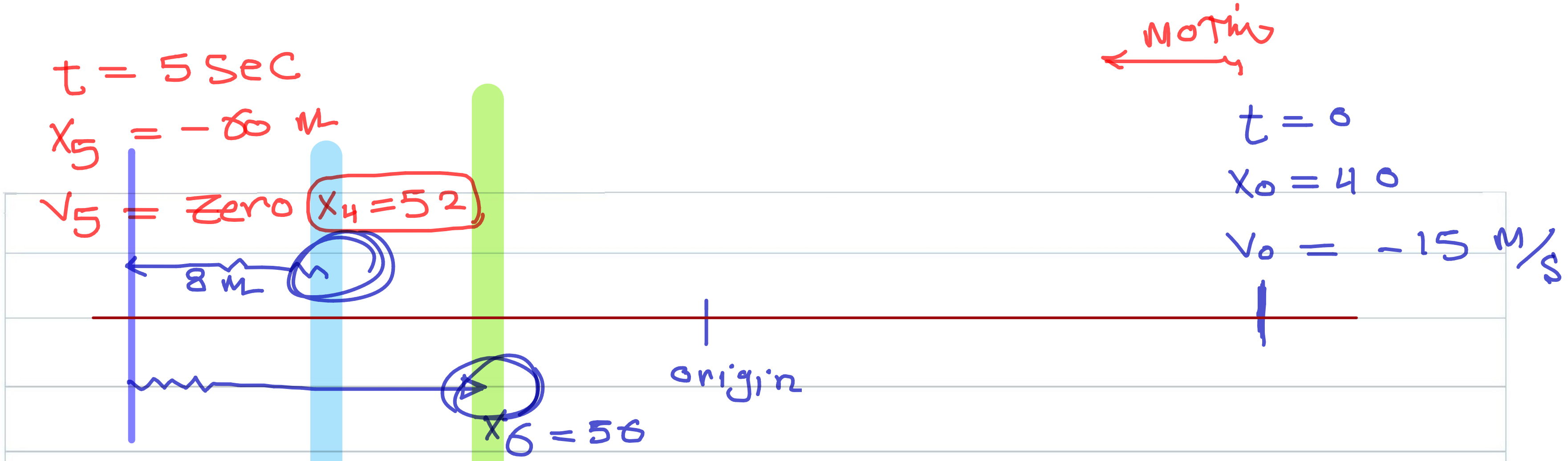
$t = -1$ sec Rejected

$\boxed{t = 5 \text{ sec}}$

(b) @ $t = 5$ sec $\qquad X_5 = ??$

$X_5 = 5^3 - 6(5)^2 - 15(5) + 40 = -60$ M

Position



@ $t_o = 0$ $\qquad X_o = 40$ M

@ $t_o = 0$ $\qquad V_o = -15$ M/s

Distance travelled = 40 + 60 = 100 M

(c) $a_5 = 6(5) - 12 = 18$ M/s²

(d) $X_4 = (4)^3 - 6(4)^2 - 15(4) + 40$ } @ t = 4sec

$= -52$ M

$X_6 = (6)^3 - 6(6)^2 - 15(6) + 40$ } @ t = 6sec

$= -50$ M

Distance travelled (From t = 4 sec → t = 5 sec)

= 60 - 52 = 8 M

Distance travelled (From t = 5 sec → t = 6 sec)

= 60 - 50 = 10 M

total distance travelled = 10 + 8 = 18 M

Position
[m]

Integration

Velocity
[m/s]

Integration

Acceleration
[m/s2]

$$x(t) = \int v(t)dt$$

$$v(t) = \int a(t)dt$$

$$a(t)$$

derrivation

x

v

a

$\int$

Position
[m]

Derivative

Velocity
[m/s]

Derivative

Acceleration
[m/s^2]

$$x(t)$$

$$v(t) = \frac{dx(t)}{dt}$$

$$a(t) = \frac{dv(t)}{dt}$$

1. $a = f(t)$. *The Acceleration Is a Given Function of t.*

$$\int_{v_0}^{v} dv = \int_0^t f(t)\, dt$$

$$v - v_0 = \int_0^t f(t)\, dt$$

2. $a = f(x)$. *The Acceleration Is a Given Function of x.*

$$\int_{v_0}^{v} v\, dv = \int_{x_0}^{x} f(x)\, dx$$

$$\tfrac{1}{2}v^2 - \tfrac{1}{2}v_0^2 = \int_{x_0}^{x} f(x)\, dx$$

3. $a = f(v)$. *The Acceleration Is a Given Function of v.*

$$dt = \frac{dv}{f(v)} \qquad dx = \frac{v\, dv}{f(v)}$$

Sample Problem 11.3 (integration)

- The brake mechanism used to reduce recoil in certain types of guns consists essentially of a piston attached to the barrel and moving in a fixed cylinder filled with oil.
- As the barrel recoils with an initial velocity v_0, the piston moves and oil is forced through orifices in the piston, causing the piston and the barrel to decelerate at a rate proportional to their velocity, that is **a = - kv**.
- Express (a) v in terms of t, (b) x in terms of t, (c) v in terms of x. Draw the corresponding motion curves

$$a = -kv$$

(a)

$$a = \frac{dv}{dt} = -kv$$

$$\frac{dv}{v} = -k\,dt$$

$$\int_{v_0}^{v} \frac{dv}{v} = -k \int_0^t dt$$

$$\boxed{\ln v \Big|_{v_0}^{v}} = -kt$$

$$\ln \frac{v}{v_0} = -kt$$

$$\frac{v}{v_0} = e^{-kt} \qquad \boxed{v = v_0 e^{-kt}}$$

(b)

$$v = \frac{dx}{dt} = v_0 e^{-kt}$$

$$\int_0^x dx = \int_0^t v_0 e^{-kt}\,dt$$

$$x = -\frac{v_0}{k}\left(e^{-kt}\right|_0^t$$

$$x = -\frac{v_0}{k}\left(e^{-kt} - 1\right)$$

(c)

$$a = \cancel{v}\frac{dv}{dx} = -k\cancel{v}$$

$$\int_{v_0}^{v} dv = -k\int_0^x dx$$

$$v - v_0 = -kx$$

$$\boxed{v = v_0 - kx}$$

* Motion with Constant acceleration :-

$$a = \frac{dv}{dt}$$

$$V = V_0 + a t \quad (\text{No} - x)$$

$$x = x_0 + V_0 t + \frac{1}{2} a t^2 \quad (\text{No} - v)$$

$$v^2 = V_0^2 + 2a(x - x_0) \quad (\text{No} - t)$$

V_0

v

x

t

a

Free Fall bodies :-

→ vertically up

→ vertically down

→ dropped

$$a_y = -9.8 \ m/s^2$$

$$V = V_0 + a_y t$$

$$y = y_0 + V_0 t + \frac{1}{2} a_y t^2$$

$$v^2 = V_0^2 + 2a_y(y - y_0)$$

$\Longrightarrow$ *