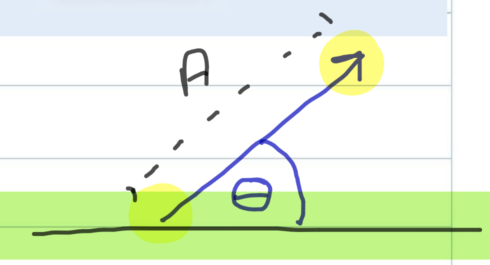
Force VECTORS

2

* Vectors } Magnitude, direction

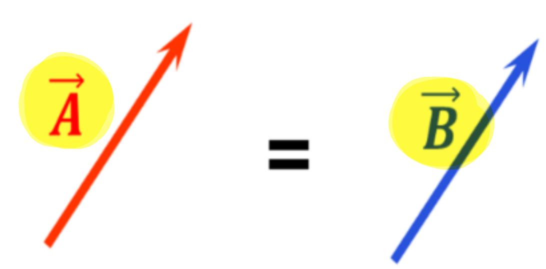


Ex. Force, velocity

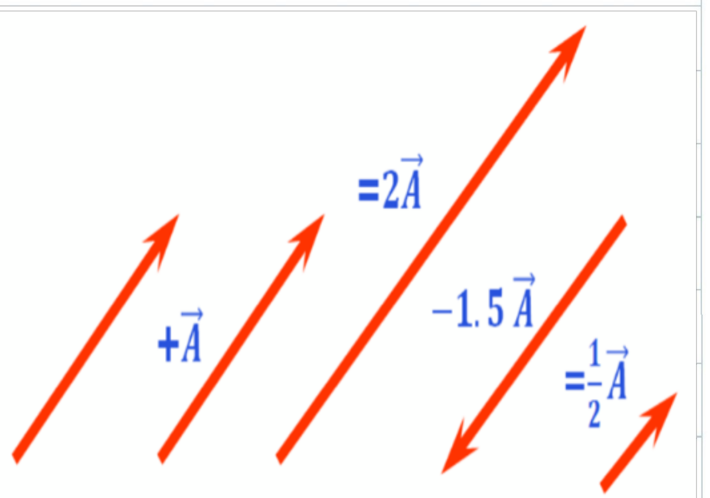
* Scalar } only Quantity (+ c -)

Ex. Mass , volume

- Vectors are **equal** when they have the same magnitude and same direction



- Vectors can be simply **added** or **subtracted**, if they have the same direction



$A = 1$ $\quad B = 2 \quad \vec{A} + \vec{B} = 3$

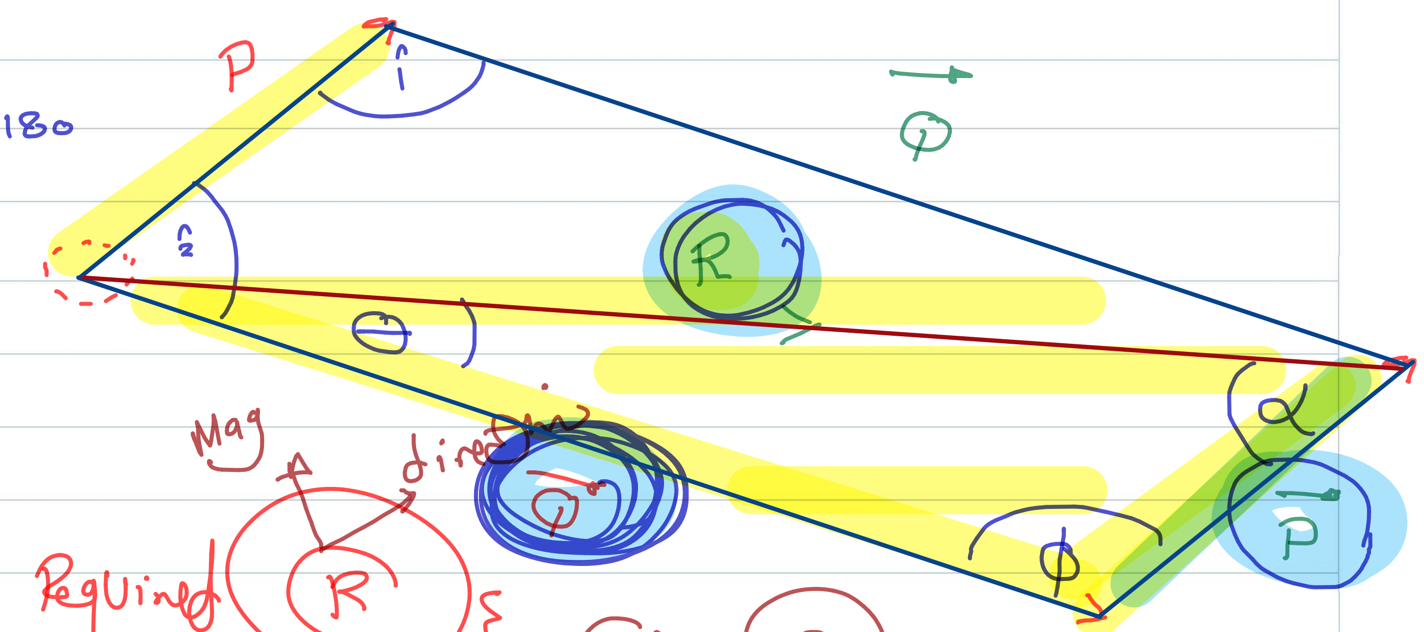
Parallelogram Law

Trigonometric method

} Resultant of two Forces

$\vec{R} = \vec{P} + \vec{Q}$

$\hat{1} + \hat{2} = 180$



Case 1

Given $\vec{P}$, $\vec{Q}$ Required (R)

[1] Conclude internal angle between P & Q [φ]

(2) Mag ⇒ Cos law

$$R = \sqrt{P^2 + Q^2 - 2PQ\cos\phi}$$

(3) Direction → sin-law

$$\frac{P}{\sin\theta} = \frac{Q}{\sin\alpha} = \frac{R}{\sin\phi}$$

Case 2

given (R) Required $\vec{P}$, $\vec{Q}$

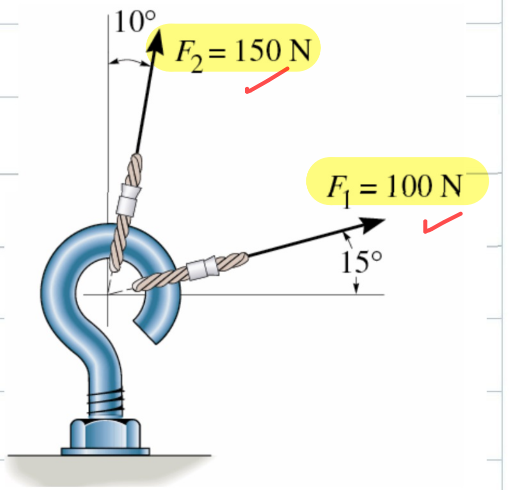
(1) Conclude all internal angle (θ, α, φ)

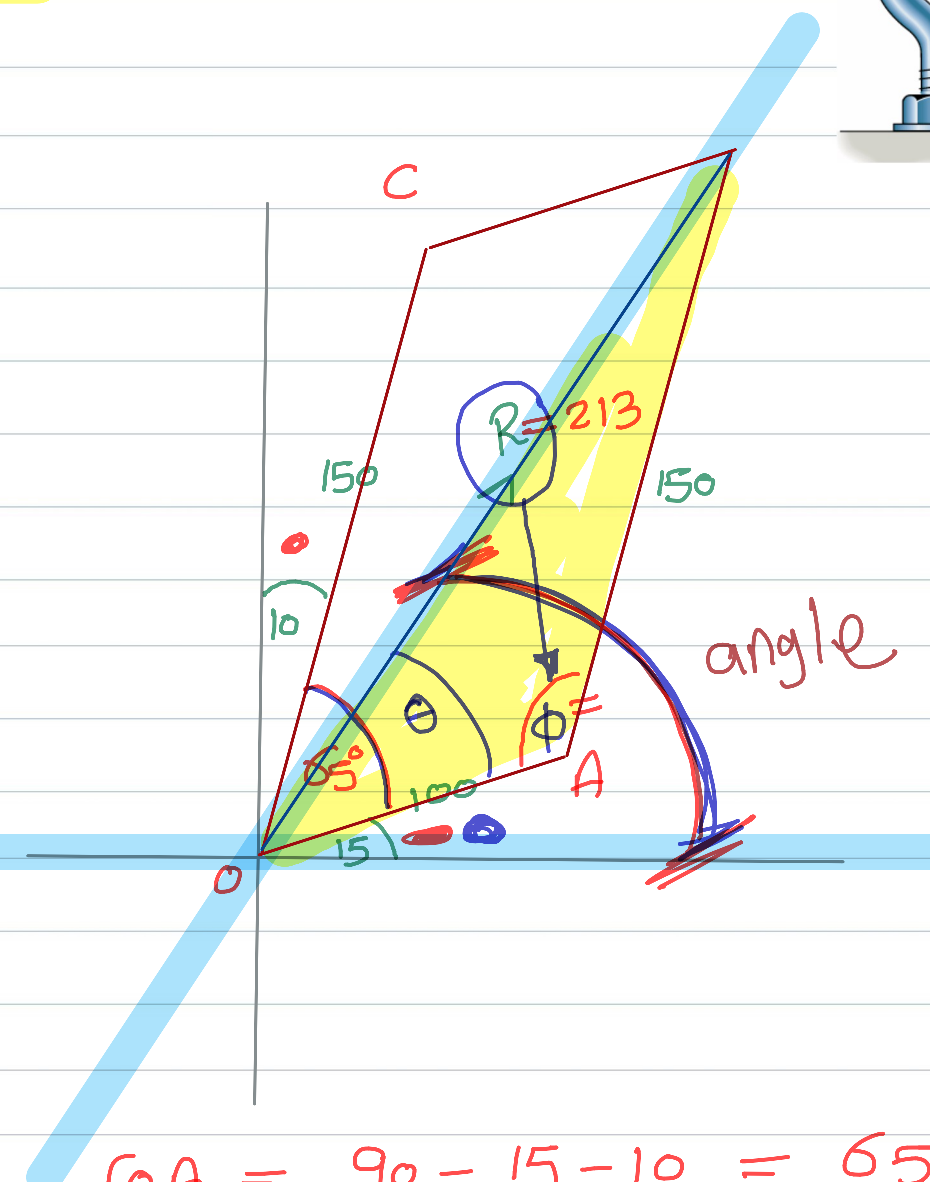
(2) sin-law

$$\frac{P}{\sin\theta} = \frac{Q}{\sin\alpha} = \frac{R}{\sin\theta}$$

Example 1:-

The screw eye in the figure at the left is subjected to two forces $\vec{F}_1$ and $\vec{F}_2$.
Determine the magnitude and direction of the resultant force.





(1) angle COA = 90 − 15 − 10 = 65°

$\phi = 180 - 65 = 115^\circ$

(2) Mag.

$$R = \sqrt{100^2 + 150^2 - 2(100)(150)\cos 115}$$

$= 213$

(3) Direction → Sin law

$$\frac{150}{\sin\theta} = \frac{213}{\sin 115}$$

$150 \sin 115 = 213 \sin\theta$

$$\sin\theta = \frac{150 \sin 115}{213}$$

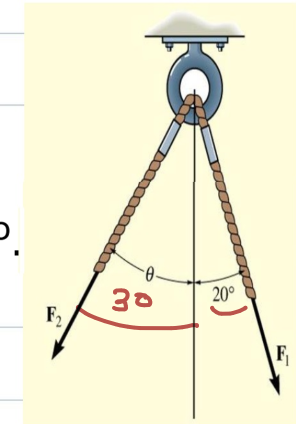
$$\theta = \sin^{-1}\frac{150 \sin 115}{213} = 39.7^\circ$$

angle = 39.7 + 15

= 54.7°

Example 2 :-

The ring below is subjected to F_1 and F_2. If we want a resultant force of 1kN and directed vertically downward, determine the magnitude of F_1 and F_2 if θ = 30°.

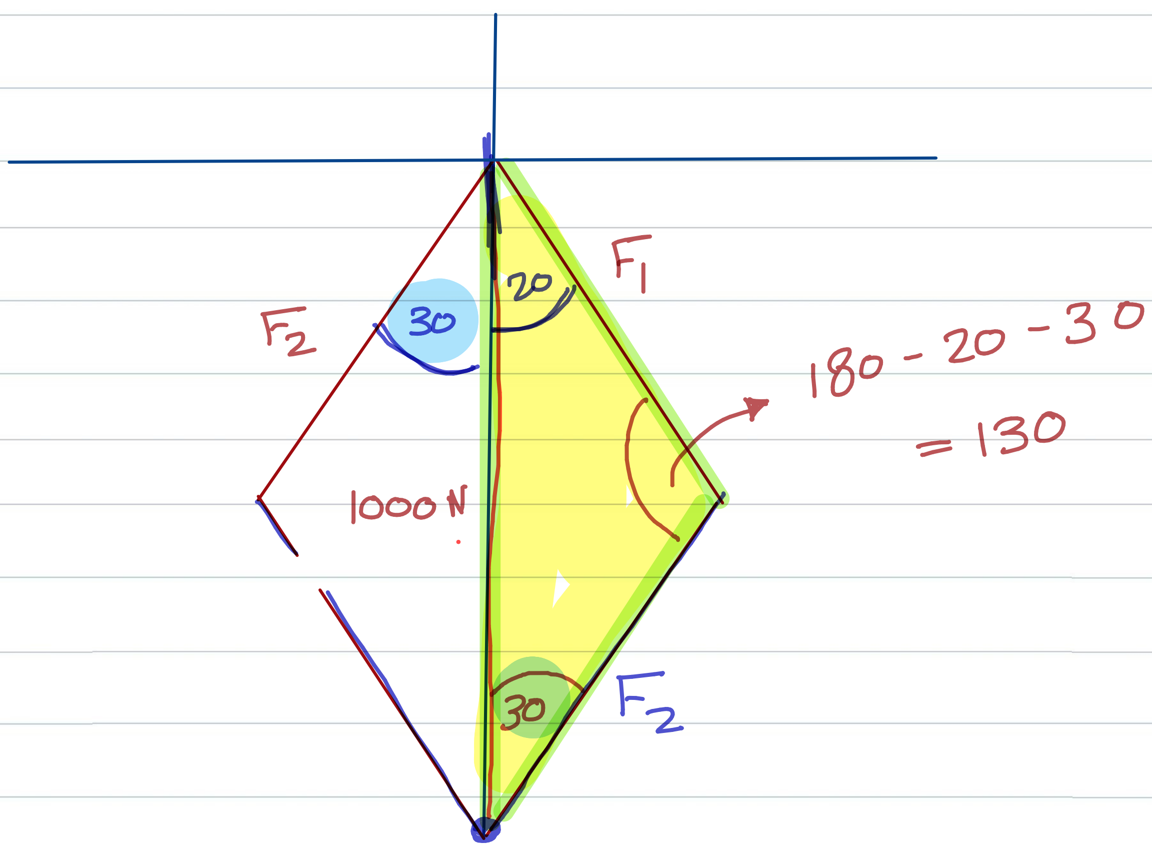


$R = 1000\ N$

$F_1 = ??$

$F_2 = ???$

Case ②

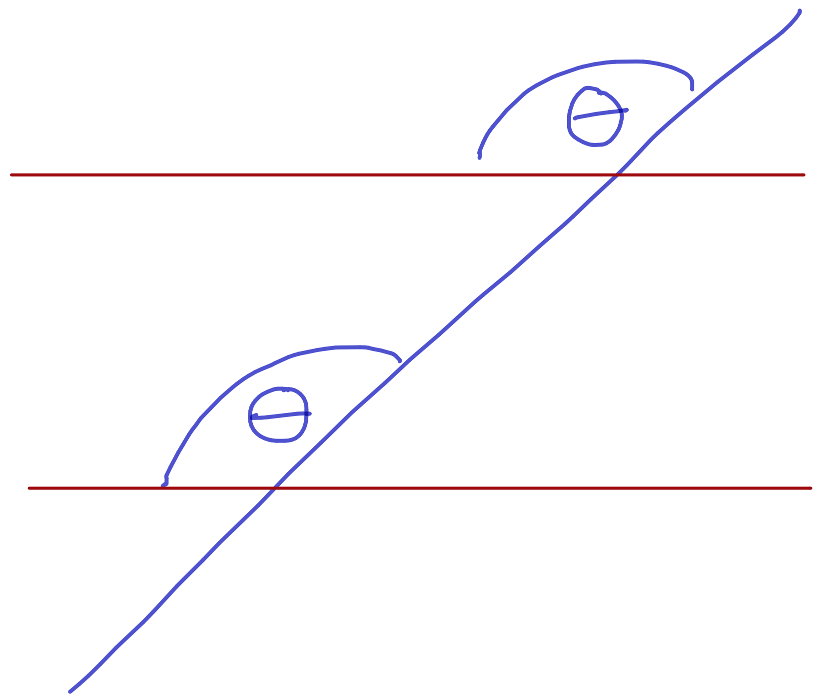


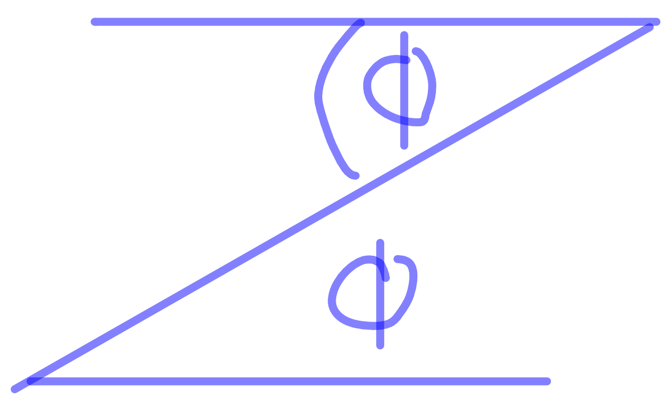
Sine - law

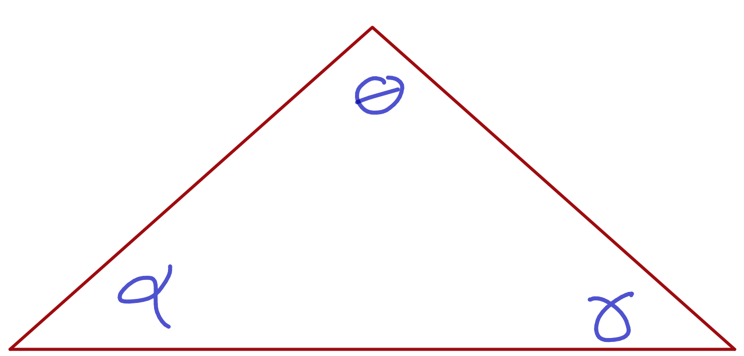
$$\frac{F_1}{\sin 30} = \frac{F_2}{\sin 20} = \frac{1000}{\sin 130}$$

$$F_1 = \frac{1000 \sin 30}{\sin 130} = 653\ N$$

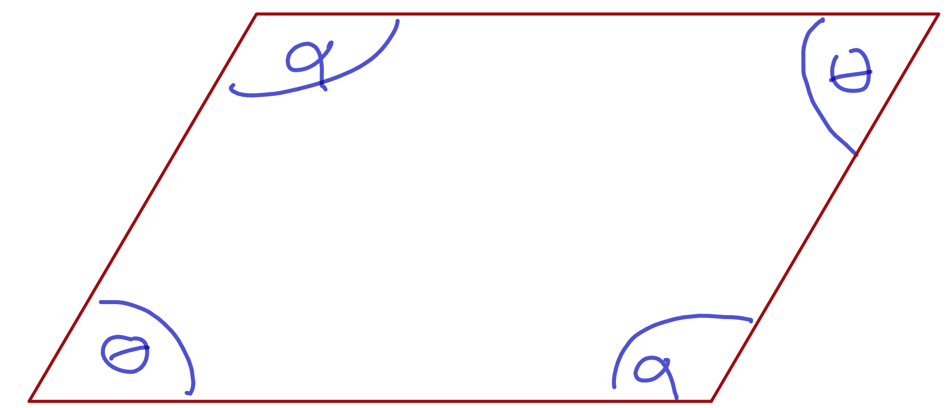
$$F_2 = \frac{1000 \sin 20}{\sin 130} = 446\ N$$







$$\theta + \alpha + \gamma = 180$$

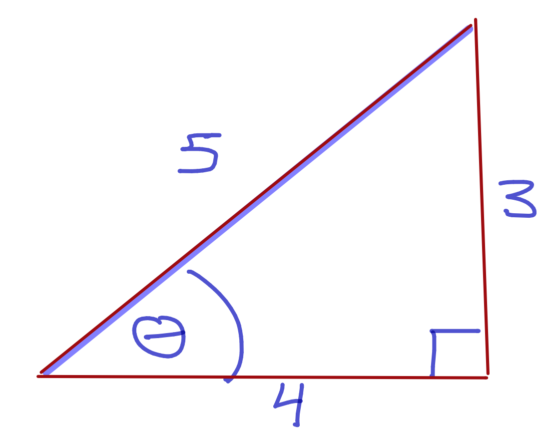


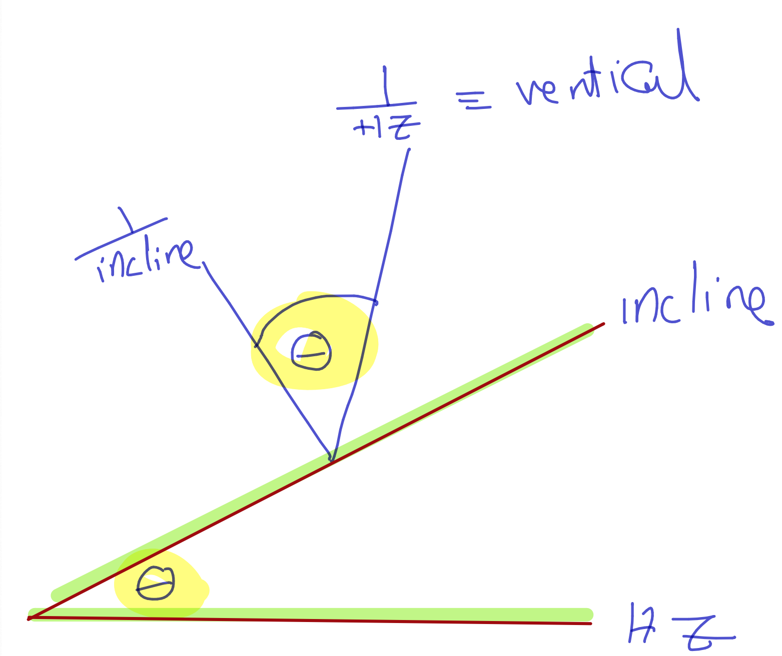
$$\theta + \alpha = 180$$

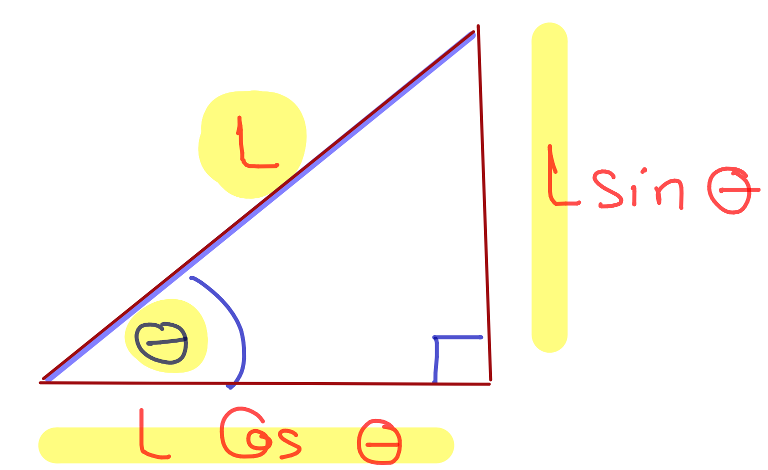
$$\sin\theta = \frac{3}{5}$$

$$\cos\theta = \frac{4}{5}$$

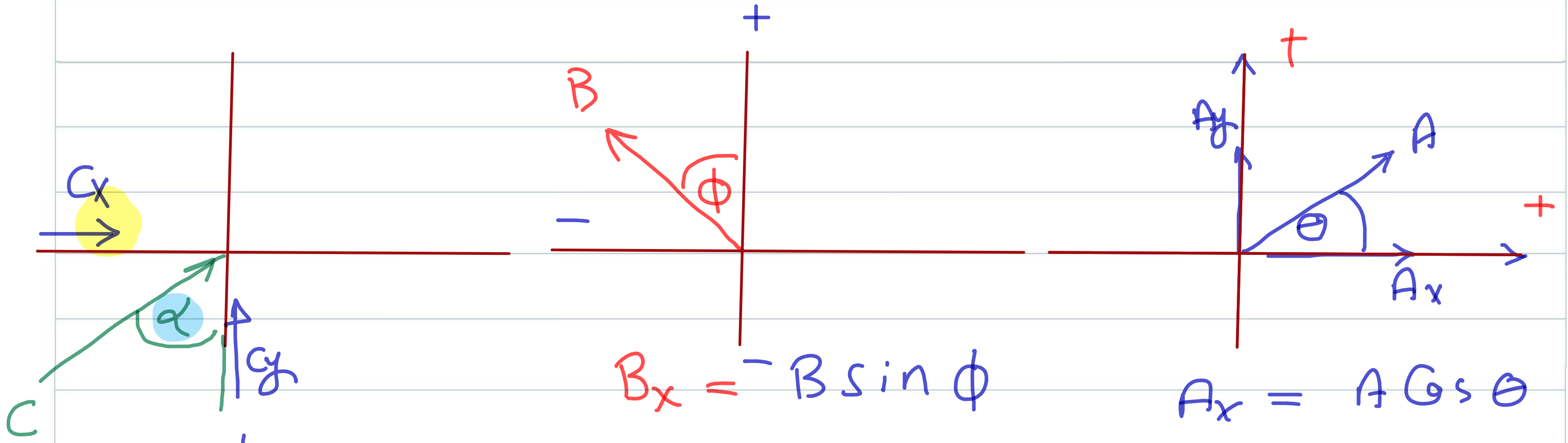
$$\tan\theta = \frac{3}{4}$$







Rectangular/Cartesian Components Method



$C_x = +C\sin\alpha$

$C_y = +C\cos\alpha$

$B_x = -B\sin\phi$

$B_y = +B\cos\phi$

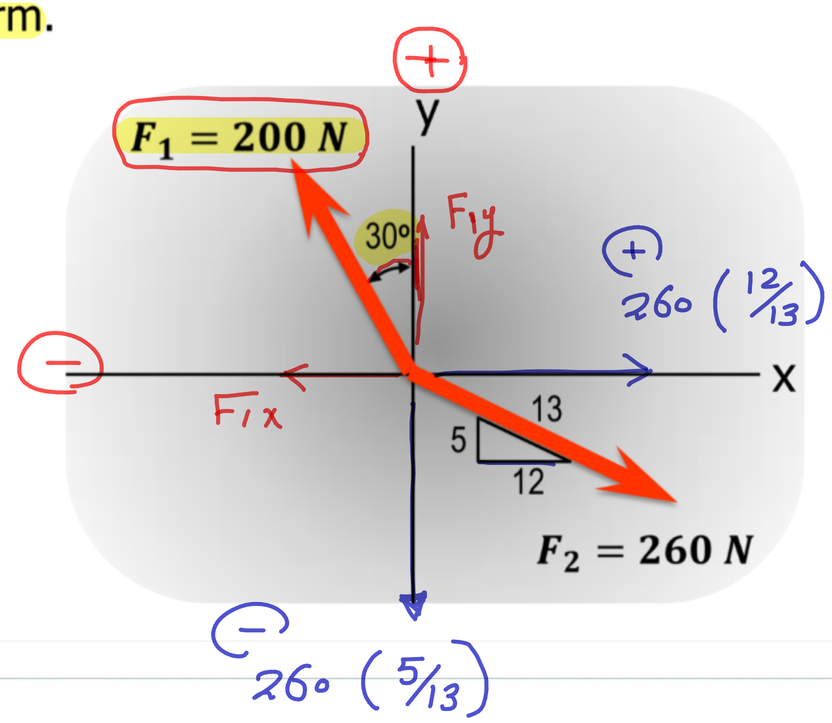
$B = (B_x)\,i + (B_y)\,j$

$A_x = A\cos\theta$

$A_y = A\sin\theta$

$A = \sqrt{A_x^2 + A_y^2}$

Determine the x and y Cartesian components of the $\mathbf{F}_1$ and $\mathbf{F}_2$ forces acting on the boom. Put each force in the Cartesian vector form.



$F_{1x} = -200\sin 30$

$= -100$

$F_{1y} = 200\cos 30$

$= 173$

$F_{2x} = 260\left(\frac{12}{13}\right)$

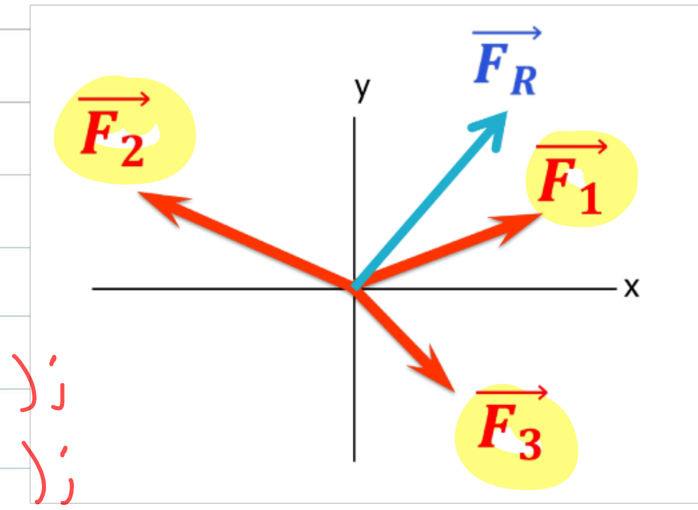
$F_{2y} = -260\left(\frac{5}{13}\right)$

$F_1 = (-100)\,\hat{i} + (173)\,\hat{j}$

Coplanar Force Resultants

More than 2 - Forces

① Resolve



F_{1x} F_{2x} F_{3x}

F_{1y} F_{2y} F_{3y}

$F_1 = (\quad)\,i + (\quad)\,j$

$F_2 = (\quad)\,i + (\quad)\,j$

② $F_R = (F_{Rx})\,i + (F_{Ry})\,j$

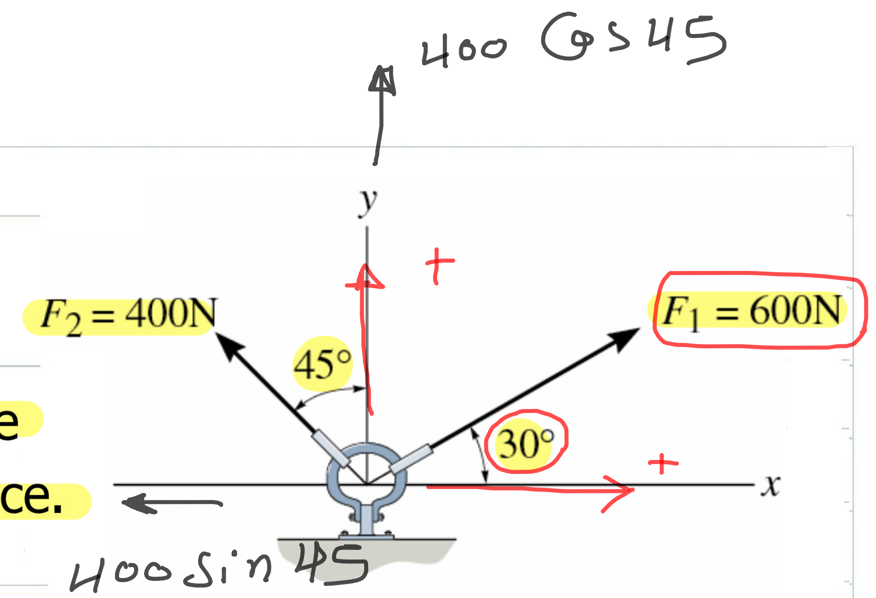
$= (F_{1x} + F_{2x} + F_{3x})\,i + (F_{1y} + F_{2y} + F_{3y})\,j$

③ $F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$

④ $\theta = \tan^{-1} \frac{F_{Ry}}{F_{Rx}}$

Example 3 :-

The link in the figure is subjected to two forces, $\mathbf{F_1}$ and $\mathbf{F_2}$. Determine the resultant magnitude and orientation of the resultant force.



① Resolve :-

$F_{1x} = 600 \cos 30 = 519.6$

$F_{1y} = 600 \sin 30 = 300$

$\vec{F_1} = (519.6)\hat{i} + (300)\hat{j}$

$F_{2x} = -400 \sin 45 = -282.8$

$F_{2y} = +400 \cos 45 = 282.8$

② $F_R = \vec{F_1} + \vec{F_2}$

$= (519.6 - 282.8)\hat{i} + (300 + 282.8)\hat{j}$

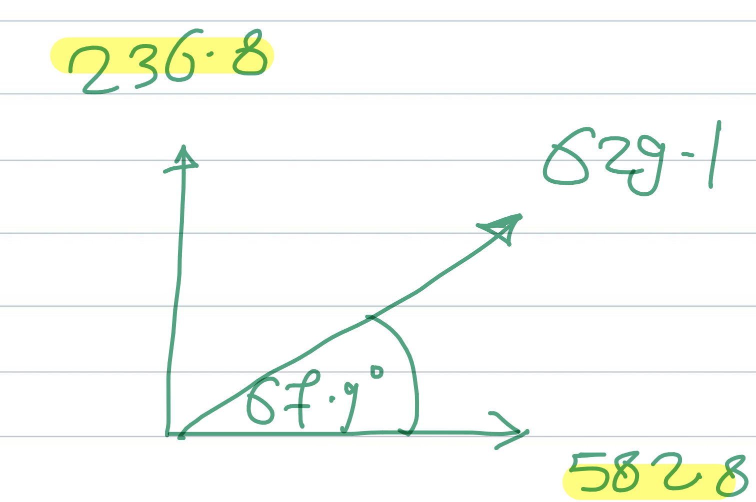
$= (236.8)\hat{i} + (582.8)\hat{j}$

③ $F_R = \sqrt{(236.8)^2 + (582.8)^2}$

$= 629.1 \ N$

④ $\theta = \tan^{-1} \frac{582.8}{236.8}$

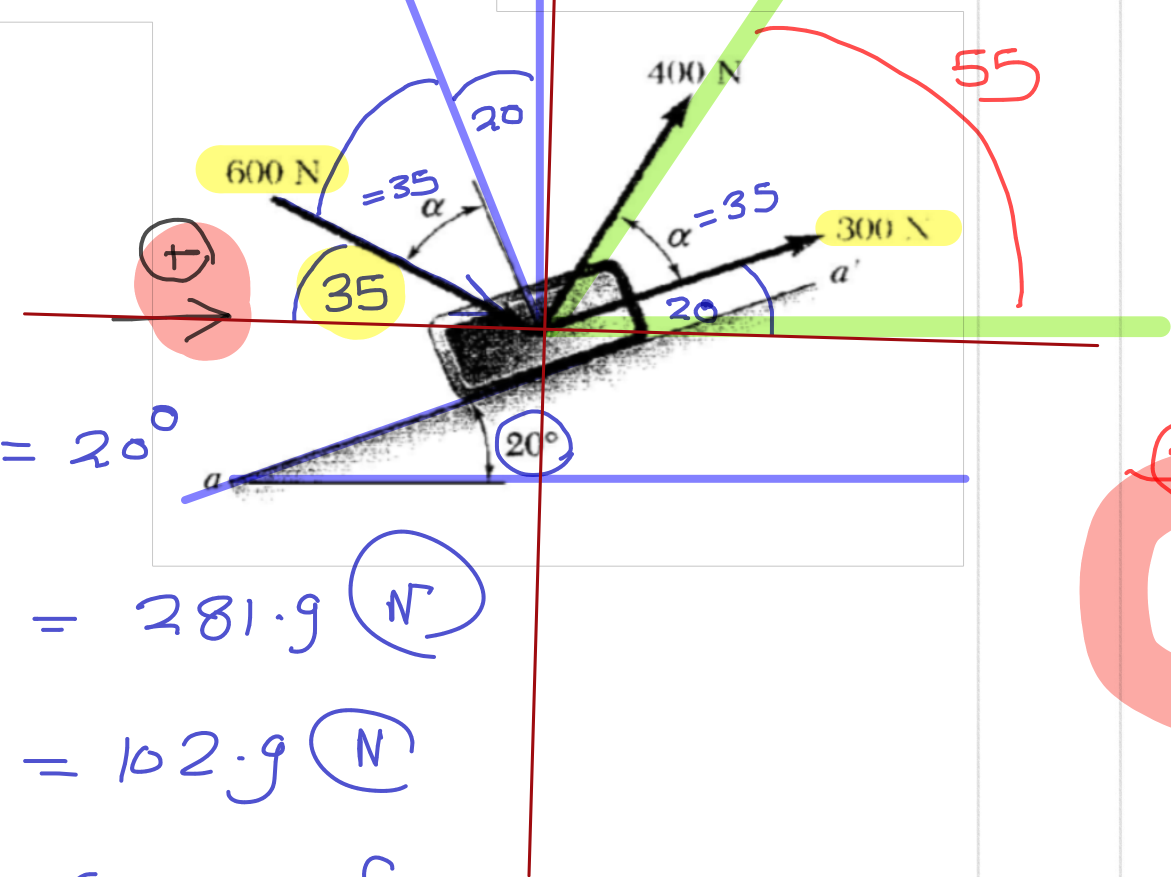
$= 67.9°$



Problem # 3

Knowing that **α = 35°**,

Determine: The resultant of the three forces shown



① Resolve :—

$F_1 = 300$

With angle With HZ = 20°

$F_{1x} = 300 \cos 20 = 281.9$ N

$F_{1y} = 300 \sin 20 = 102.9$ N

$\vec{F_1} = (281.9)\,i + (102.9)\,j$

$F_2 = 400$

With angle with HZ = 55

$F_{2x} = 400 \cos 55 = 229.4$ N

$F_{2y} = 400 \sin 55 = 327.7$ N

$F_2 = (229.4)\,i + (327.7)\,j$

$F_3 = 600$

with angle with HZ = 35

$F_{3x} = 600 \cos 35 = 491.5$

$F_{3y} = -600 \sin 35 = -344.1$

$F_3 = (491.5)\,i + (-344.1)\,j$

② $\vec{F_R} = \vec{F_1} + \vec{F_2} + \vec{F_3}$

$= (281.9 + 229.4 + 491.5)\,i + (102.9 + 327.7 - 344.1)\,j$

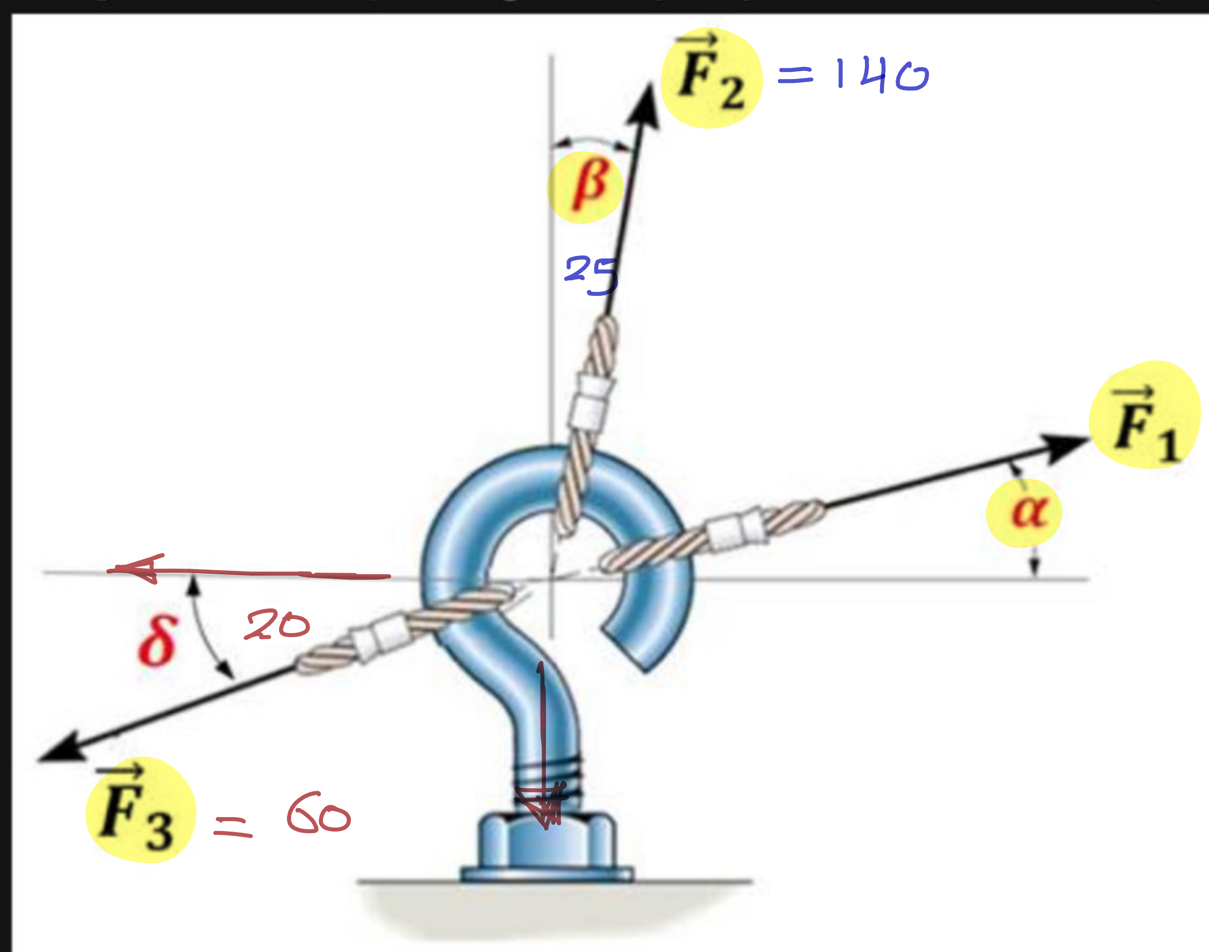
$= (1002.8)\,i + (86.2)\,j$

③ $F_R = \sqrt{(1002.8)^2 + (86.2)^2} = 1006.5$ N

④ $\theta = \tan^{-1} \frac{86.2}{1002.8} = 4.91°$

H.W

Determine the **magnitude** (**R**) and **direction** (**θ**) of the resultant force $\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$, by resolving the force vectors into **Cartesian components** (using the projection method).



$\vec{F}_1$	110 N
$\vec{F}_2$	140 N
$\vec{F}_3$	60 N
α	34 degrees
β	25 degrees
δ	20 degrees

$F_R = \sqrt{\quad}$

1 Resolve

$F_{1x} = 110 \cos 34 = 91.2$

$F_{1y} = 110 \sin 34 = 61.5$

$F_1 = (91.2)\hat{i} + (61.5)\hat{j}$

$F_{2x} = 140 \sin 25 = 59.2$

$F_{2y} = 140 \cos 25 = 126.88$

$F_2 = (59.2)\hat{i} + (126.88)\hat{j}$

$F_{3x} = -60 \cos 20 = -56.38$

$F_{3y} = -60 \sin 20 = -20.52$

$F_3 = (-56.38)\hat{i} + (-20.52)\hat{j}$

2 $\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3}$

$= (91.2 + 59.2 - 56.38)\hat{i}$

$+ (61.5 + 126.88 - 20.52)\hat{j}$

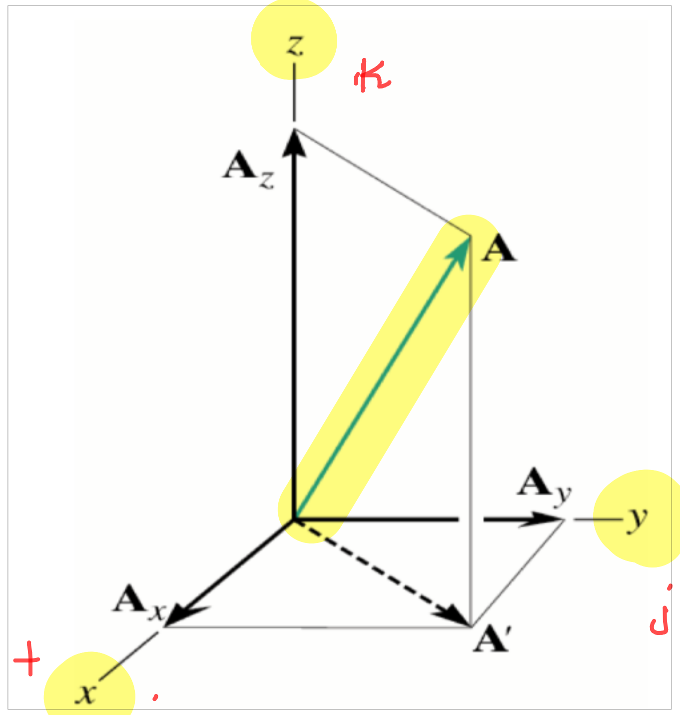
$= (\qquad)\hat{i} + (\qquad)\hat{j}$

4 $\theta = \tan^{-1} \frac{F_{Ry}}{F_{Rx}}$

2.7. Cartesian Vectors

3-D

x
y
z



Unit Vectors in Coordinate Directions:

$\vec{i}$, $\hat{i}$: Unit vector in the x-direction

$\vec{j}$, $\hat{j}$: Unit vector in the y-direction

$\vec{k}$, $\hat{k}$: Unit vector in the z-direction

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

Unit Vectors

$$|\vec{U}_A| = 1$$

$$\vec{U}_A = \frac{\vec{A}}{|A|}$$

D = 10

1

$$\vec{A} = |\vec{A}| \, \vec{U}_A$$

Magnitude

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Direction of a Cartesian Vector

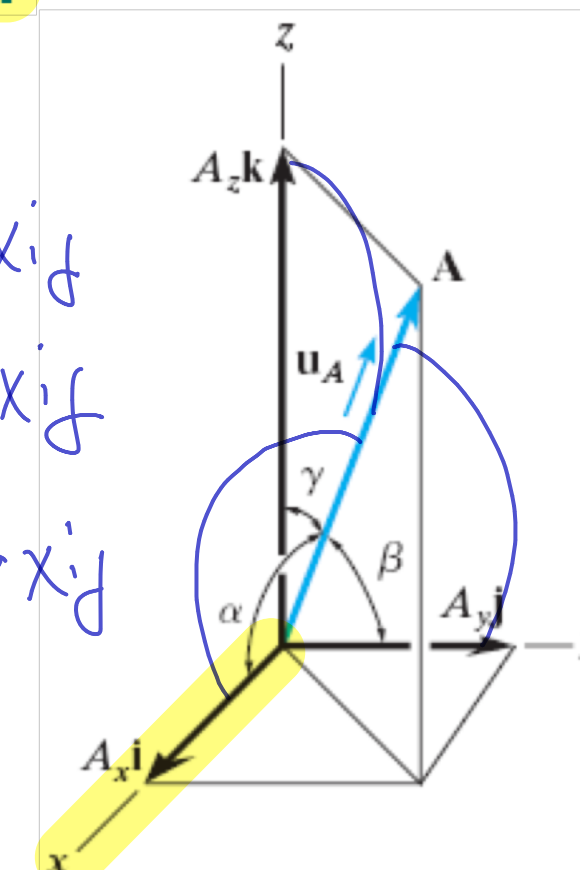
Direction angles : —

α angle with (+) x-axis

β // // (+) y-axis

γ // // (+) z-axis

α & $\beta \geq 0$ $\gamma \not< 180$



Direction Cosines of A is

$$\cos\alpha = \frac{A_x}{A}$$

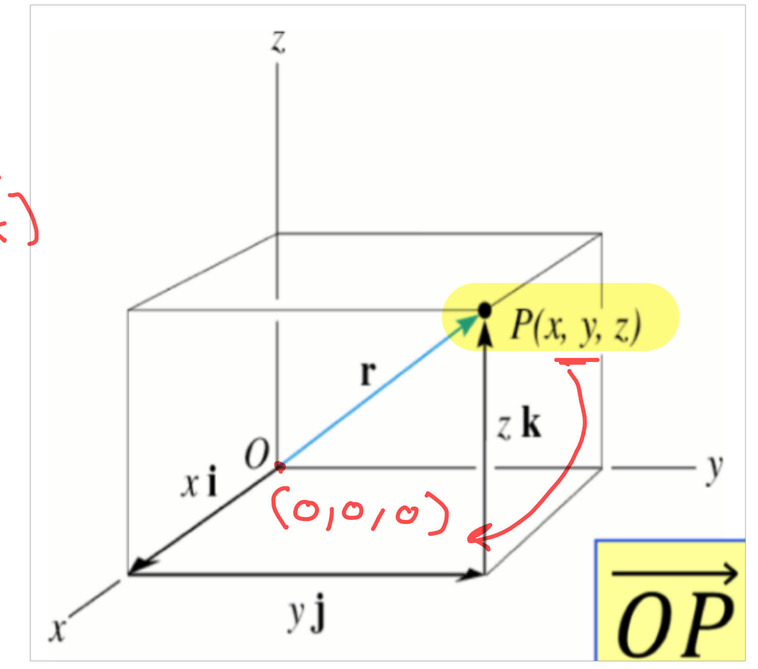
$$\cos\beta = \frac{A_y}{A}$$

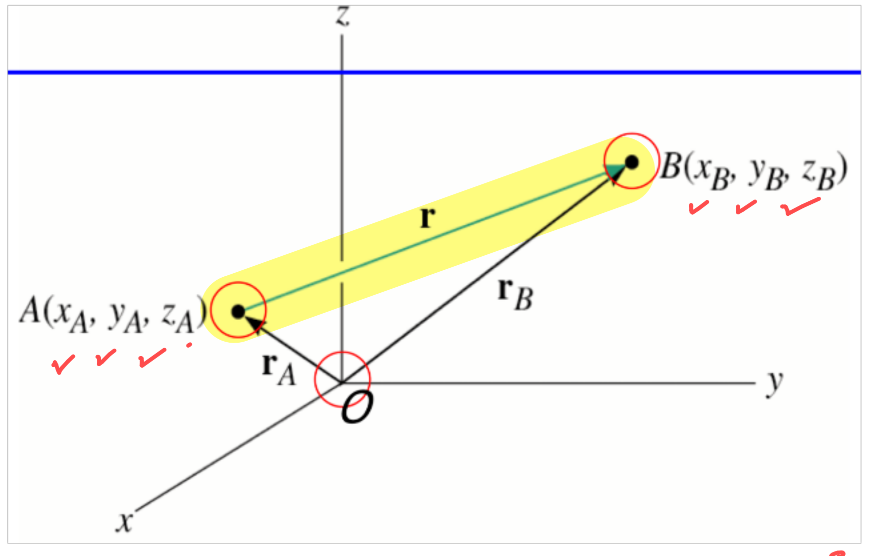
$$\cos\gamma = \frac{A_z}{A}$$

2.9. Coordinates of Relative Position Vectors

$\vec{r} = \vec{OP}$

$= (x)i + y(j) + z(\hat{k})$





$\vec{AB} = \vec{r_B} - \vec{r_A}$

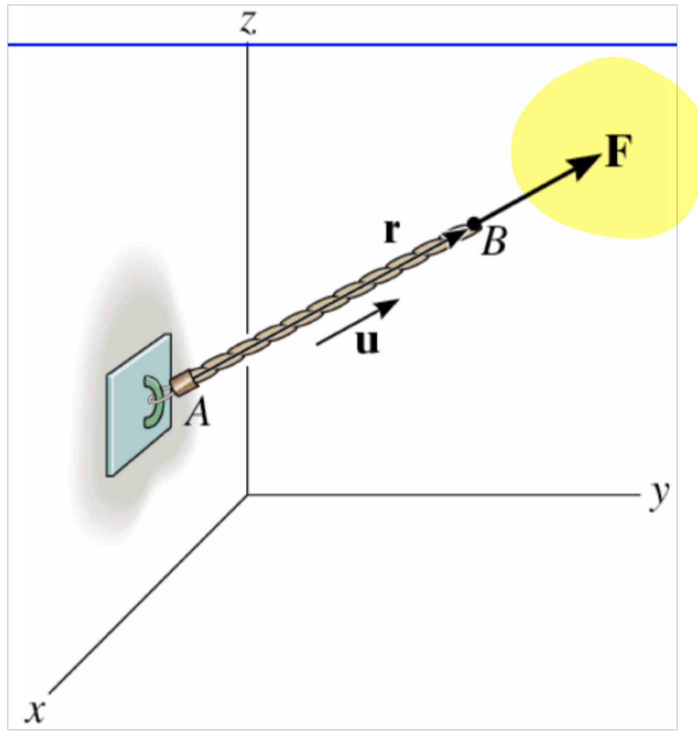
$= (x_B - x_A)i + (y_B - y_A)j + (z_B - z_A)k$

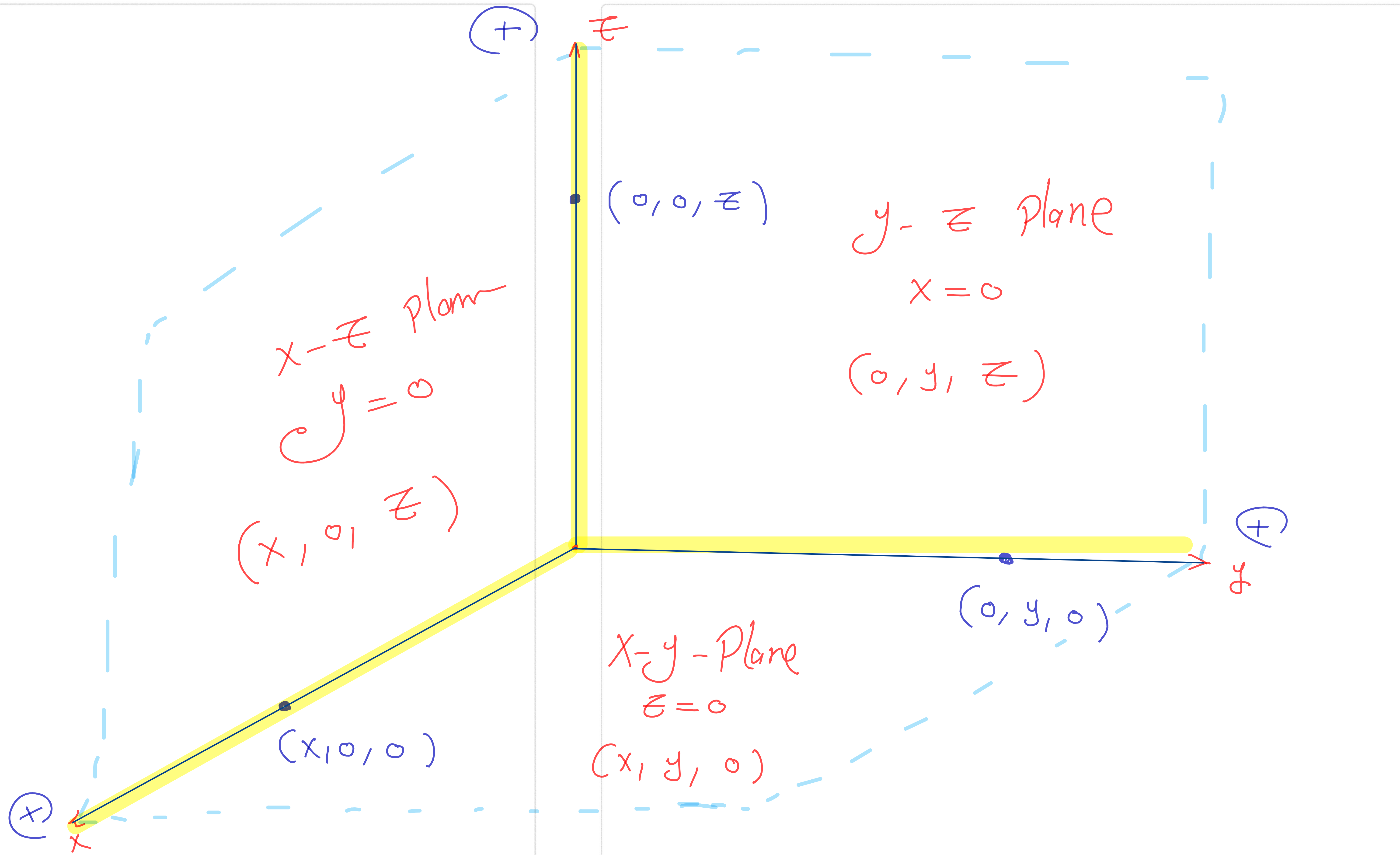
2. 10. Force Along a Line

Given F

$\vec{F} = F \vec{U_{AB}}$

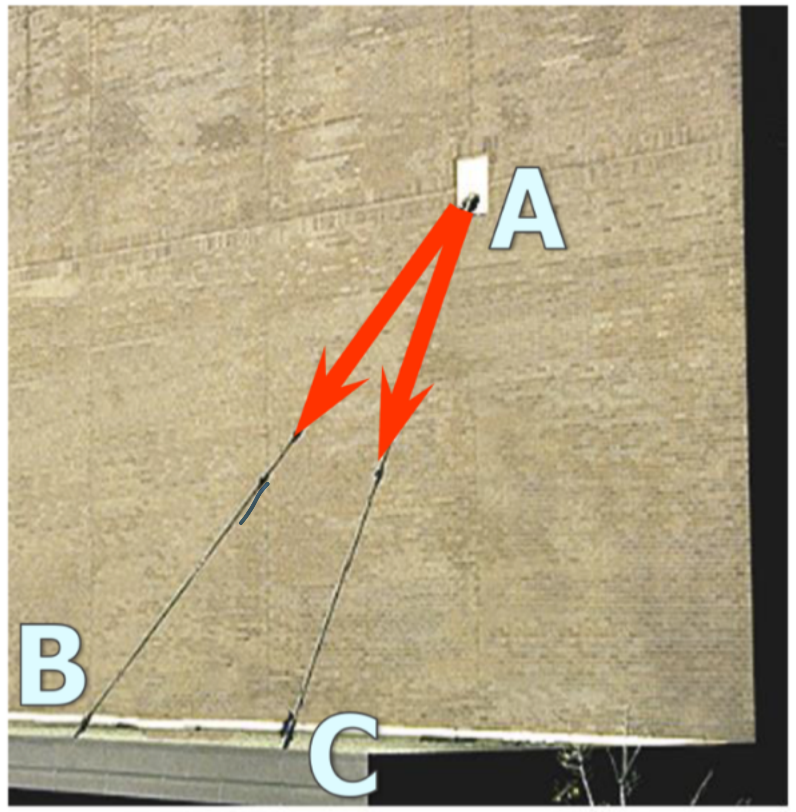
$= F \frac{\vec{AB}}{|\vec{AB}|}$





+
z
(0, 0, z)
y - z plane
x = 0
(0, y, z)
x - z plane
y = 0
(x, 0, z)
+
y
(0, y, 0)
x-y-Plane
z = 0
(x, y, 0)
(x, 0, 0)
+
x

Example



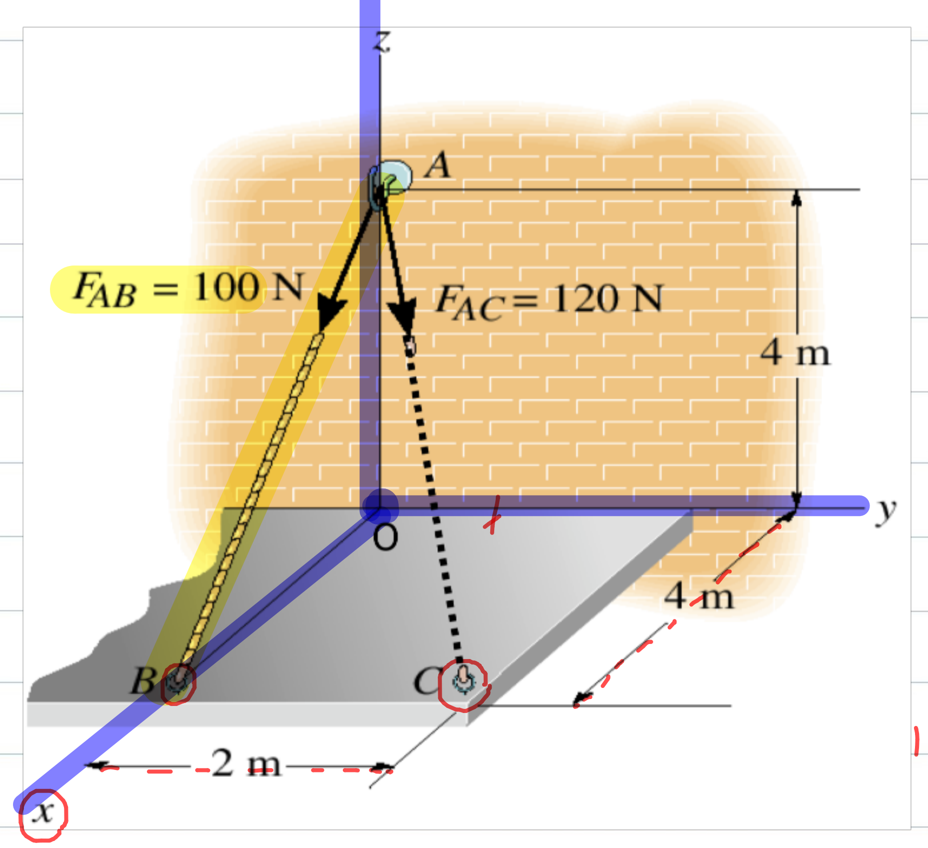
The roof is supported as shown. If the cables exert forces of F_{AB} = 100 N and F_{AC} =120 N on the wall hook at A, determine the magnitude of the resultant force acting at A.

Step (A) Identify the absolute coordinates of all points (x, y, z)

$A = (0, 0, 4)$

$B = (4, 0, 0)$

$C = (4, 2, 0)$



Step (B) Identify the absolute position vectors

$\vec{r}_A = 0\hat{i} + 0\hat{j} + 4\hat{k}$

$\vec{r}_B = 4\hat{i} + 0\hat{j} + 0\hat{k}$

$\vec{r}_C = 4\hat{i} + 2\hat{j} + 0\hat{k}$

Step (C) Identify the position vectors of the mechanical elements

$\vec{AB} = \vec{r}_B - \vec{r}_A = (4)\hat{i} + (0)\hat{j} + (-4)\hat{k}$

$|\vec{AB}| = \sqrt{4^2 + (-4)^2} = 5.66 \text{ m}$

$\vec{U}_{AB} = \dfrac{\vec{AB}}{|\vec{AB}|} = \dfrac{4\hat{i} - 4\hat{k}}{5.66}$

$= (4/5.66)\hat{i} - (4/5.66)\hat{k}$

$\vec{AC} = \vec{r}_C - \vec{r}_A = (4)\hat{i} + (2)\hat{j} + (-4)\hat{k}$

$|\vec{AC}| = \sqrt{4^2 + 2^2 + (-4)^2} = 6 \text{ m}$

$\vec{U}_{AC} = \dfrac{\vec{AC}}{|\vec{AC}|} = \dfrac{4\hat{i} + 2\hat{j} - 4\hat{k}}{6}$

$\vec{U}_{AC} = \frac{4}{6} i + \frac{2}{6} j - \frac{4}{6} k$

Step (E) **Identify the force vectors**

$$\vec{F} = F \vec{U}$$

$$\vec{F}_{AB} = F_{AB} \vec{U}_{AB} = 100 \left(\frac{4}{5.66} \hat{i} - \frac{4}{5.66} \hat{k} \right)$$

$$= 70.7 \hat{i} - 70.7 \hat{k}$$

$$\vec{F}_{AC} = F_{AC} \vec{U}_{AC} = 120 \left(\frac{4}{6} i + \frac{2}{6} j - \frac{4}{6} k \right)$$

$$= 80 i + 40 j - 80 k$$

Step (F) **Find the resultant force**

$$\vec{F}_R = \vec{F}_{AB} + \vec{F}_{AC}$$

$$= (70.7 + 80) i + (40) j + (-70.7 - 80) \hat{k}$$

$$= 150.7 i + 40 j - 150.7 k$$

Step (G) **Identify the magnitude and direction of the resultant**

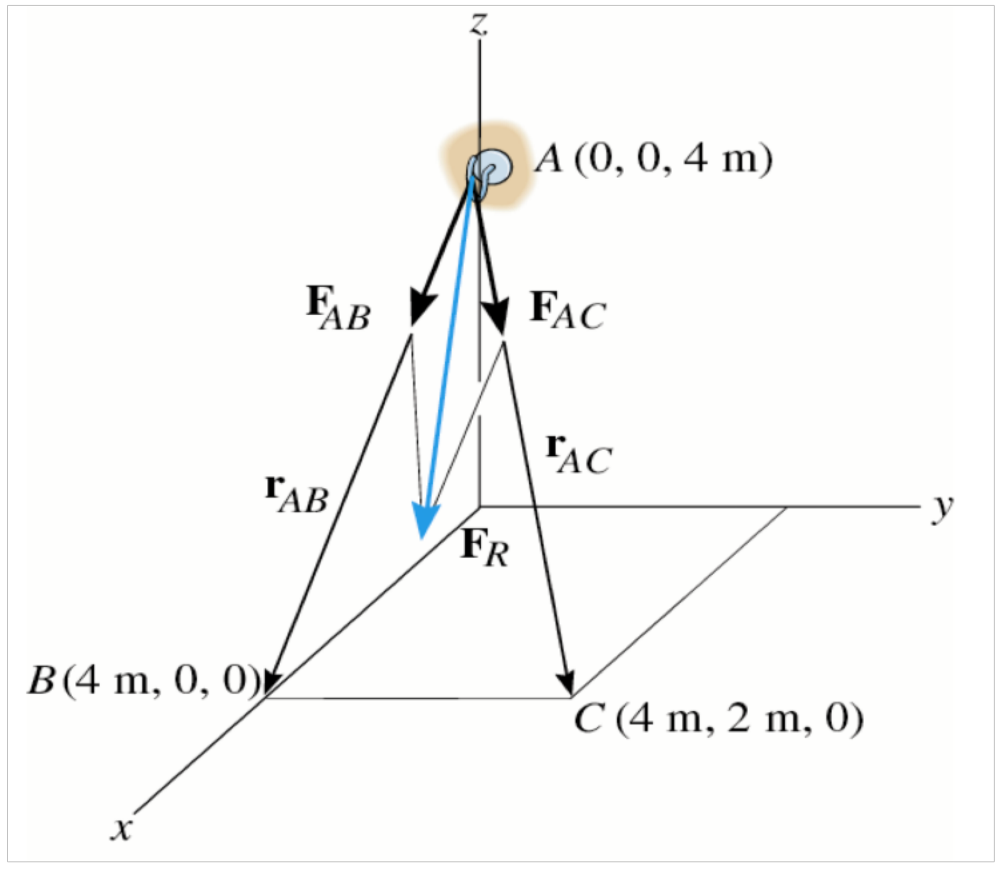
$$F_R = \sqrt{(150.7)^2 + (40)^2 + (-150.7)^2}$$

$$= 217 \text{ N}$$

$$\alpha = \cos^{-1} \frac{A_x}{A} = \cos^{-1} \frac{150.7}{217} = \checkmark$$

$$\beta = \cos^{-1} \frac{A_y}{A} = \cos^{-1} \frac{40}{217} = \checkmark$$

$$\gamma = \cos^{-1} \frac{A_z}{A} = \cos^{-1} \frac{-150.7}{217} = \checkmark$$



Ch 2