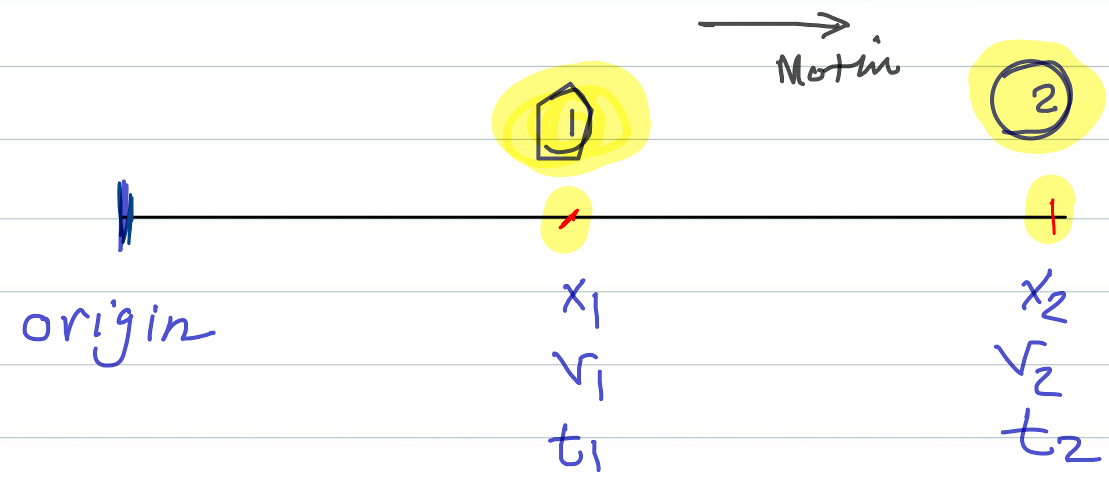
MOTION ALONG A STRAIGHT LINE

2



(1) Position :-

x_1, x_2 } Right of origin (+)

left of origin (−)

(2) Displacement (vector) :-

$$\Delta x = x_2 - x_1$$

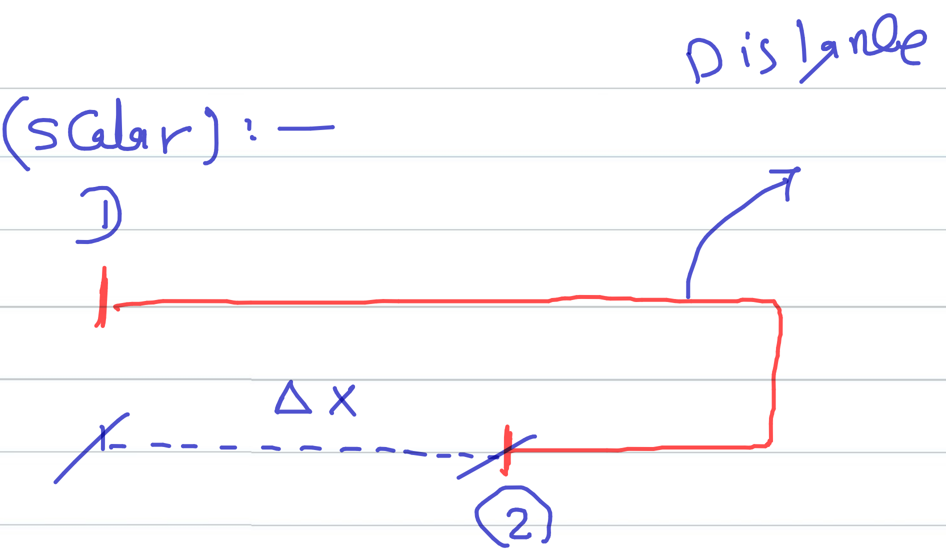
→ (+)

← (−)

(3) Distance (scalar) :-

D

always (+)

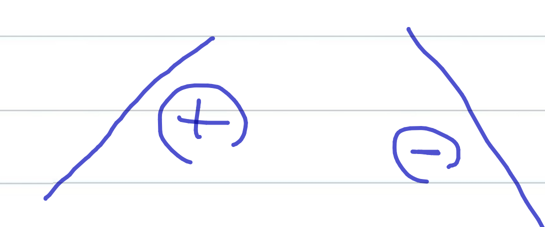


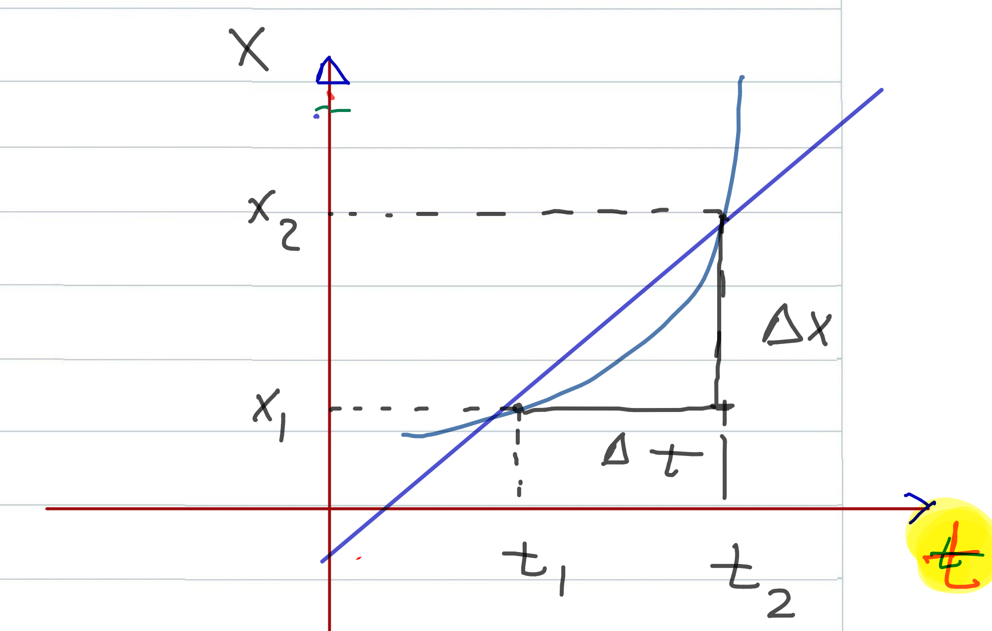
(4) average velocity :- (vector)

$$V_{av} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

+ →

− ←





x − t graph

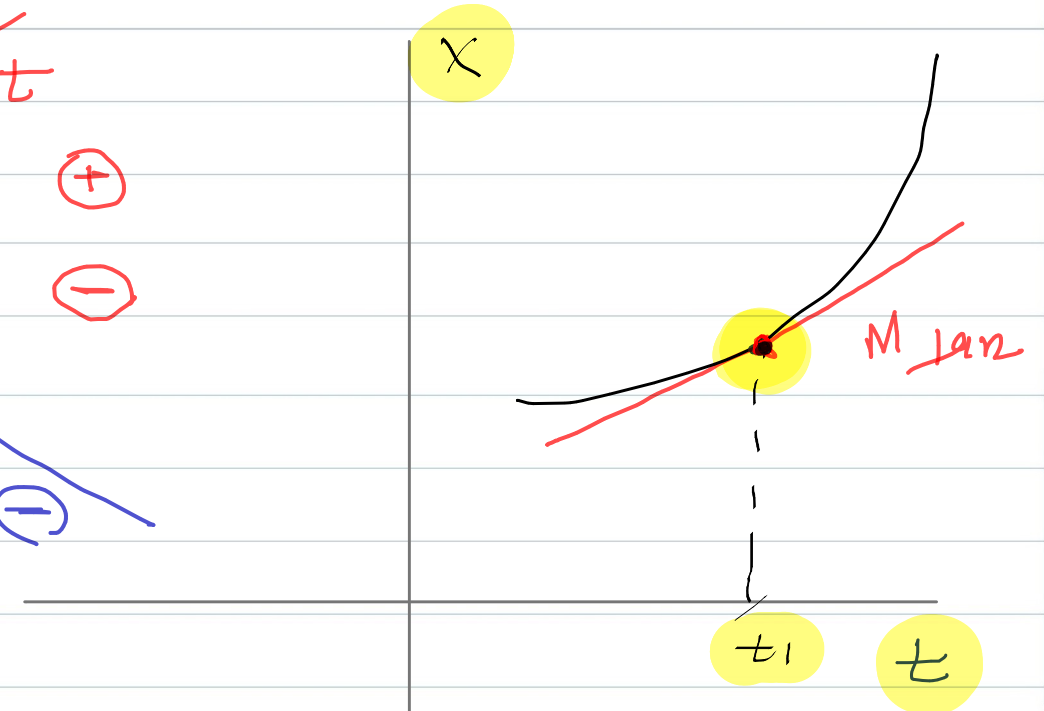
5 Instantaneous velocity :- (vector)

$$V_x = \frac{dx}{dt}$$

→ (+)

← (−)

(+) (−)



6 average speed :- (scalar)

$$\text{ave speed} = \frac{\text{Distance travelled}}{\Delta t}$$

always (+)

average speed $\neq V_{av}$

7 instantaneous speed :-

$$= |V_x|$$

8 avarage acceleration :- (vector)

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

(+) →

(−) ←

9 Instantaneous acceleration :-

$$a_x = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

+ →

− ←

(v) motion, a → velocity increase

motion (v), a ← velocity decrease

+ −

Example 2.1 Average and instantaneous velocities

A cheetah is crouched 20 m to the east of an observer (Fig. 2.6a). At time $t = 0$ the cheetah begins to run due east toward an antelope that is 50 m to the east of the observer. Dur the cheetah's coordinate x varies with time according to the equation $x = 20\text{ m} + (5.0\text{ m/s}^2)t^2$.

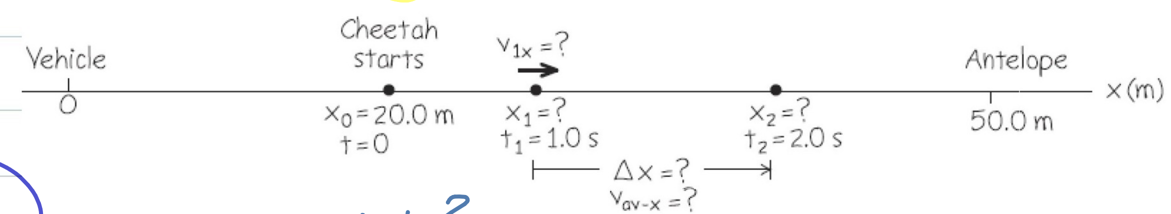
(a) Find the cheetah's displacement between $t_1 = 1.0$ s and $t_2 = 2.0$ s.

(b) Find its average velocity during that interval.

(c) Find its instantaneous velocity at $t_1 = 1.0$ sby taking $\Delta t = 0.1$ s, then 0.01 s, then 0.001 s.

(d) Derive an expression for the cheetah's instantaneous velocity as a function of time, and use it to find v_x at $t = 1.0$ s and $t = 2.0$

SOLUTION



$x = 20 + 5t^2$

a) $\Delta x = x_2 - x_1$

@ $t_1 = 1$ Sec $\Rightarrow x_1 = 20 + 5(1)^2 = 25$

@ $t_2 = 2$ Sec $\Rightarrow x_2 = 20 + 5(2)^2 = 40$

$\Delta x = x_2 - x_1 = 40 - 25 = 15$ M

b) $v_{av} = \frac{\Delta x}{\Delta t} = \frac{15}{2-1} = 15$ M/S

d) $v_x = \frac{dx}{dt} = 10t$

@ $t = 1$ Sec $\quad v_1 = 10$ M/S

@ $t = 2$ Sec $\quad v_2 = 10(2) = 20$ M/S

Example 2.2 Average acceleration

An astronaut has left an orbiting spacecraft to test a new personal maneuvering unit. As she moves along a straight line, her partner on the spacecraft measures her velocity every 2.0 s, starting at time $t = 1.0$ s:

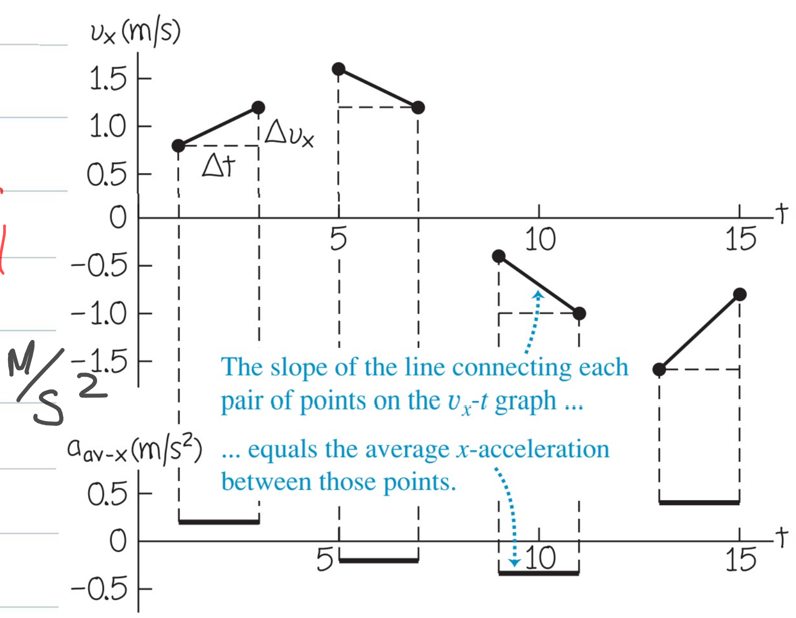
Speed = $|v_x|$

t	v_x	Speed	t	v_x	Speed
1.0 s	0.8 m/s	0.8	9.0 s	−0.4 m/s	0.4
3.0 s	1.2 m/s	1.2	11.0 s	−1.0 m/s	1 m/s
5.0 s	1.6 m/s	1.6	13.0 s	−1.6 m/s	1.6 m/s
7.0 s	1.2 m/s	1.2	15.0 s	−0.8 m/s	0.8

Find the average x-acceleration, and state whether the speed of the astronaut increases or decreases over each of these 2.0-s time intervals:

(a) $t_1 = 1.0$ s to $t_2 = 3.0$ s; (b) $t_1 = 5.0$ s to $t_2 = 7.0$ s;
(c) $t_1 = 9.0$ s to $t_2 = 11.0$ s; (d) $t_1 = 13.0$ s to $t_2 = 15.0$ s.

SOLUTION



a) $a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$

$a_{av} = \frac{1.2 - 0.8}{3 - 1} = 0.2$ M/S²

Speed increase

d) $a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$

$= \frac{-0.8 - (-1.6)}{15 - 13} = 0.4$ M/S²

Speed decrease

(d) $a_x = \frac{dv}{dt} = \frac{d}{dt}(60 + 0.5t^2)$

$= t$

@ $t = 1$ sec $\quad a_x = 1\ m/s^2$

@ $t = 3$ sec $\quad a_x = 3\ m/s^2$

2nd degree $\Rightarrow x$

1st degree Line $\Rightarrow v = \frac{dx}{dt}$

Constant $\Rightarrow a = \frac{dv}{dt}$

derivative

Example 2.3 Average and instantaneous accelerations

Suppose the x-velocity v_x of the car in Fig. 2.11 at any time t is given by the equation

$$v_x = 60\ \text{m/s} + (0.50\ \text{m/s}^3)t^2$$

(a) Find the change in x-velocity of the car in the time interval $t_1 = 1.0$ s to $t_2 = 3.0$ s.
(b) Find the average x-acceleration in this time interval.
(c) Find the instantaneous x-acceleration at time $t_1 = 1.0$ s by taking Δt to be first 0.1 s, then 0.01 s, then 0.001 s.
(d) Derive an expression for the instantaneous x-acceleration as a function of time, and use it to find a_x at $t = 1.0$ s and $t = 3.0$ s.

SOLUTION

$v_x = 60 + 0.5t^2$

(a)

@ $t_1 = 1$ sec $\quad v_1 = 60 + 0.5(1)^2 = 60.5$

@ $t_2 = 3$ sec $\quad v_2 = 60 + 0.5(3)^2 = 64.5\ m/s$

$\Delta v = v_2 - v_1 = 64.5 - 60.5$

$= 4\ m/s$

(b) $a_{av} = \frac{\Delta v}{\Delta t} = \frac{4}{3-1} = 2\ m/s$

Motion

Motion with Constant Acceleration

$a = \frac{dv}{dt}$

v change @ same rate

$v = v_0 + at$ (No - x)

$x = x_0 + v_0 t + \frac{1}{2} a t^2$ (No - v)

$v^2 = v_0^2 + 2a(x - x_0)$ (No - t)

$x - x_0 = \left(\frac{v_0 + v}{2}\right) t$ (No - a)

Free Falling Bodies

→ vertically upward or downward
dropped

$a_y = -9.8\ m/s^2$ } constant, always downward

$v = v_0 + a_y t$ (No - y)

$y = y_0 + v_0 t + \frac{1}{2} a_y t^2$ (No - v)

$v^2 = v_0^2 + 2a_y (y - y_0)$ (No - t)

$y - y_0 = \left(\frac{v_0 + v}{2}\right) t$ (No - a)

y ⊕ } above

origin

y ⊖ } below

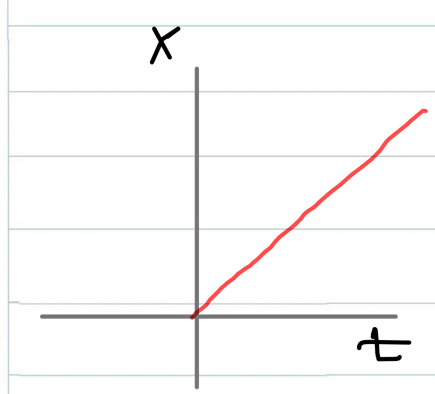
$v_y = 0$ @ Max height

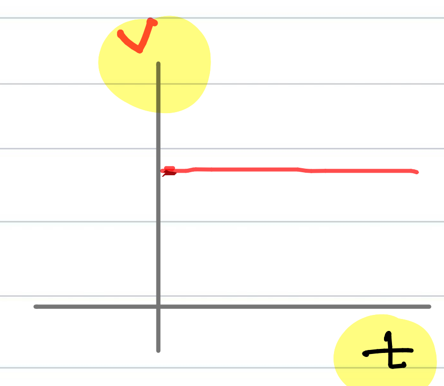
Motion

Motion with Zero acceleration :-

* $a = 0$
* Motion is uniform
* Velocity is constant
* Velocity is Max

$$\boxed{x = x_0 + vt}$$





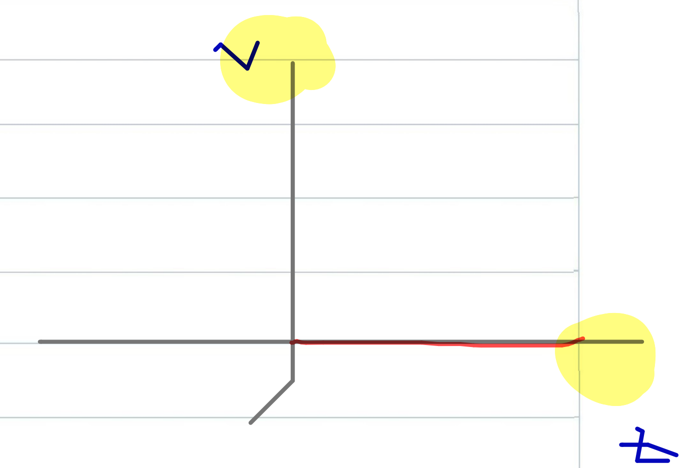
* No Motion :-

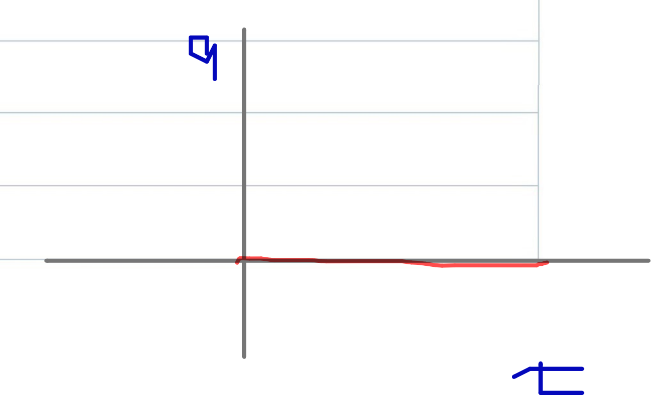
→ particle @ rest

→ x is constant

$a = 0$

$v = 0$





$V = ??$

$V_0 = 15\ M/s$

$a = 4\ M/s^2 \Rightarrow V = V_0 + a t$

$t = 2\ Sec$

$$V = 15 + (4)(2) = 23\ M/s$$

(b)

$X = ??$

$V = 25\ M/s$

$a = 4\ M/s^2$

$V_0 = 15\ M/s$

$X_0 = 5\ M$

$$V^2 = V_0^2 + 2a(X - X_0)$$

$$25^2 = 15^2 + 2(4)(X - 5)$$

$$X = 55\ M$$

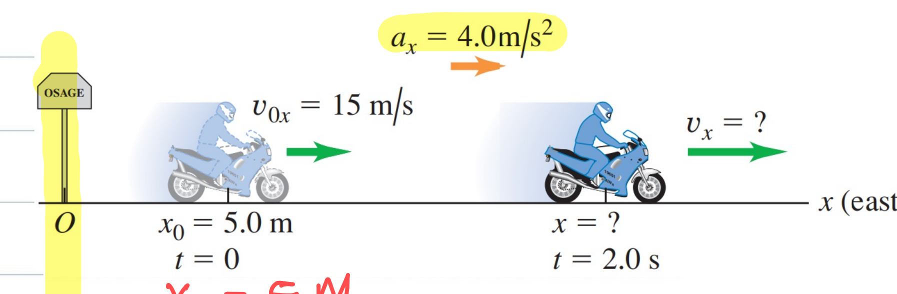
Example 2.4 Constant-acceleration calculations

A motorcyclist heading east through a small town accelerates at a constant $4.0\ \mathrm{m/s^2}$ after he leaves the city limits (Fig. 2.20). At time $t = 0$ he is 5.0 m east of the city-limits signpost, moving east at 15 m/s.

(a) Find his position and velocity at $t = 2.0$ s.

(b) Where is he when his velocity is 25 m/s?

SOLUTION



$X_0 = 5\ M$

$V_0 = 15\ M/s$

(a)

$X = ??$

$X_0 = 5\ M$

$V_0 = 15\ M/s$

$t = 2\ Sec$

$a = 4\ M/s^2$

$$X = X_0 + V_0 t + \frac{1}{2} a t^2$$

$$= 5 + (15)(2) + \frac{1}{2}(4)(2)^2$$

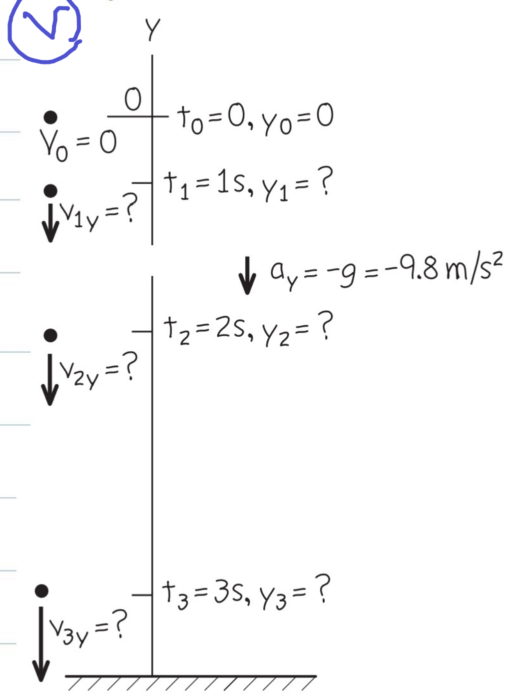
$$= 43\ M$$

Example 2.6 A freely falling coin

A one-euro coin is dropped from the Leaning Tower of Pisa and falls freely from rest. What are its position and velocity after 1.0 s, 2.0 s, and 3.0 s?

(y) (v)

SOLUTION



$y = ??$

$y_0 = 0$
$V_0 = 0$
$a_y = -9.8\ M/s^2$
$t = 1\ sec$

$$y = \cancel{y_0} + \cancel{V_0 t} + \frac{1}{2} a_y t^2$$
$$= \frac{1}{2}(-9.8)(1)^2$$
$$= -4.9\ M$$

$V_0 = 0$
$y_0 = 0$
origin

$a_y = -9.8\ M/s^2$
$t = 1\ sec$
$y = ??$
$V = ??$

$V = ??$

$y_0 = 0$
$V_0 = 0$
$a_y = -9.8\ M/s^2$
$t = 1\ sec$

$$V = \cancel{V_0} + a_y (t)$$
$$= -8.8(1) = -9.8\ M/s$$

Example 2.7 Up-and-down motion in free fall

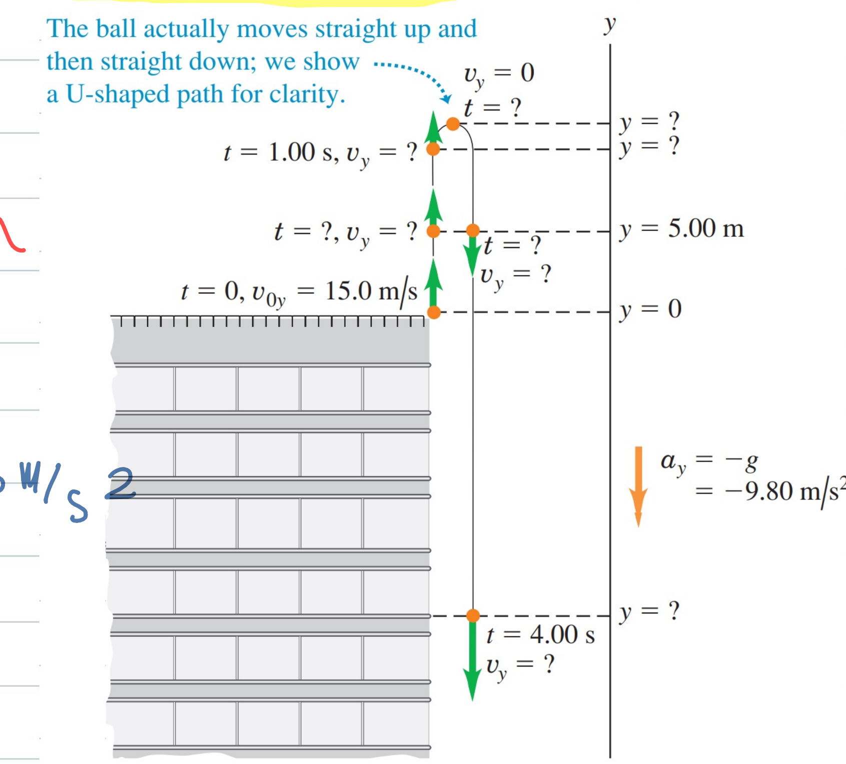
$a_y = -9.8\ M/s^2$

You throw a ball vertically upward from the roof of a tall building. The ball leaves your hand at a point even with the roof railing with an upward speed of 15.0 m/s; the ball is then in free fall. On its way back down, it just misses the railing. Find (a) the ball's position and velocity 1.00 s and 4.00 s after leaving your hand; (b) the ball's velocity when it is 5.00 m above the railing; (c) the maximum height reached; (d) the ball's acceleration when it is at its maximum height.

SOLUTION

$y = 5\ M$

$y_0 = 0$
origin
$V_0 = 15\ M/s$
$a_y = -9.8\ M/s^2$



(a)

$y = ??$

$y_0 = 0$
$V_0 = 15\ M/s$
$a_y = -9.8\ M/s^2$
$t = 4\ Sec$

$$\Rightarrow y = y_0 + V_0 t + \frac{1}{2} a_y t^2$$

$$y = 0 + 15(4) + \frac{1}{2}(-9.8)(4)^2 = -18.4\ M$$

below rail

$V = ??$ $\left\{ \begin{array}{l} y_0 = 0 \\ V_0 = 15\ M/s \\ a_y = -9.8\ M/s^2 \\ t = 4\ sec \end{array} \right.$ $\Longrightarrow V = V_0 + a_y t$

$$V = 15 - 9.8(4) = -24.2\ M/s \downarrow$$

(b)

$V = ??$ $\left\{ \begin{array}{l} y = 5\ M \\ y_0 = 0 \\ V_0 = 15\ M/s \\ a_y = -9.8\ M/s^2 \end{array} \right.$

$$V^2 = V_0^2 + 2a_y(y - y_0)$$

$$V^2 = 15^2 + 2(-9.8)(5 - 0)$$

$$= 127\ M^2/s^2$$

$$V_y = \pm 11.3\ M/s$$

$V_y = 11.3\ M/s$ as we go up

$V_y = -11.3\ M/s$ as we go down

(c)

$y_{Max} = ??$ $\left\{ \begin{array}{l} V_y = 0 \\ V_0 = 15\ M/s \\ a_y = -9.8\ M/s^2 \\ y_0 = 0 \end{array} \right.$

$$V^2 = V_0^2 + 2a_y(y - y_0)$$

$$0 = 15^2 + 2(-9.8)(y_{Max} - 0)$$

$$y_{Max} = \frac{15^2}{2(9.8)} = 11.5$$

(d) a_y is always Constant and downward

$$a_y = -9.8\ M/s^2$$

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Example 2.8 **Two solutions or one?**

At what time after being released has the ball in Example 2.7 fallen 5.00 m below the roof railing?

SOLUTION

$\boxed{t = ??}$

$y = -5\ M$

$\boxed{y_o = 0}$

$\boxed{V_o = 15\ M/s}$

$\boxed{a_y = -9.8\ M/s^2}$

No − V

$$y = y_o + V_o t + \frac{1}{2} a_y t^2$$

$$-5 = 0 + 15t + \underbrace{\frac{1}{2}(-9.8)}_{4.9} t^2$$

$$4.9 t^2 - 15t - 5 = 0$$

$t = 3.36$ Sec ✓

$t = -0.3$ ~~Sec~~ Reject

$$V^2 = V_o^2 + 2 a_y$$

Review

→ Position (x)

→ $\Delta x = x_2 - x_1$

→ $v_{av} = \dfrac{\Delta x}{\Delta t}$

→ $v_x = \dfrac{dx}{dt}$

+ →

← −

→ Speed $= |v_x|$

→ $a_{av} = \dfrac{\Delta v}{\Delta t} = \dfrac{v_2 - v_1}{t_2 - t_1}$

→ $a_x = \dfrac{dv}{dt}$

+ →

← −

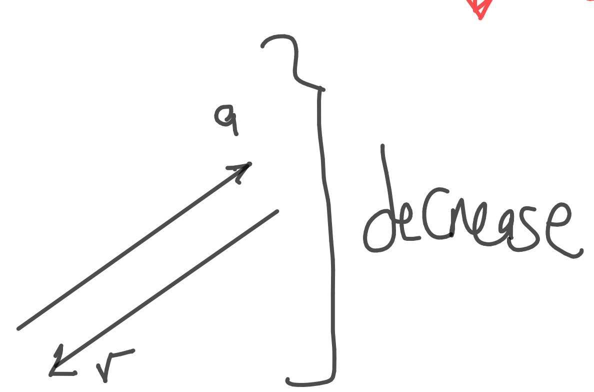
derivation

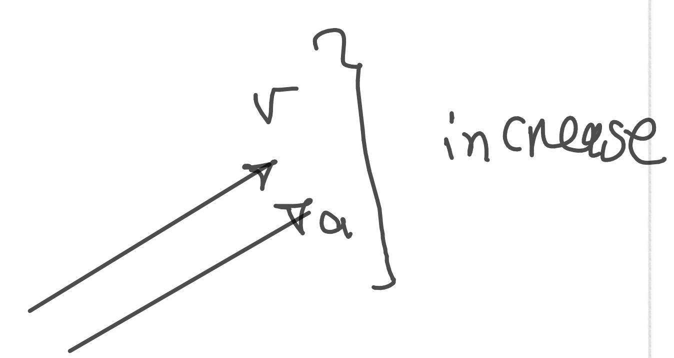
x

v

a

$\int$





Motion

Motion with Constant acceleration :-

$a = \dfrac{dv}{dt}$

$v = v_0 + a\,t$ (No $-\,x$)

$x = x_0 + v_0 t + \frac{1}{2} a t^2$ (No $-\,v$)

$v^2 = v_0^2 + 2a\,(x - x_0)$ (No $-\,t$)

$x - x_0 = \left(\dfrac{v_0 + v}{2}\right) t$ (No $-\,a$)

Free Fall

\# $a_y = -9.8\ m/s^2$

@ Max height

$v_y = 0$

Motion with Zero acceleration :-

\# $\boxed{a = 0}$

→ Velocity is Constant

→ v_{Max}

→ Uniform Motion

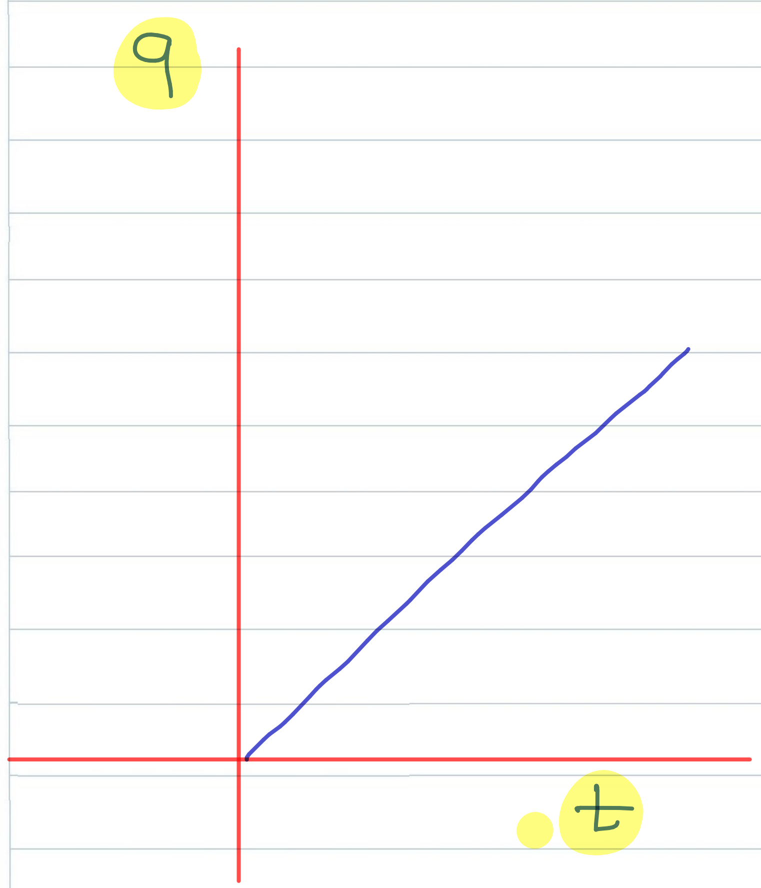
$x = x_0 + v\,t$

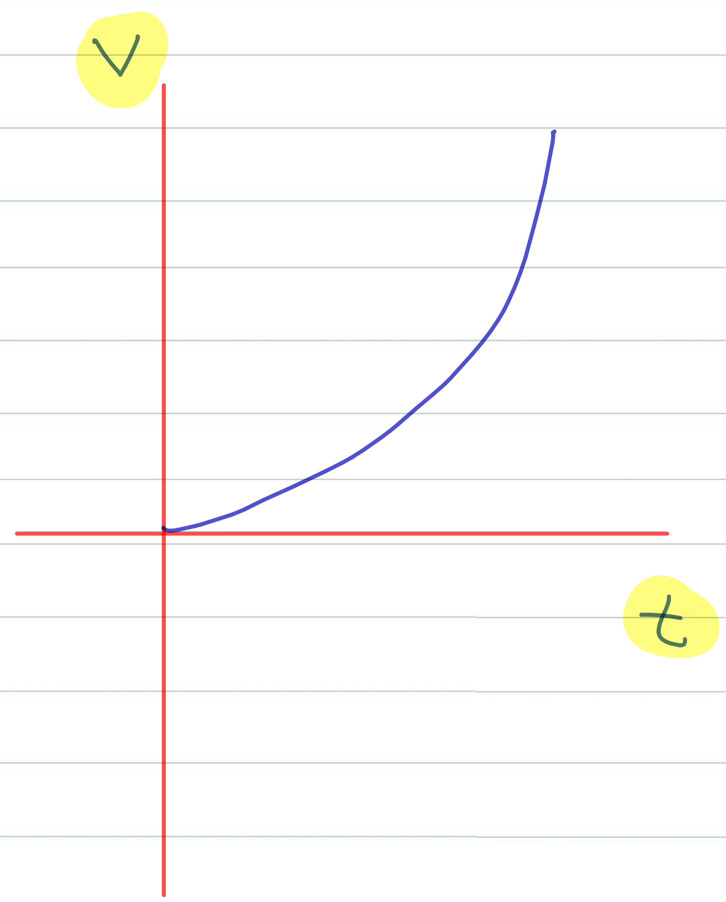
Motion with not constant acceleration:-

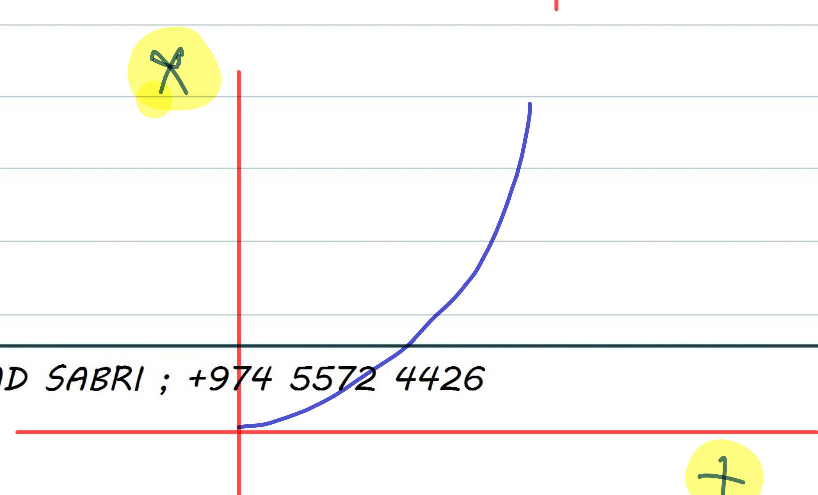
$$a = F(t)$$

$$v = v_0 + \int a \, dt$$

$$\underbrace{x = x_0}_{\Delta x} + \int v \, dt$$







derivation

x — 3rd degree

$v \Rightarrow \int$ 2nd degree

a — 1st-degree

Displacement $= \Delta x = \int v \, dt$

$= \Sigma$ Area above $- \Sigma$ Area below

@ $v - t$ graph

Distance $= \Sigma$ Area above $+ \Sigma$ Area below

$$x = 50 + 10t + t^2 - \underbrace{\frac{0.05}{3}}t^3$$

$$X = 50 + 10t + t^2 - 0.016\, t^3$$

(b) $t = ??$ @ V_{Max} $\}$ $a = 0$

$$0 = 2 - 0.1\,t \Rightarrow t = 20 \text{ sec}$$

(c) $V_{Max} = ??$

$$V_{Max} = 10 + 2(20) - 0.05(20)^2$$

$$= 30 \text{ M/S}$$

(d) $X = ??$

$$X = 50 + 10(20) + (20)^2 - 0.016(20)^3$$

$$X = 517 \text{ (M)}$$

Example 2.9 Motion with ch changing acceleration

Sally is driving along a straight hig highway in her 1965 Mustang. At $t = 0$, when she is moving at 10 m/s in the positive x-direction, she passes a signpost at $x = 50$ m. Her x-acceleration as a function of time is

$$a_x = 2.0 \text{ m/s}^2 - (0.10 \text{ m/s}^3)t$$

(a) Find her x-velocity v_x and position x as functions of time.
(b) When is her x-velocity greatest? V_{Max}
(c) What is that maximum x-velocity?
(d) Where is the car when it reaches that maximum x-velocity?

SOLUTION

@ $t = 0$ $\quad V_0 = 10$ M/S

$X_0 = 50$ M

$$a_x = 2 - 0.1\,t$$

(a) $$V_x = V_0 + \int a\, dt$$

$$= 10 + \int (2 - 0.1\,t)\, dt$$

$$V = 10 + 2t - \underbrace{\underbrace{\frac{0.1}{2}}_{}}_{0.05} t^2$$

$$X = X_0 + \int V\, dt$$

$$= 50 + \int (10 + 2t - 0.05\,t^2)\, dt$$

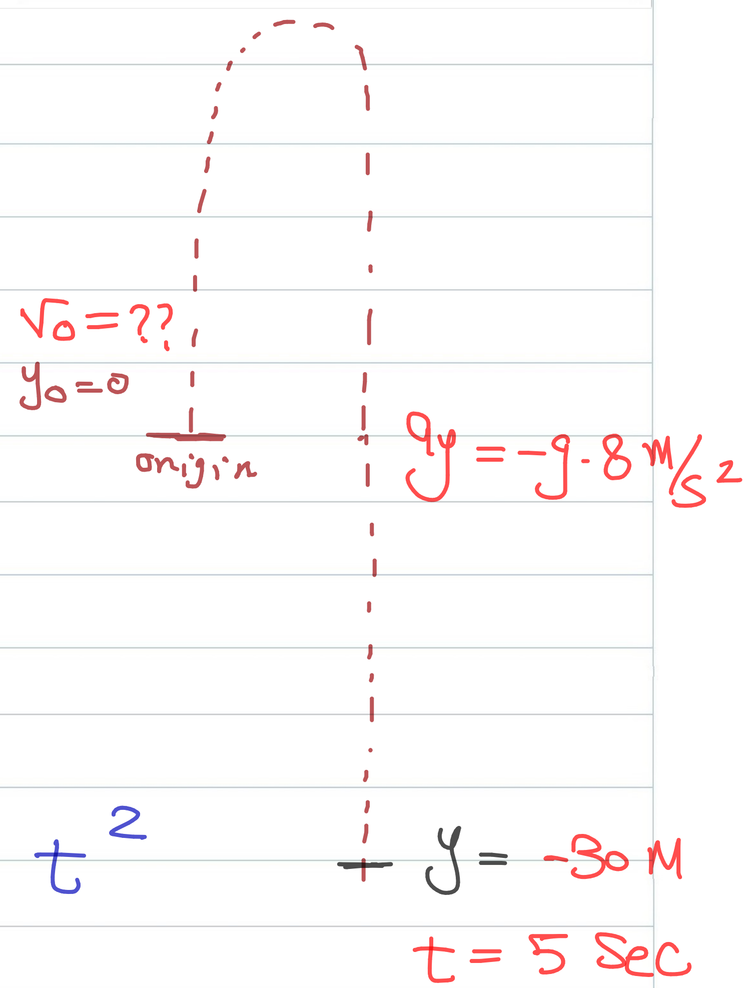
negative

Free Fall

An egg is thrown nearly vertically upward from a point near the cornice of a tall building. The egg just misses the cornice on the way down and passes a point 30.0 m below its starting point 5.00 s after it leaves the thrower's hand. Ignore air resistance. (a) What is the initial speed of the egg? (b) How high does it rise above its starting point? (c) What is the magnitude of its velocity at the highest point? (d) What are the magnitude and direction of its acceleration at the highest point? (e) Sketch a_y-t, v_y-t, and y-t graphs for the motion of the egg.

(a)

$V_0 = ??$ $\left\{ \begin{array}{l} y_0 = 0 \\ y = -30\,M \\ t = 5\ sec \\ a_y = -9.8\ M/s^2 \end{array} \right.$



$$y = y_0 + V_0 t + \frac{1}{2} a_y t^2$$

$$-30 = 0 + V_0(5) + \frac{1}{2}(-9.8)(5)^2$$

$$V_0 = 18.5\ M/s$$

(b)

$y_{Max} = ??$ $\left\{ \begin{array}{l} V_y = 0 \\ a_y = -9.8\ M/s^2 \\ y_0 = 0 \qquad V_0 = 18.5\ M/s \end{array} \right.$

$$V^2 = V_0^2 + 2 a_y (y_{Max} - y_0)$$

$$0 = 18.5^2 + 2(-9.8)(y_{Max} - 0)$$

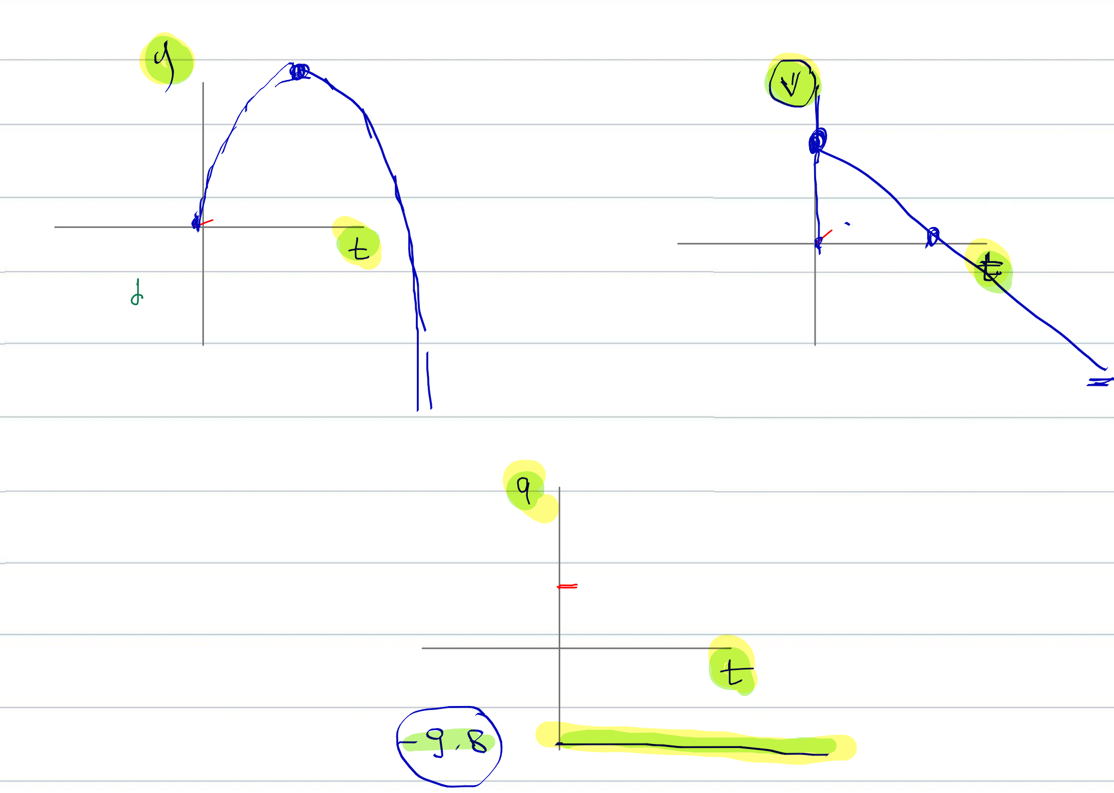
$$y_{Max} = \frac{18.5^2}{2 \times 9.8} = 17.5\ M$$

(c) @ Max height

$$V_y = 0$$

(d) $a_y = -9.8\ M/s^2$ } Constant always down ward

(e)



Question 5: (1 pt) A car accelerates from 10.0 m/s to 30.0 m/s at a rate of 3.00 m/s^2. How far does the car travel while accelerating? 2015

V_0 V a Δx

A) 80.0 m
B) 133 m
C) 226 m
D) 399 m
E) 0 m

$$v^2 = v_0^2 + 2a(x - x_0)$$
$$30^2 = 10^2 + 2(3)(\Delta x)$$
$$\Delta x = 133 \text{ m}$$

Question 6: (1 pt) A ball is thrown directly upward and experiences no air resistance. Which one of the following statements about its motion is correct? 2015

A) The acceleration of the ball is upward while it is traveling up and downward while it is traveling down.
B) The acceleration of the ball is downward while it is traveling up and upward while it is traveling down.
C) The acceleration is downward during the entire time the ball is in the air.
D) The acceleration of the ball is downward while it is traveling up and downward while it is traveling down but is zero at the highest point when the ball stops.
E) None of the above

$a = -0.1 \text{ m/s}^2$

Question 7: (1 pt) A ball rolls across a floor with an acceleration of 0.100 m/s² in a direction opposite to its velocity. The ball has a velocity of 4.00 m/s after rolling a distance 6.00 m across the floor. What was the initial speed of the ball? 2015

v x $x_0 = 0$ V_0

A) 4.15 m/s
B) 5.85 m/s
C) 4.60 m/s
D) 5.21 m/s
E) 3.85 m/s

$$v^2 = v_0^2 + 2a(x - x_0)$$
$$(4)^2 = v_0^2 + 2(-0.1)(6 - 0)$$
$$V_0 = 4.15 \text{ m/s}$$

Question 8: (1 pt) The acceleration of an object as a function of time is given by $a(t) = (3.00 \text{ m/s}^3)t$, where t is in seconds. If the object is at rest at time $t = 0.00$ s, what is the velocity of the object at time $t = 6.00$ s? 2015

A) 18.0 m/s
B) 54.0 m/s
C) 0.00 m/s
D) 15.0 m/s
E) 108 m/s

$a = 3t$
$v_0 = 0$

$$v = v_0 + \int 3t \, dt$$
$$= 0 + \frac{3t^2}{2}$$
$$= \frac{3t^2}{2}$$

@ $t = 6$ s $v = \frac{3}{2}(6)^2$
$= 54$ m/s

x — M
$\int$ v — m/s
a — m/s²

2. The velocity of an object which is initially at the origin and moving in the positive x-direction is given by

$$v(t) = 2.00\ m/s + (3.00\ m/s)t - (1.0\ m/s^2)\,t^2$$

A. Determine the acceleration of the object at $t = 5.00$ s. (2 points)
B. Determine the position of the object at $t = 5.00$ s. (2 points)

x
V
a

$$v(t) = 2 + 3t - t^2$$

a

$$a = \frac{dv}{dt} = 3 - 2t$$

@ $t = 5$ sec $a = 3 - 2(5) = -7 \text{ m/s}^2$

b

$$x = x_0 + \int v \, dt$$
$$x = 0 + \int 2 + 3t - t^2 \, dt$$
$$x = 2t + \frac{3t^2}{2} - \frac{t^3}{3}$$

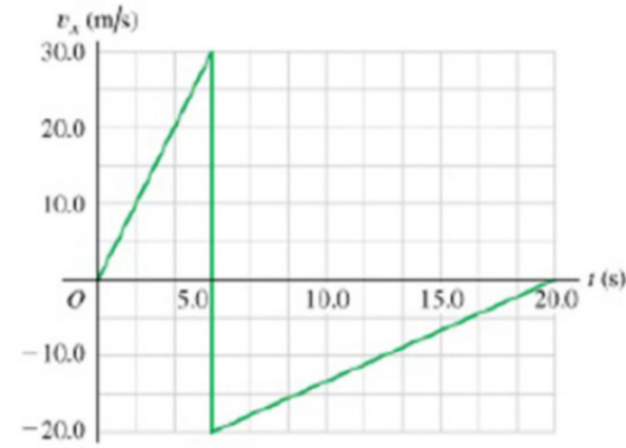
@ $t = 5$ sec

$$x_5 = 2(5) + \frac{3}{2}(5)^2 - \frac{(5)^3}{3}$$
$$= \frac{35}{6} \text{ M}$$

Problem 2: **(3 pts)** A rigid ball traveling in a straight line (the x-axis) hits a solid wall and suddenly rebounds during a brief instant. The graph in in the figure below shows this ball's velocity as a function of time. During the first 20.0 s of its motion, find:

2015

a) The total distance the ball moves
b) Its displacement.
c) Sketch a graph of for this ball's motion (x-t graph).



a) Total Distance = area under graph $v_x - t$

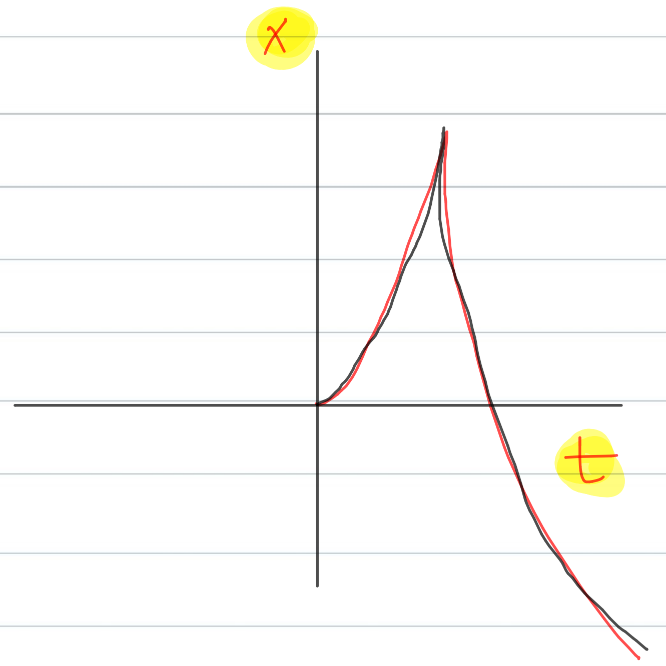
$= A_1 + A_2$

$= \frac{1}{2}(5)(30) + \frac{1}{2}(15)(20)$

$= 75 + 150 = 225$ m

b) $\Delta x = \int v_x \, dt = A_1 - A_2$

$= 75 - 150 = -75$ m

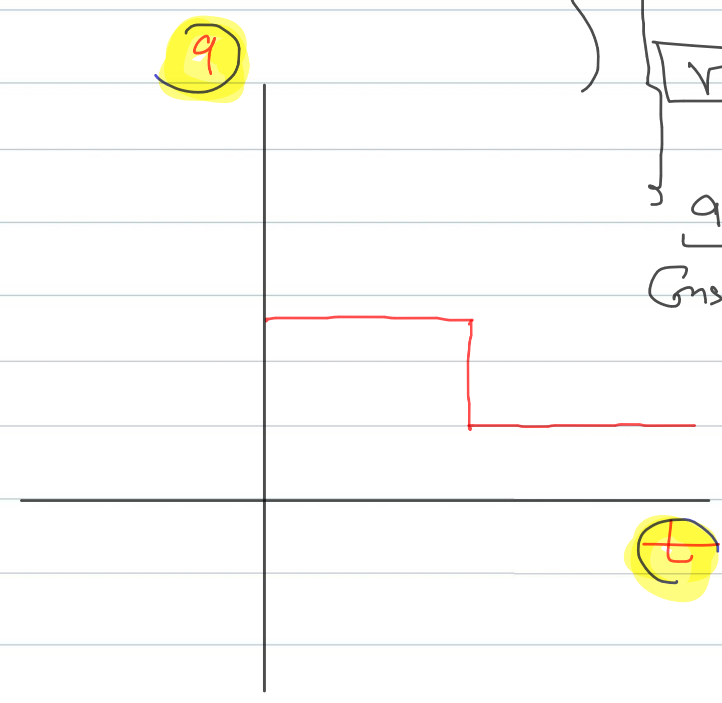


x

$\int$ $\boxed{v}$ ($\frac{d}{dt}$)

a

Constant



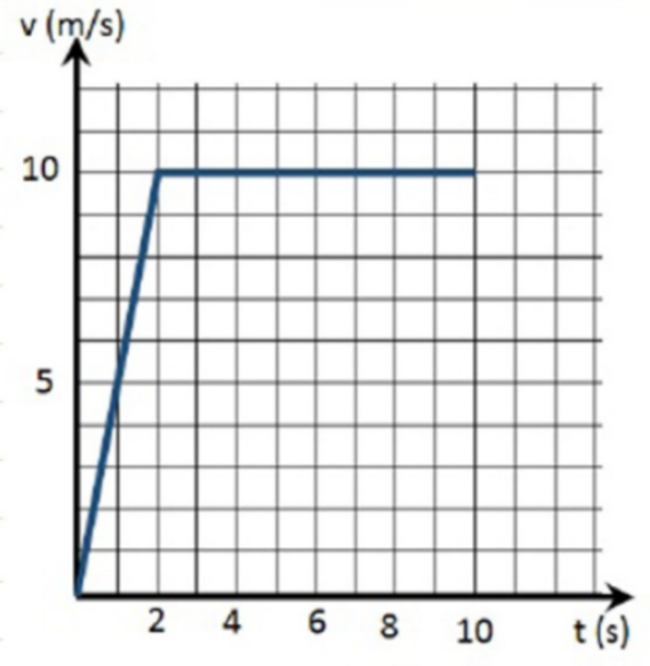
Problem 2: **(3 pts)** A typical runner can maintain his maximum acceleration for 2.0 s and his maximum speed is 10 m/s. After reaching this maximum speed, he runs at constant speed as shown in the figure.

(a) Calculate the instantaneous acceleration at t = 1.0 s of the motion.

fall 2016

(b) Determine the runner displacement during the first 5.0 s of his motion

(c) Determine the average velocity during the period from t = 0 to t = 5.0 s.



a

$a = \frac{dv}{dt}$ = slope @ point

$= \frac{10 - 0}{2 - 0}$

$= 5 \ m/s^2$

b

$\Delta x = \int_0^5 v \, dt$ = Area under curve

$= A_1 + A_2$

$= \frac{1}{2}(2)(10) + 3 * 10 = 40 \ m$

c

$a_{av} = \frac{\Delta x}{\Delta t} = \frac{40}{5 - 0} = 8 \ m/s$

Example 2.5 Two bodies with different accelerations

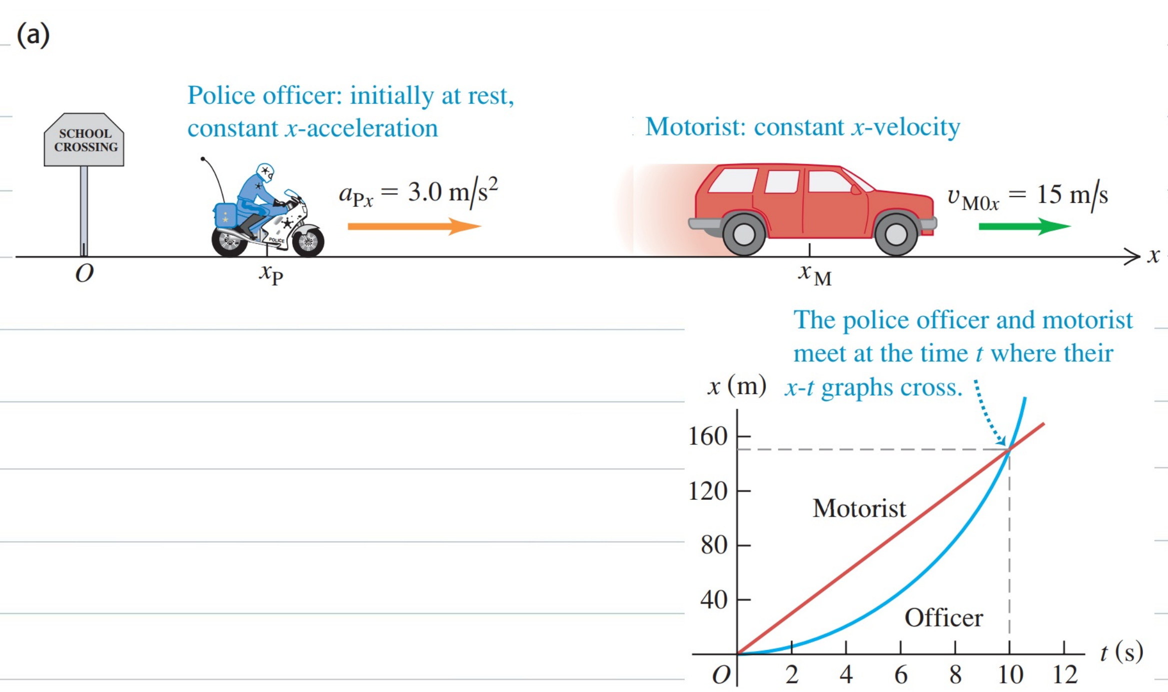
A motorist traveling with a constant speed of 15 m/s (about 34 mi/h) passes a school-crossing corner, where the speed limit is 10 m/s (about 22 mi/h). Just as the motorist passes the school-crossing sign, a police officer on a motorcycle stopped there starts in pursuit with a constant acceleration of 3.0 m/s^2 (Fig. 2.21a).

(a) How much time elapses before the officer passes the motorist?

(b) What is the officer's speed at that time?

(c) At that time, what distance has each vehicle traveled?

SOLUTION



H.W